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Multi-strategy Enhanced Black Kite Algorithm for Constrained Engineering Optimization Problems

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Abstract: *To overcome the premature convergence issue and enhance both convergence speed and accuracy of the Black Kite Algorithm (BKA), this paper proposes a multi-strategy enhanced Black Kite Algorithm (MBKA). Firstly, the Lens Imaging Opposition-Based Learning (LOBL) strategy is used to mutate the leader to prevent premature convergence of the algorithm; Secondly, in the attack phase, the position update conditions are changed and incorporating information disparity in early iterations and leader guidance in later iterations to improve the algorithm's convergence accuracy; Finally, in the migration phase, changing the position updating method and corresponding conditions to accelerate the algorithm's convergence speed and accuracy. Based on 16 benchmark functions and 7 comparison algorithms, the effectiveness of each improved strategy of MBKA is verified, and the convergence accuracy, convergence behavior, and statistical test results are compared and analyzed. Experimental results show that MBKA ranks first in the average ranking, and the p-value of the Wilcoxon test confirms that MBKA has a significant difference from other algorithms. Moreover, when examined on three engineering design problems, MBKA achieved a superior average solution compared to the original BKA. These comprehensive results confirm that MBKA offers excellent convergence characteristics and strong robustness.*

Keywords: *Black Kite Algorithm; LOBL strategy; Information discrepancy; Bisection method; Engineering design problems*

1. Introduction

Swarm intelligence (SI) optimization algorithms [1,2,3]are a class of algorithms inspired by group behavior in nature, which simulate the cooperative behavior of biological groups in nature to solve complex optimization problems [4]. However, they generally have shortcomings such as easily falling into local optimal solutions and slow convergence speed.

Black-winged Kite Algorithm (BKA) [5]is a swarm intelligence optimization algorithm proposed in 2024, which simulates the migration and predation behaviors of black-winged kite populations, and exhibits strong adaptability, few adjustable parameters, and high convergence accuracy, but it still has the problems of slow convergence speed, susceptibility to local optima trapping, and low convergence precision when handling high-dimensional complex problems. At present, there are fewer improvements about BKA, among the existing works, Chen et al. [6]employed Sine mapping for population initialization, implemented differential mutation with three distinct individuals, and introduced adaptive inertia weight factors to enhance optimization capability and convergence speed. Li et al. [7]introduced Levy flight dynamics and adaptive t-distribution (with tunable degrees of freedom) to modify the black-winged kite's position updating, achieving enhanced global search in initial iterations while maintaining superior local optima avoidance in later optimization stages. Zhang et al. [8]combined logical chaotic mapping with the Osprey Optimization Algorithm to enhance BKA's search performance. Du et al. [9]developed a complex-valued encoded BKA (CBKA) demonstrating superior stability, precision, and convergence speed in global optimization. Fu et al.[10]integrated optimal point set strategies, Gompertz growth models, and Levy flight to modify BKA. Zhao et al.[11]proposed an improved Black-winged Kite algorithm based on chaos mapping and adversarial learning, which firstly uses the Tent chaos map to achieve a uniform initial solution distribution, introduces Beta random distribution and optimizes the nonlinear factor in the attack stage to make the search trend more in line with the demand, and introduces an adversarial learning mechanism to expand the search direction, the results show that the optimization

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accuracy of the algorithm is significantly better than that of the comparison algorithm. Zhao et al. [12]cited the Morlet wavelet variant strategy, which increased the population diversity, helped the algorithm jump out of the local optimum, improved the leader's judgment conditions in the migration stage, and improved the convergence speed and accuracy of the algorithm. Mu et al. [13]proposed a Multi-Strategy Improved Black Kite Algorithm (MSBKA) for feature selection in natural disaster tweet classification. By utilizing enhanced loop mapping, integrated hierarchical inverse learning, and combined with the Nelder-Mead method to improve BKA, the results show that the MSBKA-SVM model improves the accuracy of classification. Li et al. [14]proposed a black-winged kite optimization algorithm that fuses the Osprey optimization algorithm and cross-line enhancement (DKCBKA), firstly, in the attack stage, the Osprey optimization algorithm is fused to enhance the global search ability and convergence speed of the algorithm, in the migration stage, the random difference mutation strategy is used to enhance the diversity of the population, and balance the local development and global exploration ability, finally, the convergence accuracy of the algorithm is improved through horizontal and vertical strategies. Xue et al.[15] integrated the Black-winged Kite Algorithm (BKA) and the Artificial Rabbit Optimization (ARO) algorithm, and introduced a good point set strategy to initialize the population to improve the optimization efficiency and accuracy of the algorithm. In general, existing improvements to the BKA have primarily focused on three directions: optimizing population initialization methods, optimizing the update method of individual position, and integrating other optimization algorithms.

To overcome the issues of slow convergence speed and premature convergence in the Black-winged Kite Algorithm (BKA), this paper proposes a multi-strategy enhanced Black Kite Algorithm (MBKA). Firstly, the Lens Imaging Opposition-Based Learning (LOBL) strategy is used to mutate the leader to prevent premature convergence f the algorithm; Secondly, in the attack phase, the position update conditions are changed and incorporating information disparity and leaders in the early and late iterations, respectively, to improve the algorithm's convergence accuracy; Finally, in the migration stage, the position update method and its selection conditions are changed to improve convergence speed and precision. The main contributions of this study can be summarized as:

- Proposing a Multi-strategy enhanced Black-winged Kite Algorithm (MBKA).
- Validation of Improved strategies: Experiments verify the efficacy of each strategy.
- Comprehensive Benchmark: Based on 8 unimodal and 8 multimodal benchmark functions, MBKA is experimentally compared with 7 other algorithms, and verified in 3 engineering design problems
- Statistical Analysis: Comprehensive Friedman and Wilcoxon rank-sum tests across all 16 benchmark functions conclusively demonstrate the algorithm's efficiency and scientific reliability
- Discussion of limitations and future work: The study acknowledges current limitations of MBKA and proposes potential directions for future research.

This paper is structured as follows: Section 2 presents the standard Black-winged Kite Algorithm (BKA). Section 3 describes the principles and algorithm flow of the proposed MBKA. Section 4: comparative experiments and analysis of results. Finally, Section 5 summarizes, highlights the limitations of MBKA, and suggests areas for future research efforts.

2. Black-winged kite algorithm

As a new type of swarm intelligence optimization algorithm, the black-winged kite algorithm is inspired by the predation and migration habits of the black-winged kite, and the black-winged kite algorithm relies on the attack and migration behavior of the black-winged kite to simulate the local exploration and global search of the swarm intelligence optimization algorithm [5]. The population evolves through aggressive and migratory behaviors until reaching termination criteria, ultimately converging toward the global optimum. The Black-winged Kite algorithm is mainly implemented through the following three steps.

2.1. Initialization

Firstly, a matrix was used to initialize the position of the entire population of Black-winged Kites, and the position of each Black-winged Kite was evenly distributed by equation (1).

$$X_i = BK_{lb} + rand(BK_{ub} - BK_{lb}) \tag{1}$$

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Where i is a black-winged kite in the population; BK_{lb} and BK_{ub} represent the lower and upper limits of the position of the black-winged kite, respectively; rand is a random number between [0,1]. At initialization, BKA identifies X_L (the individual with optimal fitness value) as the population leader. X_L is calculated as follows:

$$f_{best} = \min(f(X_i)) \tag{2}$$

$$X_L = X\left(find(f_{best} = f(X_i))\right) \tag{3}$$

2.2. Attacking behavior

During the flight, the black-winged kite relies on the wind speed and direction to adjust the angle of its wings and tail, quietly hovers to observe the prey, and then quickly dives to attack, so the attack behavior of the black-winged kite algorithm includes two different behavior modes (hovering and diving). The attack behavior of black-winged kites is formulated as:

$$y_{t+1}^{i,j} = \begin{cases} y_t^{i,j} + n^*(1+\sin(r))^* y_t^{i,j}, p < r \\ y_t^{i,j} + n^*(2r-1)^* y_t^{i,j}, else \end{cases} \tag{4}$$

$$n = 0.05 * e^{-2*(\frac{t}{T})^2} \tag{5}$$

Where, $y_t^{i,j}$ and $y_{t+1}^{i,j}$ denote the position of the i^{th} black-winged kite in the j^{th} dimension at the t^{th} and $(t+1)^{th}$ iteration, and r is a random number between [0,1]; p is the constant 0.9, T is the maximum number of iterations, and t is the current number of iterations.

