

Simulation of spiral motion based on collision detection

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Abstract: *This study starts from solving the position and velocity of the dragon head and the handles in front and behind the dragon body of the dragon dance team. The equations of motion are established. Considering the geometric relationship between each handle and the length of the board, solve for the position and velocity of the handles in front of the dragon's head. Recursively derive the parameter of motion at each point thereafter. In this paper, in order to solve the critical condition of non-collision between the benches of the dragon dance team. With the help of rotation matrix and graphical transformation, the complex collision problem is transformed into a simple problem of whether the points fall into the danger zone. It improves the performance success rate and spectacle of the bench dragon.*

Keywords: *Iterative Algorithms, Solenoidal Equations, Discrete Models*

1. Introduction

The Bench Dragon is a traditional folkloric performance in the Zhejiang and Fujian regions of China. Its name comes from the main props used during the performance - the benches. During the performance, people hold benches to simulate the dynamics and demeanor of the dragon, and they put several hundreds of benches together to form a winding dragon, so it is called a bench dragon. When the dragon is coiled, the dragon's head is in front of the leader, the dragon's body and the dragon's tail follow each other in a circle, and the whole is in the shape of a disk. In this paper, the bench dragon model consists of 223 sections of benches, which are composed of a 341cm-long section of dragon head, a 220cm-long section of 221 sections of dragon body, and a section of dragon tail, and the two adjacent benches are connected by handles.

The trajectory of the Bench Dragon is similar to the Archimedean spiral of the double-screw combination type [1]. Archimedean spiral (Archimedean curve) [2], also known as "isotropic spiral". When point P moves along a moving ray OP with equal velocity, the ray rotates around point O with equal angular velocity, and the trajectory of point P is called the Archimedean spiral [3]. It was first defined by Archimedes in his work On Solenoids. Because of their excellent properties, solenoids are used in a wide variety of machines and equipment. Their advantages determine directly or indirectly better results in the struggle for survival of living organisms. Taking the shortest length of the screw, it is measured that the screw is the shortest in the curve between two points in the column surface over the column surface, so it needs to consider how it can consume less material, less energy and get better utilization efficiency, which is crucial. In the various curves, the screw plays a role in saving materials and energy consumption, and its application in engineering is very wide [4]. The Dragon Dance is more efficient, safer, and more enjoyable to watch when it follows the trajectory of the solenoid.

Bench dragon performance has always relied on the leader of the dragon head to control the spacing according to experience, with a certain degree of inaccuracy, and sometimes the spacing will be inaccurate to grasp, the occurrence of the dragon body collision can't move forward [5-6]. Therefore, there is an urgent need for an algorithm to accurately calculate the performance distance in advance.

2. Simulation of motion trajectories by iterative loop collision detection algorithms

2.1 Establishment of iterative loop collision detection algorithm

First of all, a model is assumed as an example for simulation: the dragon dance team is coiled clockwise along the equidistant screw line with a pitch of 55 cm, and the center of each handle is located on the

screw line. The traveling speed of the handle in front of the dragon's head is always 1 m/s. The simulated bench dragon consists of 223 benches, which are composed of a 341-cm-long section of the dragon's head, a 220-cm-long section of the 221-section dragon's body, and a section of the dragon's tail, and the two neighboring benches are connected by the handles.

The first step is to establish the helix equation of this simulation model and calculate the trajectory of each point by this equation to simulate the movement of the dragon dance and check whether the benches are in contact or not after each time step. If there is no contact, then it continues; otherwise, it stops moving [7]. The calculations in this paper are realized with the help of MATLAB software.

The trajectory of the dragon dance team is a spiral line, which can be expressed by the spiral equation. Assuming that before the dragon head enters point A, the team has been arranged in a spiral line, to establish the equation:

$$r = k\theta = \frac{d}{2\pi} \theta \quad (1)$$

where k is The Helix equation, θ represent The Width of the bench.

The relationship between the arc length ds and the angle $d\theta$ can be obtained from the arc length formula:

$$\begin{aligned} ds &= \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \\ &= \sqrt{k^2\theta^2 + k^2} d\theta \\ &= k\sqrt{1 + \theta^2} d\theta \end{aligned} \quad (2)$$

From the assumed model, it knows that the traveling speed of the first handle of the faucet is always 1m/s. The derivation of equation (2) is associated with the initial angle θ_0 :

$$\begin{aligned} v = \frac{ds}{dt} &= -k\sqrt{1 + \theta^2} \frac{d\theta}{dt} = -1 \\ \begin{cases} \frac{d\theta}{dt} &= \frac{-1}{k\sqrt{1 + \theta^2}} \\ \theta_0 &= 2\pi \times 16 \end{cases} \end{aligned} \quad (3)$$

From this, the relationship between the angle θ and the time t can be derived, and then the polar coordinates can be converted to right-angle coordinates to determine the position of the first handle of the faucet at each time point:

$$\begin{cases} x_1 = r_1(\theta(t))\cos\theta_1(t) \\ y_1 = r_1(\theta(t))\sin\theta_1(t) \end{cases} \quad (4)$$

Subsequently, the positions of the back handle of the head and each handle of the dragon's body and tail are determined for each point in time. The coordinates of the front handles are (x_1 , y_1) while the position of the rear handles can be derived by solving the following nonlinear equation:

$$(k\theta \cos\theta - x_1)^2 + (k\theta \sin\theta - y_1)^2 = d^2 \quad (5)$$

Through a nested loop, distinguishing between faucets and other parts of the bench due to its different lengths, for each point in time, the position information of all handle holes was calculated. The angular value of the next handle hole and the position in right-angle coordinates were found.