2.3. Migration behavior

The migration of the black-winged kite is guided by leaders X_L. The black-winged kite algorithm can dynamically select leaders. If the current fitness value of the black-winged kite is less than the fitness value of the random black-winged kite, the leader will give up the leadership and join the migratory population. Conversely, the current Black-winged Kite's fitness value is greater than that of the random Black-winged Kite, and the leader will continue to lead the current Black-winged Kite. The migration behavior of black-winged kites is formulated as:

$$y_{t+1}^{i,j} = \begin{cases} y_t^{i,j} + C(0,1) * (y_t^{i,j} - L_t^j), F_i < F_{ri} \\ y_t^{i,j} + C(0,1) * (L_t^j - m * y_t^{i,j}), else \end{cases} \tag{6}$$

$$m = 2 * \sin(r + \pi / 2) \tag{7}$$

Where, L_t^j denotes the position of the leader X_L in the j^{th} dimension of the t^{th} iteration; F_i indicates the fitness value of the i^{th} Black-winged Kite in the t^{th} iteration; F_{ri} indicates the fitness value of a random black-winged kite in the t^{th} iteration; C(0,1) represents the Cauchy mutation, whose probability density function is denoted by Eq. (8), and when u =0, δ =1, its probability density function becomes the standard form, denoted by Eq. (9).

$$f(x,\delta,u) = \frac{1}{\pi} \frac{\delta}{\delta^2 + (x-u)^2} \quad , -\infty < x < \infty \tag{8}$$

$$f(x,\delta,u) = \frac{1}{\pi} \frac{\delta}{1+x^2} \quad , -\infty < x < \infty \tag{9}$$

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3. MBKA

3.1. Apply the LOBL strategy to the Leader

In the original BKA, the migratory behavior of the black-winged kite was led by the leader, but the leader would also fall into the local optimum due to the limitations of his own exploration, and lead other black-winged kites to gather around him, so that the population fell into the local optimum. The LOBL strategy is an inverse learning strategy that combines the principle of optical lens imaging, which improves the global exploration ability of the algorithm by dynamically generating the reverse solution[16]. MBKA employs the LOBL strategy to mutate the leader and generate a new reverse solution, which helps the leader escape local optima, thus enabling the population to escape premature convergence. The new solution generated by the leader based on the LOBL strategy mutation is represented by Eq. (11).

$$k = 1 + \sqrt{(t/T)} \tag{10}$$

$$L_t^{new} = (ub + lb)/2 + (ub + lb)/2k - L_t/k \tag{11}$$

where k is the dynamic scaling factor; t is the current number of iterations; T is the total number of iterations; lb and ub are the lower upper limits of the black-winged kite's locomotion space; L_t Indicates the position of the leader in the t^{th} iteration; L_t^{new} is a new leadership position formed based on the LOBL strategy. To ensure that the leader's mutation toward more optimal solutions, the position update is based on a greedy strategy, and the corresponding mathematical model is:

$$L_t = \begin{cases} L_t^{new} & , F_{L_t^{new}} < F_{L_t} \\ L_t & , \quad else \end{cases} \tag{12}$$

Where F_{L_t} indicates the fitness value of the leader at the t^{th} iteration; $\mathrm{F}_{L_t^{new}}$ denotes the fitness value of the mutated leader at the t^{th} iteration.

3.2. Attacking behavior based on information discrepancy and the leader

In this strategy, inspired by the bisection method, the position update criteria of the original BKA attack behavior is changed to two stages: the early and late stages of iteration, and the attack behavior of the original BKA only depends on the position of the previous generation of black-winged kites, considering that there is information exchange between black-winged kites, MBKA incorporates other individual information in the attack stage. In the early stage of iteration, the position difference vector between any two randomly selected individuals in the population is introduced to simulate the information discrepancy among individuals, and the nonlinear decreasing parameter w is incorporated to adjust the amplitude of the information difference, so as to simulate the process of the information discrepancy between individuals changing from large to small with iteration. In the later stage of iteration, the leader is incorporated to realize the local exploitation of the black-winged kite around the leader, and accelerate the convergence of the algorithm. The mathematical model corresponding to the improved attacking behavior is shown in Eq. (15).

$$w = 0.1 * (1 - t/T) \tag{13}$$

$$step = w * (y_t^{m,j} - y_t^{n,j}) \tag{14}$$

$$y_{t+1}^{i,j} = \begin{cases} y_t^{i,j} + n * (1 + \sin(r)) * y_t^{i,j} + step, t < T/2 \\ (y_t^{i,j} + L_t)/2 + n * (2r - 1) * y_t^{i,j}, else \end{cases} \tag{15}$$

Where $y_t^{m,j}$ and $y_t^{n,j}$ are the corresponding positions of the two black-winged kites randomly selected in the population; n reference to Equation (5); r is a random number between [0,1].

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3.3. Migration behavior based on median position and median fitness

In the migration phase, the original BKA determines migration behavior by comparing its fitness value with that of a random individual in the entire population, which is inevitably inefficient and fluctuating due to randomness. To improve the convergence speed and precision of BKA, inspired by the bisection method, the selection condition is changed as its fitness value compared with the median fitness value of the population. When its fitness value exceeds the population's median fitness value, indicating that its previous generation position is disadvantageous and should be discarded, the position update relies on the position corresponding to the median fitness value. Conversely, if its fitness is below the median, position updates rely on the previous generation's position. The improved migration behavior is formulated as:

$$y_{t+1}^{i,j} = \begin{cases} y_t^{i,j} + C(0,1) * (y_t^{i,j} - L_t^j), F_i < F_{mid} \\ y_t^{mid,j} + C(0,1) * (L_t^j - m * y_t^{mid,j}), else \end{cases} \tag{16}$$

$$m = 2 * \sin(r + \pi / 2) \tag{17}$$

Where $y_t^{mid,j}$ represents the j^{th} dimension position corresponding to the median of the fitness value in the population; F_{mid} represents the median value of population fitness; C(0,1) represents the Cauchy mutation, whose probability density function refers to equation (9).

3.4. MBKA algorithm flow

The algorithmic implementation of MBKA is detailed in the pseudocode below, with Figure 1 providing its corresponding flowchart.

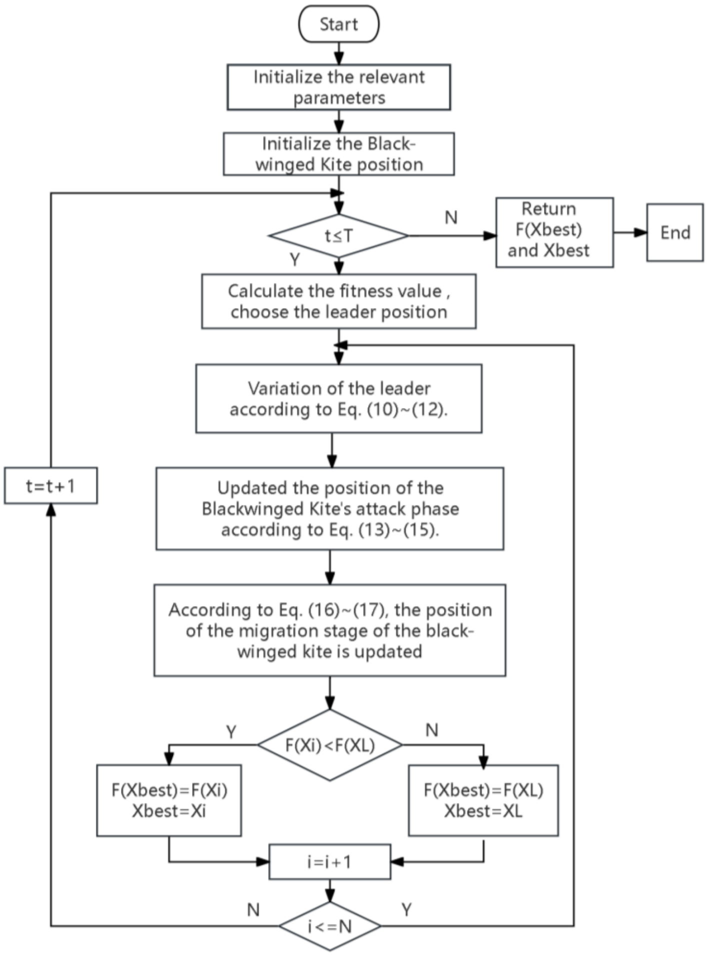

Figure 1: Flow chart of the MBKA.

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Algorithm: **MBKA**

Input: The population size pop N, the maximum number of iterations T, and the variable dimension dim.
Output: The best quasi-optimal solution obtained by MBKA for a given optimization problem.
1. **Initialization phase**
2. Initialization of the position of Black-winged kites and evaluation of the objective function.
3. Calculate the fitness value of each Black-winged kite
4. while (t< T) do
5. Select the Black-winged kite with the best position L_t
/*Apply the LOBL strategy to Leader*/
6. $L_t = (ub+lb)/2 + (ub+lb)/2k - L_t/k$
/* Attacking behavior */
7. if t <T/2
8. $y_{t+1}^{i,j} = y_t^{i,j} + n*(1+\sin(r))*y_t^{i,j} + step$
9. else if do
10. $y_{t+1}^{i,j} = (y_t^{i,j} + L_t)/2 + n*(2r-1)*y_t^{i,j}$
11. end if
/* Migration behavior */
12. if $F_i < F_{mid}$
13. $y_{t+1}^{i,j} = y_t^{i,j} + C(0,1)*(y_t^{i,j} - L_t^j)$
14. else if do
15. $y_{t+1}^{i,j} = y_t^{mid,j} + C(0,1)*(L_t^{new,j} - m*y_t^{mid,j})$
16. end if
/*Select the best individual*/
17. if $F(y_{t+1}^i) < F(L_t)$
18. $\mathrm{X}_{\mathrm{best}} = y_{t+1}^i$, $F(\mathrm{X}_{\mathrm{best}}) = F(y_{t+1}^i)$
19. else if do
20. $\mathrm{X}_{\mathrm{best}} = L_t$, $F(\mathrm{X}_{\mathrm{best}}) = L_t$
21. end if
22. end while
23. **Return** $\mathrm{X}_{\mathrm{best}}$ and $F(\mathrm{X}_{\mathrm{best}})$

4. Experiments and Analysis

4.1. Test functions

To evaluate the optimization ability and convergence ability of the MBKA, 8 unimodal test functions and 8 multimodal test functions [5] shown in Table 1 and Table 2 were selected to test, among them, F1-F8 is a single-peak function with only a single solution to evaluate the local search ability of the algorithm, and F9-F16 is a multimodal function with multiple local optimal solutions, which is used to evaluate the algorithm's local optima avoidance and global exploration capacity.

Table 1: Unimodal test functions.

Name	Function	range	min
Sphere	$F_1(x) = \sum_{i=1}^{D} x_i^2$	[-100,100]	0

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Table 1(continued).

Name	Function	range	min
Schwfel2.22	$F_2(x)=\sum_{i=1}^{\dim}\left[y_i^2-10\cos(2\pi y_i)+10\right]$ $y_i=\{x_i,\lvert x_i<0.5\rvert\}$	[-10,10]	0
Schwefel1.2	$F_3(x)=\sum_{i=1}^{D}\left(\sum_{j=1}^{i}x_j\right)^2$	[-100,100]	0
Schwefel2.21	$F_4(x)=\max\{\lvert x_i\rvert,1\le x_i\le D\}$	[-10,10]	0
Quartic	$F_5(x)=\sum_{i=1}^{Dim}Dim\times x_i^2+rand(0,1)$	[-1.28,1.28]	0
Sum power	$F_6=\sum_{i=1}^{Dim}\lvert x_i\rvert^{(i+1)}$	[-1,1]	0
Sum squares	$F_7(x)=\sum_{i=1}^{Dim}Dim\times x_i^2$	[-10,10]	0
Zakharov	$F_8(x)=\sum_{i=1}^{dim}x_i^2+\left(\sum_{i=1}^{\dim}0.5ix_i\right)^2+\left(\sum_{i=1}^{\dim}0.5ix_i\right)^4$	[-5,10]	0

Table 2: Multimodal Test Functions.

Name	Function	range	min
Rastrigin	$F_9(x)=\sum_{i=1}^{\dim}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	[-5.12,5.12]	0
NCRastrigin	$F_{10}(x)=\sum_{i=1}^{\dim}\left[y_i^2-10\cos(2\pi y_i)+10\right]$ $y_i=\begin{cases}x_i,\lvert x_i\rvert<0.5\\ round(2x_i)/2,\lvert x_i\rvert>0.5\end{cases}$	[-5.12,5.12]	0
Ackley	$F_{11}(x)=-20\exp\left(-0.2\sqrt{\frac{1}{\dim}\sum_{i=1}^{\dim}x_i^2}\right)+\exp\left(\frac{1}{\dim}\sum_{i=1}^{\dim}\cos(2\pi x_i)\right)+20\_\exp(1)$	[-50,50]	0
Griewank	$F_{12}(x)=\frac{1}{4000}\sum_{i=1}^{\dim}x_i^2-\prod_{i=1}^{\dim}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	[-600,600]	0
Alpine	$F_{13}(x)=\sum_{i=1}^{\dim}\lvert x_i\times\sin(x_i)+0.1x_i\rvert$	[-10,10]	0
Solomon	$F_{14}(x)=1-\cos\left(2\pi\sqrt{\sum_{i=1}^{Dimx_i^2}}\right)+0.1\sqrt{\sum_{i=1}^{Dim}x_i^2}$	[-100,100]	0
Bohachevsky	$F_{15}(x)=\sum_{i=1}^{D-1}\begin{bmatrix}x_i^2+2x_{i+1}^2-0.3\cos(3\pi x_i)\\ -0.4\cos(4\pi x_{i+1})+0.7\end{bmatrix}$	[-10,10]	0
Generalized schaffer	$F_{16}=0.5+\pi\left(\left(\mathrm{Sin}\left(\sum_{i=1}^{D}x_i^2\right)\right)^2-0.5\right)\times\left(1+0.001\left(\sum_{i=1}^{D}x_i^2\right)\right)^{-2}$	[-100,100]	0

4.2. Experimental environment and parameter setting

The operating system selected for this experiment: Windows 10 64-bit; Processor: Intel (R) Core (TM)

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i7-75002.70 GHz, memory is 4 GB, simulation software: MatlabR2020b.

To verify the effectiveness of MBKA, the effectiveness of three improvement strategies is verified first, then seven algorithms are selected for comparison, and finally verified on three engineering design problems. The comparison algorithms include Harris Eagle Optimization (HHO) [17], Gray Wolf optimization (GWO) [18], Whale Optimization (WOA) [19], Kepler optimization algorithm (KOA) [20], Sine Cosine Algorithm (SCA) [21], Particle Swarm optimization (PSO) [22], and Black-winged Kite algorithm (BKA) [5]. Table 3 shows the parameters of each algorithm, where t is the current number of iterations and T is the maximum number of iterations. To ensure the fairness of the experiment, in all subsequent experiments, all algorithms were configured identically: maximum iterations T=500, population size N=30, with 30 independent runs. Three metrics (mean, standard deviation, and best value) were recorded, where bold denotes optimal results.

Table 3: Algorithm parameter settings.

Name	Parameter settings
MBKA	/
HHO	β=1.5 scale=0.01
GWO	a=2-2*t/T C=2*rand
WOA	a=2-t*(2/T) b=1
KOA	Tc=3 M0=0.1 lambda=15
SCA	a=2
PSO	w=0.9 c1=1.5 c2=1.5;
BKA	P=0.9

4.3. Improved strategy effectiveness analysis

Based on the 30D F1-F16, a comparison was made between the standard BKA, MBKA-1(integrated with the LOBL strategy), MBKA-2 (with improved attack behavior), MBKA-3(with improved migration behavior), and the comprehensive MBKA. The experimental results are shown in Table 4.

Compared with BKA, MBKA-1 obtains better convergence accuracy and greater robustness in all other functions, except for the worse mean and standard deviation in F5; Compared with BKA, MBKA-2 achieves superior convergence accuracy and greater robustness in all test functions except for F5; MBKA-3 is the most effective improvement strategy, especially in F3 and F6-F8, where MBKA-3 can converge to the theoretical optimal value of 0; MBKA with three strategies shows the best performance, especially in F1, F3, F6-F8, and F14, where MBKA can converge to the theoretical optimal value of 0. Collectively, the experimental results demonstrate that all three proposed strategies - whether applied individually or comprehensively - significantly improve convergence accuracy and robustness compared to the BKA, with only F5 exceptions.

Table 4: Experimental results of 30-dimensional test function.

Function	Index	BKA	MBKA-1	MBKA-2	MBKA-3	MBKA
F1	Avg.	1.59E-86	1.89E-159	9.48E-168	3.72E-307	**0.00E+00**
	Std.	7.19E-86	9.11E-159	0.00E+00	0.00E+00	**0.00E+00**
	Best	5.44E-106	9.41E-187	3.60E-202	0.00E+00	**0.00E+00**
F2	Avg.	1.38E-39	4.99E-82	9.84E-59	1.41E-164	**1.60E-288**
	Std.	7.31E-39	1.67E-81	5.39E-58	0.00E+00	**0.00E+00**
	Best	1.77E-53	5.00E-93	4.50E-103	2.23E-249	**0.00E+00**
F3	Avg.	6.72E-74	1.43E-160	2.86E-104	0.00E+00	**0.00E+00**
	Std.	3.68E-73	6.71E-160	1.57E-103	0.00E+00	**0.00E+00**
	Best	2.39E-106	1.85E-192	2.90E-199	0.00E+00	**0.00E+00**
F4	Avg.	4.81E-42	2.16E-78	9.29E-75	2.42E-173	**4.51E-290**
	Std.	2.29E-41	1.18E-77	4.90E-74	0.00E+00	**0.00E+00**
	Best	2.17E-53	3.05E-93	4.83E-100	7.21E-216	**0.00E+00**
F5	Avg.	2.34E-04	2.21E-04	3.34E-04	1.45E-04	**8.70E-05**

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	Std.	1.23E-04	1.76E-04	4.62E-04	1.80E-04	**1.05E-04**
	Best	3.36E-05	2.81E-05	2.67E-05	2.57E-06	**9.07E-06**

Table 4(continued).

Function	Index	BKA	MBKA-1	MBKA-2	MBKA-3	MBKA
F6	Avg.	1.29E-117	1.19E-170	3.09E-189	0.00E+00	**0.00E+00**
	Std.	6.54E-117	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**
	Best	5.89E-132	2.56E-200	5.91E-264	0.00E+00	**0.00E+00**
F7	Avg.	1.12E-76	1.34E-155	9.35E-110	0.00E+00	**0.00E+00**
	Std.	6.11E-76	7.34E-155	5.12E-109	0.00E+00	**0.00E+00**
	Best	8.41E-106	4.51E-188	4.61E-201	0.00E+00	**0.00E+00**
F8	Avg.	2.38E-73	2.19E-82	4.36E-160	0.00E+00	**0.00E+00**
	Std.	9.97E-73	1.20E-81	2.37E-159	0.00E+00	**0.00E+00**
	Best	1.03E-105	2.15E-104	5.01E-205	0.00E+00	**0.00E+00**
F9	Avg.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F10	Avg.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F11	Avg.	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Best	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
F12	Avg.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F13	Avg.	3.35E-37	9.44E-83	1.13E-58	4.10E-158	**3.56E-282**
	Std.	1.83E-36	3.64E-82	6.17E-58	2.24E-157	**0.00E+00**
	Best	1.87E-54	4.33E-102	3.05E-104	9.89E-237	**0.00E+00**
F14	Avg.	1.33E-02	1.23E-74	3.63E-46	1.99E-152	**0.00E+00**
	Std.	3.45E-02	6.54E-74	1.99E-45	1.09E-151	**0.00E+00**
	Best	8.37E-48	6.81E-93	3.23E-97	**0.00E+00**	**0.00E+00**
F15	Avg.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F16	Avg.	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Best	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01

4.4. Accuracy comparison and analysis

The comparison results of the convergence precision of MBKA and 7 other algorithms on the 30D test functions are shown in Table 5. For unimodal functions, MBKA is the only algorithm with a mean value converging to the theoretical optimal value of 0 in F1, F3, and F6-F8. In F2, F4, and F5, the mean values of MBKA are1.41E-292,2.72E-289 and 5.84E-05, respectively, which are significantly better than those of BKA (1.19E-39,2.10E-41, and 3.17E-04) and HHO (8.43E-48,4.07E-49, and 1.49E-04). For multimodal functions, in function F13, while the BKA algorithm is trapped in a local optimum (Avg. =4.81E-50), the MBKA algorithm can escape the local optimum (Avg. =1.42E-288, Best =0). In F14, MBKA is the only algorithm with three metrics all equal to 0. Moreover, the standard deviation of MBKA is the smallest among all algorithms, indicating that MBKA has strong robustness.

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Table 1: Experimental results of 30-dimensional unimodal function.

Function	Index	MBKA	HHO	GWO	WOA	KOA	SCA	PSO	BKA
F1	Avg.	**0.00E+00**	6.66E-96	1.02E-27	1.50E-72	1.47E+04	2.06E+01	4.89E+00	8.75E-79
	Std.	**0.00E+00**	2.93E-95	1.77E-27	7.63E-72	3.43E+03	2.94E+01	1.34E+00	4.79E-78
	Best	**0.00E+00**	4.83E-113	2.07E-29	2.60E-83	9.00E+03	9.72E-03	2.59E+00	2.84E-103
F2	Avg.	**1.41E-292**	8.43E-48	1.09E-16	1.62E-50	2.02E+03	1.35E-02	8.31E+00	1.19E-39
	Std.	**0.00E+00**	4.56E-47	6.45E-17	5.42E-50	9.87E+03	1.86E-02	1.24E+00	6.37E-39
	Best.	**0.00E+00**	2.73E-57	2.63E-17	9.59E-56	4.24E+01	3.86E-05	5.83E+00	2.96E-54
F3	Avg.	**0.00E+00**	3.04E-79	3.38E-07	4.59E+02	4.25E+02	1.01E+02	3.28E+01	9.58E-83
	Std.	**0.00E+00**	1.65E-78	1.34E-06	1.32E+02	1.38E+02	6.23E+01	1.16E+01	5.25E-82
	Best	**0.00E+00**	8.05E-99	9.05E-11	2.27E+02	1.65E+02	1.43E+00	1.42E+01	3.25E-106
F4	Avg.	**2.72E-289**	4.07E-49	8.98E-08	4.48E+00	5.56E+00	3.64E+00	1.13E+00	2.10E-41
	Std.	**0.00E+00**	1.49E-48	8.55E-08	2.78E+00	6.65E-01	1.25E+00	1.42E-01	1.15E-40
	Best	**0.00E+00**	1.45E-57	5.82E-09	2.46E-02	4.45E+00	1.05E+00	8.04E-01	6.76E-54
F5	Avg.	**5.84E-05**	1.49E-04	2.33E-03	5.07E-03	7.64E+00	1.83E-01	1.24E+01	3.17E-04
	Std.	**5.64E-05**	1.58E-04	1.43E-03	5.20E-03	3.09E+00	2.98E-01	7.77E+00	3.56E-04
	Best	**4.20E-06**	6.14E-06	3.77E-04	1.97E-04	1.60E+00	9.24E-03	3.17E+00	3.31E-05
F6	Avg.	**0.00E+00**	7.46E-123	8.98E-94	2.37E-104	1.35E-02	6.34E-05	1.17E-02	2.20E-111
	Std.	**0.00E+00**	3.65E-122	4.84E-93	1.10E-103	1.21E-02	1.99E-04	9.03E-03	1.21E-110
	Best	**0.00E+00**	5.40E-150	3.69E-106	5.28E-139	1.17E-03	9.22E-11	3.91E-04	2.26E-135
F7	Avg.	**0.00E+00**	3.36E-96	1.24E-28	2.63E-75	1.84E+03	1.58E+00	6.69E+01	5.43E-77
	Std.	**0.00E+00**	1.83E-95	1.87E-28	8.10E-75	4.80E+02	4.12E+00	2.08E+01	2.98E-76
	Best	**0.00E+00**	2.37E-118	2.21E-30	3.40E-91	1.12E+03	2.75E-03	2.51E+01	2.63E-103
F8	Avg.	**0.00E+00**	4.03E-91	2.35E-28	1.16E-71	1.70E+04	1.50E+01	1.27E+02	1.03E-76
	Std.	**0.00E+00**	2.20E-90	3.60E-28	6.32E-71	6.96E+03	2.23E+01	4.86E+01	5.62E-76
	Best	**0.00E+00**	5.89E-113	5.98E-30	7.94E-86	6.53E+03	2.03E-04	4.13E+01	8.35E-105

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Table 5 (continued).

Function	Index	MBKA	HHO	GWO	WOA	KOA	SCA	PSO	BKA
F9	Avg.	**0.00E+00**	**0.00E+00**	3.43E+00	3.79E-15	2.93E+02	3.02E+01	2.02E+02	**0.00E+00**
	Std.	**0.00E+00**	**0.00E+00**	4.48E+00	1.44E-14	2.15E+01	3.66E+01	2.70E+01	**0.00E+00**
	Best	**0.00E+00**	**0.00E+00**	5.68E-14	0.00E+00	2.49E+02	7.77E-01	1.52E+02	**0.00E+00**
F10	Avg.	**0.00E+00**	**0.00E+00**	8.19E+00	**0.00E+00**	2.54E+02	8.06E+01	1.70E+02	**0.00E+00**
	Std.	**0.00E+00**	**0.00E+00**	6.06E+00	**0.00E+00**	3.50E+01	3.59E+01	2.47E+01	**0.00E+00**
	Best.	**0.00E+00**	**0.00E+00**	5.33E-14	**0.00E+00**	1.75E+02	2.20E+01	9.43E+01	**0.00E+00**
F11	Avg.	8.88E-16	8.88E-16	4.16E+00	4.80E-15	2.00E+01	2.03E+01	3.05E+00	8.88E-16
	Std.	0.00E+00	0.00E+00	8.47E+00	2.53E-15	2.70E-05	7.12E-02	2.96E-01	0.00E+00
	Best	8.88E-16	8.88E-16	7.55E-14	8.88E-16	2.00E+01	2.02E+01	2.43E+00	8.88E-16
F12	Avg.	**0.00E+00**	**0.00E+00**	4.61E-03	4.33E-03	1.28E+02	9.80E-01	3.47E-01	**0.00E+00**
	Std.	**0.00E+00**	**0.00E+00**	8.88E-03	2.37E-02	2.72E+01	4.36E-01	7.09E-02	**0.00E+00**
	Best	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	7.78E+01	2.61E-02	2.51E-01	**0.00E+00**
F13	Avg.	**1.42E-288**	6.93E-07	3.73E-04	1.74E-47	3.36E+01	1.16E+00	7.29E+00	4.81E-50
	Std.	**0.00E+00**	3.80E-06	5.27E-04	9.54E-47	3.91E+00	2.05E+00	1.70E+00	1.18E-49
	Best	**0.00E+00**	4.63E-60	1.13E-16	1.26E-59	2.36E+01	7.98E-03	3.90E+00	7.36E-54
F14	Avg.	**0.00E+00**	2.55E-51	1.73E-01	1.17E-01	9.66E-01	2.57E-01	4.11E-01	9.99E-03
	Std.	**0.00E+00**	9.33E-51	5.21E-02	7.91E-02	1.13E-01	6.78E-02	4.64E-02	3.05E-02
	Best	**0.00E+00**	7.66E-60	9.99E-02	4.76E-44	6.93E-01	9.99E-02	3.00E-01	2.95E-49
F15	Avg.	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	3.63E+01	1.34E-02	2.99E+01	**0.00E+00**
	Std.	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	4.21E+00	2.99E-02	4.71E+00	**0.00E+00**
	Best	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.86E+01	2.46E-04	1.83E+01	**0.00E+00**
F16	Avg.	3.98E-01	3.98E-01	3.98E-01	3.98E-01	5.34E-01	3.99E-01	3.98E-01	3.98E-01
	Std.	**0.00E+00**	9.35E-06	1.29E-04	1.49E-05	1.28E-01	2.23E-03	1.89E-07	2.49E-08
	Best	3.98E-01	3.98E-01	3.98E-01	3.98E-01	4.02E-01	3.98E-01	3.98E-01	3.98E-01

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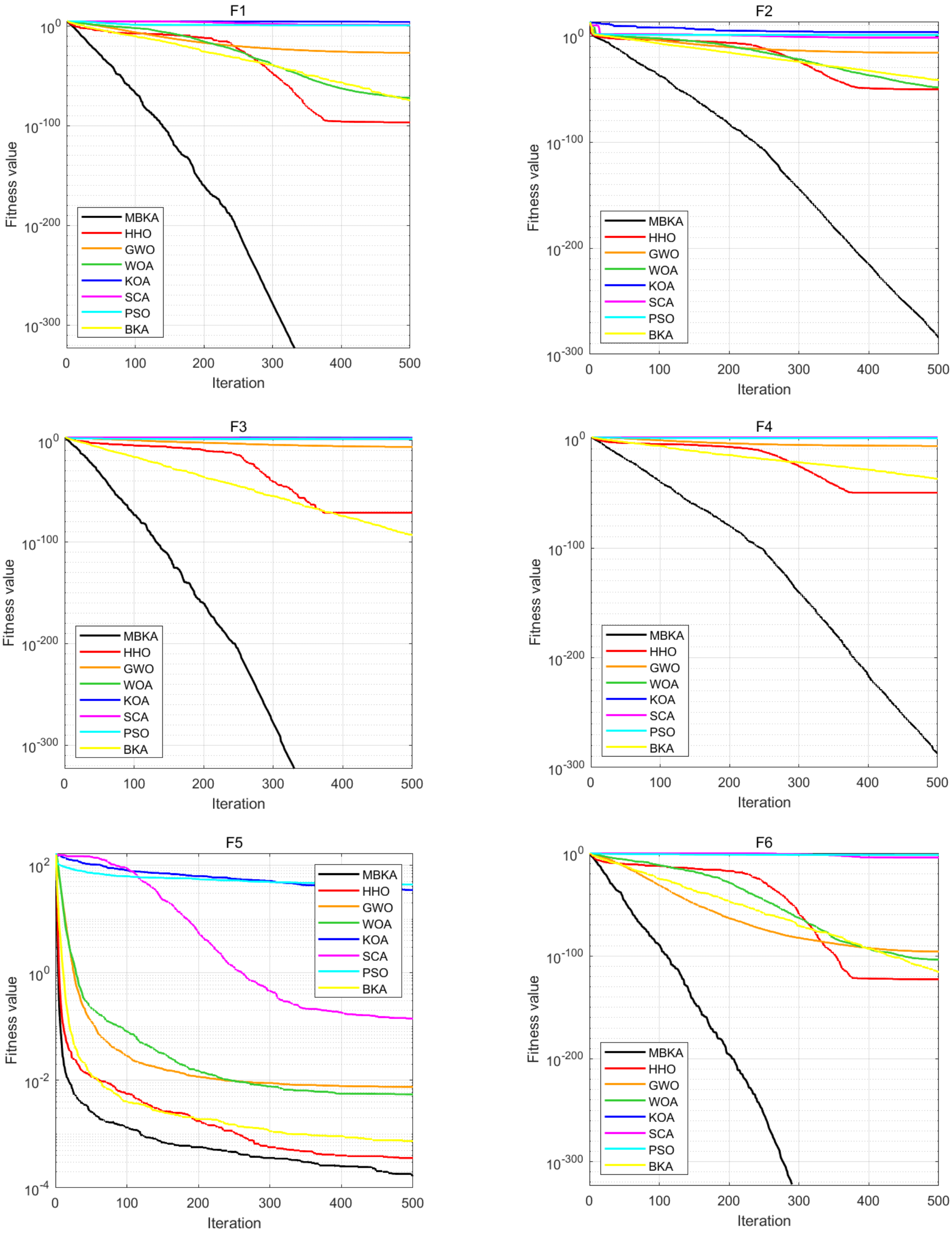
4.5. Convergence analysis

Figures 2 and 3 show the average convergence curves of MBKA and seven other algorithms on 30-dimensional test functions, clearly demonstrating MBKA's superior optimization performance. For unimodal functions, MBKA convergence speed is the fastest, especially F1, F3, and F6-F8; MBKA converges to the theoretical optimal value of 0. While MBKA's convergence speed in F5 is not much different from that of HHO and BKA, it still maintains the highest solution accuracy. Among all the multimodal test functions, MBKA has the fastest convergence speed and is the only algorithm in F14 that converges to the theoretical optimal value of 0. The above experimental results demonstrate that MBKA exhibits a fast convergence speed and high convergence accuracy, and can escape from local optima and gradually approach the global optimum.



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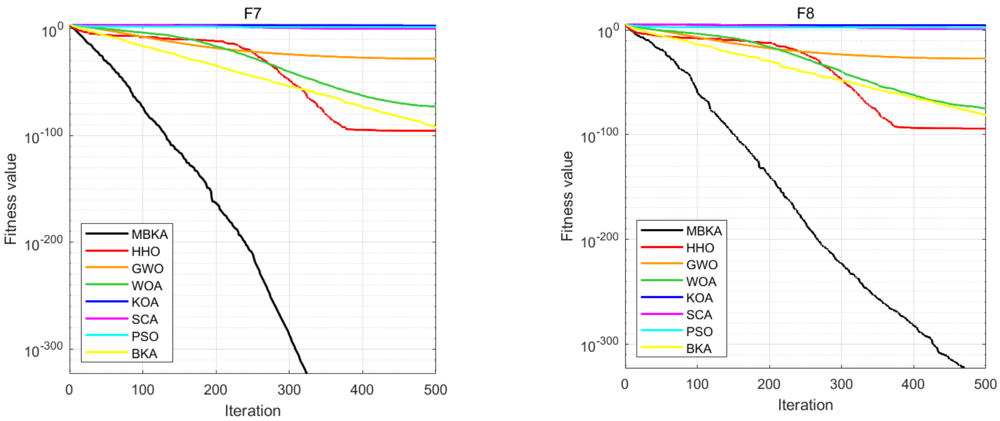
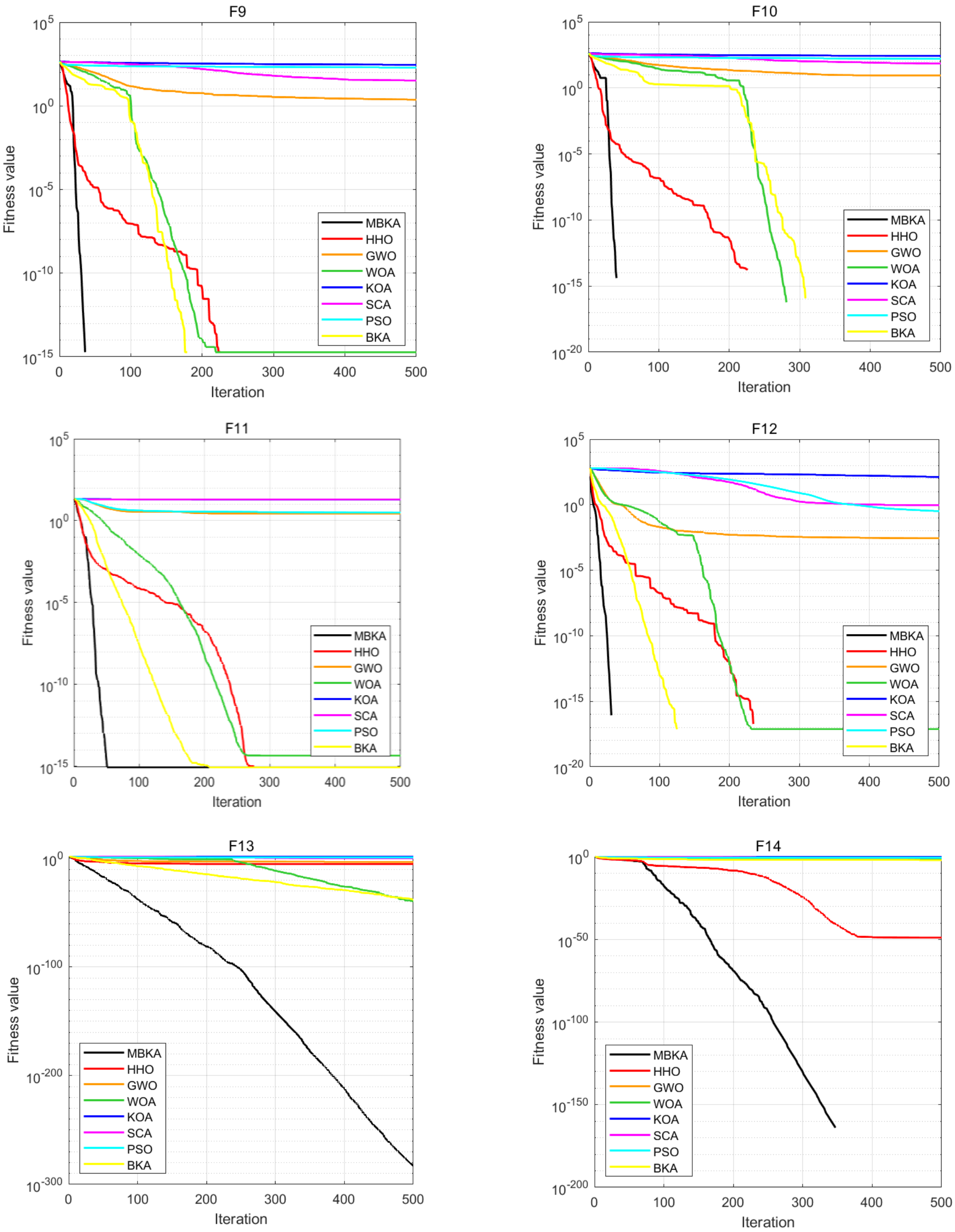


Figure 2: Convergence curve of the average fitness value of the unimodal function.



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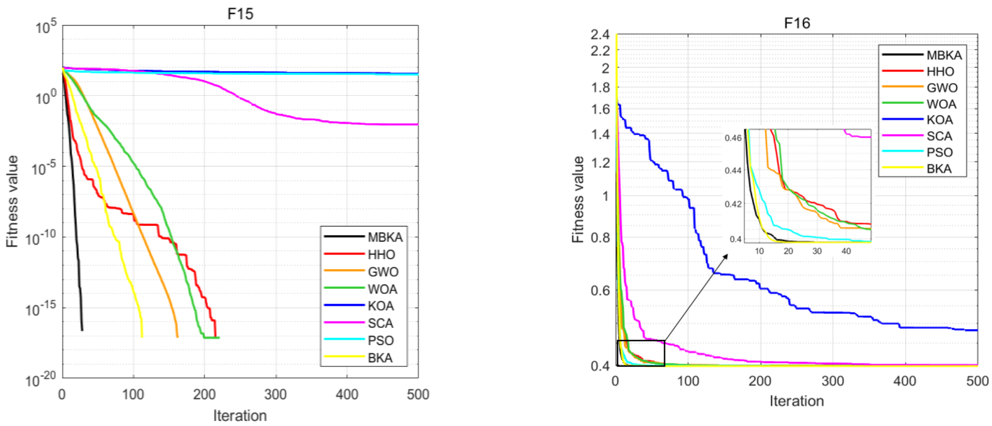

Figure 3: Convergence curve of the average fitness value of the multimodal function.

4.6. Statistical Analysis

The Friedman test, a nonparametric statistical method for detecting significant differences among multiple population distributions through rank analysis, was employed to evaluate algorithm performance. The Wilcoxon rank-sum test is another widely used nonparametric statistical method for evaluating the significance of performance differences between two metaheuristic algorithms [5]. Table 6 presents the Friedman rankings and average ranks of eight algorithms across 16 benchmark functions in 30D spaces. We can see that the MBKA ranks first in most benchmark functions and first in average rankings. Table 7 presents the p-values of the Wilcoxon rank-sum tests for MBKA with the seven competing algorithms in 30D spaces. Except for F5 when compared with HHO, and F9, F10, and F12 when compared with WOA, where MBKA's p-values > 0.05 (indicating no statistically significant differences), the remaining results demonstrate that MBKA exhibits statistically significant differences (at the significance level $\alpha = 0.05$) against the other seven algorithms.

Table 6: Friedman statistical test results of the algorithm.

Function	MBKA	HHO	GWO	WOA	KOA	SCA	PSO	BKA
F1	1.00	2.20	5.00	3.90	8.00	6.43	6.57	2.90
F2	1.00	2.60	5.00	2.60	8.00	6.00	7.00	3.80
F3	1.00	2.77	4.00	7.50	7.50	5.90	5.10	2.33
F4	1.00	2.23	4.00	6.97	7.37	6.47	5.20	2.77
F5	1.43	1.73	4.80	3.90	7.37	6.00	7.63	3.13
F6	1.00	2.10	4.93	3.93	7.43	6.00	7.57	3.03
F7	1.00	2.27	5.00	4.00	8.00	6.00	7.00	2.73
F8	1.00	2.07	5.00	4.00	8.00	6.03	6.97	2.93
F9	2.50	2.50	5.07	2.50	8.00	5.93	7.00	2.50
F10	2.50	2.50	4.93	2.57	8.00	6.07	6.93	2.50
F11	2.13	2.13	5.50	3.60	6.83	7.83	5.83	2.13
F12	2.83	2.83	3.57	2.93	8.00	6.97	6.03	2.83
F13	1.00	2.67	5.00	2.53	8.00	6.00	7.00	3.80
F14	1.00	2.00	4.80	4.23	8.00	6.13	6.80	3.03
F15	3.00	3.00	3.00	3.00	7.93	6.00	7.07	3.00
F16	1.53	5.03	4.93	4.60	8.00	6.97	3.40	1.53
Average	1.44	2.41	4.81	3.97	7.88	6.31	6.31	2.88
Ranking	1	2	5	4	8	6	6	3

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Table 7: The p-values obtained by the Wilcoxon rank-sum test for MBKA against other algorithms.

Function	HHO	GWO	WOA	KOA	SCA	PSO	BKA
F1	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12
F2	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
F3	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12
F4	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
F5	**1.12E-01**	3.02E-11	8.48E-09	3.02E-11	3.02E-11	3.02E-11	5.57E-10
F6	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12
F7	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12
F8	1.72E-12	1.72E-12	1.72E-12	1.72E-12	1.72E-12	1.72E-12	1.72E-12
F9	NaN	1.17E-12	NaN	1.21E-12	1.21E-12	1.21E-12	NaN
F10	NaN	4.57E-12	**3.34E-01**	1.21E-12	1.21E-12	1.21E-12	NaN
F11	NaN	1.20E-12	1.01E-08	8.73E-14	1.21E-12	1.21E-12	NaN
F12	NaN	1.37E-03	**3.34E-01**	1.21E-12	1.21E-12	1.21E-12	NaN
F13	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
F14	1.21E-12	1.21E-12	1.20E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12
F15	NaN	NaN	NaN	1.21E-12	1.21E-12	1.21E-12	NaN
F16	1.66E-11	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	NaN

** p-values > 0.05 appear in bold; NaN represents undefined results from the Wilcoxon test.*

4.7. MBKA for solving engineering problems

Engineering optimization problems contain a large number of equations and inequality constraints, reflecting the complexity and diversity of real-world engineering tasks [4]. This section evaluates MBKA's practical performance by verifying its effectiveness in solving three complex engineering design problems. The experiment was independently repeated 30 times for each problem, with a population size of 30 and a maximum iteration of 500, recording both optimal and average values.

4.7.1. Welded beam design problem

The goal of this problem is to minimize the manufacturing cost of a welded beam while satisfying 4 constraints. There are four design variables, and this problem is mathematically described as follows:

$$\text{Consider variable } H = [h_1, h_2, h_3, h_4] = [h, l, t, b]$$

$$\text{Minimize: } f(H) = 1.10471 h_2 h_1^2 + (14 + h_2) \times 0.04811 h_3 h_4 \tag{18}$$

$$l_1(H) = -\tau_{\max} + \tau(h) \le 0 \tag{19}$$

$$l_2(H) = -\sigma_{\max} + \sigma(h) \le 0 \tag{20}$$

$$l_3(H) = -h_4 + h_1 \le 0 \tag{21}$$

$$l_4(H) = -5 + 0.10471 h_1^2 + (14 + h_2) \times 0.04811 h_3 h_4 \le 0 \tag{22}$$

$$l_5(H) = -h_1 + 0.125 \le 0 \tag{23}$$

$$l_6(H) = -\delta_{\max} + \delta(h) \le 0 \tag{24}$$

$$l_7(H) = -P_c(h) + P \le 0 \tag{25}$$

$$\tau(h) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{h_2}{2R} + (\tau'')^2} \tag{26}$$

$$\tau' = \frac{P}{\sqrt{2} h_1 h_2}, \tau'' = \frac{MR}{J} \tag{27}$$

$$M = P\left(L + \frac{h_2}{2}\right), R = \sqrt{\frac{h_2^2}{4} + (\frac{h_1 + h_3}{2})^2}, \delta(h) = \frac{4PL^3}{E h_3^3 h_4} \tag{28}$$

$$J = 2\left[\sqrt{2} h_1 h_2 \left\{\frac{h_2^2}{12} + \left(\frac{h_1 + h_3}{2}\right)^2\right\}\right], \sigma(h) = \frac{6PL}{h_4 h_3^2} \tag{29}$$

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$$P_c(h) = \frac{4.013E\sqrt{\left(h_4^6 h_3^2\right)/36}}{L^2}\left(1 - \frac{h_3}{2L}\sqrt{E/4G}\right) \tag{30}$$

Variables range: P=6000, L=14, E=30×106, G=12×106

$$\tau_{max} = 13{,}000, \sigma_{max} = 30{,}000, \delta_{max} = 0.25, 0.1 \le h_1, h_4 \le 2, 0.1 \le h_2, h_3 \le 10$$

4.7.2. Speed reducer design problem

The goal of this problem is to minimize the weight of the reducer while satisfying 11 constraints. There are seven design variables, and the problem is mathematically described as follows:

variable Consider $H = [h_1, h_2, h_3, h_4, h_5, h_6, h_7] = [b, m, p, l_1, l_2, d_1, d_2]$

Minimize: $f(H) = 0.7854h_1h_2^2(3.333h_3^2 + 14.9334h_3 - 43.0934) - 1.508h_1(h_6^2 + h_7^2)7.477(h_6^3 + h_7^3) + 0.7854(h_4h_6^2 + h_5h_7^2)$ (31)

$$l_1(H) = \frac{27}{(h_1h_2^2h_3)} - 1 \le 0 \tag{32}$$

$$l_2(H) = \frac{397.5}{(h_1h_2^2h_3^2)} - 1 \le 0 \tag{33}$$

$$l_3(H) = \frac{1.93h_4^3}{(h_1h_3h_6^4)} - 1 \le 0 \tag{34}$$

$$l_4(H) = \frac{1}{(110h_6^3)} \times \sqrt{16.9 \times 10^6 + \left(\frac{745h_4}{h_2h_3}\right)^2} - 1 \le 0 \tag{35}$$

$$l_5(H) = \frac{1.93h_5^3}{(h_2h_3h_7^4)} - 1 \le 0 \tag{36}$$

$$l_6(H) = \frac{1}{(85h_7^3)} \times \sqrt{157.5 \times 10^6 + \left(\frac{745h_5}{h_2h_3}\right)^2} - 1 \le 0 \tag{37}$$

$$l_7(H) = \frac{h_2h_3}{40} - 1 \le 0 \tag{38}$$

$$l_8(H) = 5 \times \frac{h_2}{h_1} - 1 \le 0 \tag{39}$$

$$l_9(H) = \frac{h_1}{12h_2} - 1 \le 0 \tag{40}$$

$$l_{10}(H) = \frac{1.5h_6 + 1.9}{h_4} - 1 \le 0 \tag{41}$$

$$l_{11}(H) = \frac{1.1h_7 + 1.9}{h_5} - 1 \le 0 \tag{42}$$

Variables range: $2.6 \le h_1 \le 3.6, 0.7 \le h_2 \le 0.8, 17 \le h_3 \le 28, 7.3 \le h_4 \le 8.3$

$$7.3 \le h_5 \le 8.3, 2.9 \le h_6 \le 3.9, 5 \le h_7 \le 5.5$$

4.7.3. Three-bar truss design problem

The goal of this problem is to build the smallest model size with 2 design variables while satisfying 3 constraints, and this problem is mathematically described as follows:

variable Consider $H = [h_1, h_2] = [x_1, x_2]$

$$\text{Minimize:} f(H) = \left(2\sqrt{2h_1} + h_2\right) \times l \tag{43}$$

$$l_1(H) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \le 0 \tag{44}$$

$$l_2(H) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \le 0 \tag{45}$$

$$l_3(H) = \frac{1}{\sqrt{2}x_2 + x_1}P - \sigma \le 0 \tag{46}$$

Variables range: $l = 100cm, P = 2KN/cm^2, \sigma = 2KN/cm^2, 0 \le x_i \le 1, i = 1,2$

Tables 8,9, and 10 present the optimal solutions, corresponding design variables, and mean values obtained from 30 independent runs of MBKA and comparison algorithms. For the welded beam design

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problem (Table 8), MBKA achieved the optimal function value f(H)=1.693370, with structural variables H = (0.205393,3.242820,9.036580,0.205733), and a mean function value of f(H)=1.697271. In the speed reducer design (Table 9), MBKA obtained the optimal function value f(H)=2999.704341 with structural variables H = (3.50054,0.7,17,7.53094,7.71967,3.35862,5.2878). Although this optimal value is slightly worse than BKA's best solution, MBKA's mean performance(f(H) = 3009.346454) is better than BKA's (f(H) = 3010.403670). For the three-bar truss design (Table 10), MBKA reached the optimal value f(H)= 263.895844, with structural variables H=(0.788656, 0.408403) and a mean value of f(H)= 263.898156, showing similar performance to BKA's optimal value of 263.895845, this result indicates that while MBKA's performance in certain high-dimensional or heavily-constrained structural design problems may be similar to BKA's, the comprehensive analysis of these three engineering design problems proves MBKA's superior reliability and efficiency in practical engineering applications.

Table 8: The best solutions to the welded beam design problem using various algorithms.

Algorithm	Optimal values for variables				Optimal cost	Average cost
	h	l	t	b		
MBKA	0.205393	3.242820	9.036580	0.205733	**1.693370**	**1.697271**
HHO	0.197357	3.437040	9.104590	0.205393	1.716641	2.071610
GWO	0.205645	3.243560	9.041870	0.205714	1.694599	1.706803
WOA	0.170130	4.050220	9.888030	0.201842	1.862667	2.563548
KOA	0.184369	4.548740	8.662130	0.248081	2.088452	2.930900
SCA	0.205935	3.312260	8.987980	0.211549	1.738837	1.862256
PSO	0.204354	3.271110	9.037060	0.205728	1.695725	1.717534
BKA	0.205590	3.239900	9.035550	0.205779	1.693426	1.751204

Table 9: The best solutions to the speed reducer design problem using various algorithms.

Algorithm	Optimal values for variables							Optimal cost	Average cost
	b	m	p	l_1	l_2	d_1	d_2		
MBKA	3.50054	0.7	17	7.53094	7.71967	3.35862	5.2878	2999.704341	**3009.346454**
HHO	3.50076	0.7	17	7.37537	7.73431	3.38522	5.28666	3004.879493	3717.239768
GWO	3.50469	0.7	17	7.49753	7.78504	3.35305	5.29327	3004.522206	3012.626355
WOA	3.5	0.7	17	7.3	7.73905	3.35124	5.29978	3003.614938	3608.144382
KOA	3.52983	0.7	17	7.34447	8.3	3.45536	5.31754	3066.921771	3202.124052
SCA	3.57922	0.7	17	8.3	8.18289	3.49631	5.31102	3099.813514	3154.393904
PSO	3.50056	0.7	17	7.3	8.3	3.35023	5.28687	3007.665749	3066.304898
BKA	3.50001	0.7	17	7.35487	7.72053	3.35367	5.28798	**2996.794835**	3010.403670

Table 10: The best solutions to the three-bar truss design problem using various algorithms.

Algorithm	Optimal values for variables		Optimal weight	Average weight
	x_1	x_2		
MBKA	0.788656	0.408303	**263.895844**	**263.898156**
HHO	0.788587	0.408498	263.895849	264.042195
GWO	0.787812	0.410696	263.896460	263.904782
WOA	0.787531	0.411493	263.896811	265.404447
KOA	0.776671	0.445179	264.193752	265.294069
SCA	0.789783	0.405171	263.901422	265.996493
PSO	0.788957	0.407457	263.896410	263.904058
BKA	0.788723	0.408114	263.895845	263.932087

5. Conclusions

5.1. Contributions

To prevent premature convergence of the Black Kite Algorithm (BKA) and improve its convergence speed and accuracy, this paper proposes a multi-strategy enhanced Black Kite Algorithm (MBKA). First, the Lens Imaging Opposition-Based Learning (LOBL) strategy is used to mutate the leader to avoid premature convergence. Second, in the attack phase, the position update conditions are changed and incorporate information disparity in early iterations and leader guidance in later iterations to enhance convergence accuracy. Finally, in the migration phase, changing the position updating method and corresponding conditions to accelerate convergence speed and improve convergence precision. The comprehensive experimental results propose that MBKA shows stronger robustness, faster convergence speed, and higher convergence accuracy. Finally, the performance of MBKA is verified in three practical engineering design problems, which proves the effectiveness and feasibility of MBKA in solving real-world application problems.

5.2. Limitations

Despite its demonstrated effectiveness, MBKA still has several limitations. First, the dimensionality was limited to 30D, and the high dimensions are not verified, which prevents definitive conclusions about MBKA's effectiveness for high-dimensional complex optimization. Second, the adoption of median fitness and T/2 as partition thresholds was inspired by the bisection method, although experimental results confirm the efficacy of this partition threshold, finer adjustments to these partition points may potentially enhance performance further.

5.3. Future work

A potential future direction is to integrate BKA with other classical algorithms to further enhance its performance, with applications in fields such as machine learning and UAV path planning.

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