Finally, the speed of the dragon dance team is calculated by the central difference method, the so-called central difference method is a commonly used numerical calculation method, using the function value of the function at two points near the point to approximate the derivative of the function at a certain point, which is a discrete form of the concept of derivative in calculus.

For the first point of the dragon's head, since there is no data before, the forward difference method is used to calculate the velocity, i.e., the angle of the second point is subtracted from the angle of the first point, which is substituted into the velocity equation in equation (3) to get the velocity of the first point of the dragon's head. For the last point of the dragon's tail, since there is no data afterward, the backward difference method is used to calculate the velocity, i.e., the angle of the last point is subtracted from the angle of the second-to-last point, and substituted into the velocity equation in equation (3) to obtain the velocity of the last point of the dragon's tail.

$$\frac{d\theta}{dt} = \frac{\theta_i - \theta_{i-1}}{\Delta t} \quad (6)$$

For the second to the penultimate point, the velocity is calculated using the center difference method, i.e., subtracting the angle of the next point from the angle of the previous point and substituting it into the velocity equation in equation (3) to obtain the velocity at these points.

$$\frac{d\theta}{dt} = \frac{(\theta_{i+1} - \theta_i) + (\theta_i - \theta_{i-1})}{2\Delta t} \quad (7)$$

From this, the velocity of each handle of the dragon dance team at each time point was calculated, and the motion simulation of the dragon dance team from 0s to 300s was realized.

In order to realize collision detection, the range of the dangerous benches is first determined. Here it assumes that the faucet is adjacent to the outer ring with the location of the dangerous bench as shown in Figure 1.

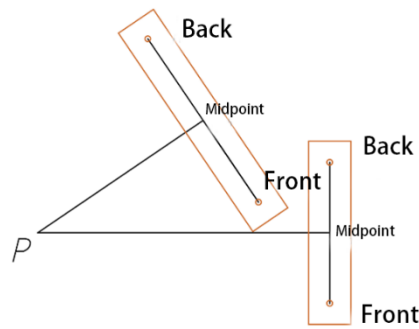


Figure 1 Schematic diagram of bench position

The lengths of the two benches are L_1 and L_2 . The coordinates of the two handle points in the first bench are (x_{11}, y_{11}) and (x_{12}, y_{12}) , and the coordinates of the two handle points in the second bench are (x_{21}, y_{21}) and (x_{22}, y_{22}) . Calculate the slope of the center line of the two benches:

$$k_1 = \frac{y_{11} - y_{12}}{x_{11} - x_{12}} \quad (8)$$

$$k_2 = \frac{y_{21} - y_{22}}{x_{21} - x_{22}} \quad (9)$$

This gives the slope of the vertical line of the bench centerline:

$$k_{1-} = \frac{-1}{k_1} \quad (10)$$

$$k_{2-} = \frac{-1}{k_2}$$

Separately over the center of the bench can be obtained from the intersection of the perpendicular line, to get the new coordinate origin P, expressed in code as follows: $P = A \setminus [k_{1-} * X1_center(1) - X1_center(2); k_{2-} * X2_center(1) - X2_center(2)]$. θ is the angle between the center vertical lines of the two benches,

assuming that the faucet bench is rotated clockwise by an angle θ to the point where the two center vertical lines coincide, the two coordinate ranges can be expressed as respectively:

$$\begin{cases} |x - d_2| \leq \frac{d}{2} \\ |y - 0| \leq \frac{L_2}{2} \end{cases} \quad (11)$$

$$\begin{cases} |x - x_{p1}| \leq \frac{d}{2} \\ |y - 0| \leq \frac{L_1}{2} \end{cases} \quad (12)$$

where L_1 is The faucet bench length and L_2 means The length of the dragon's body and the length of the tail bench.

The points of the leading bench are uniformly discretized by rotating through the matrix $T = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ to rotate all the discrete points by coordinates to the coordinate system in which bench 2 is located, thus establishing an iterative loop until the two bench coordinate functions have overlap, the run is terminated, and the collision termination is dishd in.

The collision detection model combines the geometric and motion laws of the dancing dragon for comprehensive consideration, which has a certain degree of scientific, not only better restore the motion simulation of the dancing dragon team but also well demonstrate its internal geometric relationship.

2.2 Calculation of the minimum spacing for bench dragon movement

The question can be introduced to find the pitch must be smaller than 55cm, and the minimum angle $\theta_{\min}$ change is monotonic, that is, as the pitch increases and decreases. There are two methods in this paper:

①First seek the faucet can reach the distance from the center of the circle at 4.5m when the angle, and then use the nested loop in the problem one can get when the faucet reaches the edge of the turnaround space when the position of each handle, and then through the collision detection algorithm to detect whether the collision, and then look for not colliding with the smallest pitch, this method is a fast and small amount of calculation, but is not precise enough, there is no collision in the 4.5m, but in the case of collision before.

②Use the iterative loop collision detection algorithm to find the angle of collision at different pitches, and the angle when reaching the center of the circle at 4.5m without considering the collision, and compare the two to take the smallest pitch that meets the conditions. This method is highly accurate but computationally intensive.

It uses the first method to determine the general range and then the second method to find the exact value.

The faucet is able to reach the center of the circle 4.5m from the angle and at this time the position of each handle, to determine whether the collision.

$$r = k\theta = \frac{l}{2\pi}\theta$$

$$\text{when } r = 4.5m : \theta = \frac{9\pi}{l} \quad (13)$$

Dichotomy: Choose 0.42m as the test pitch, if not then choose the middle value of 0.42m~0.55m, and vice versa to take a smaller one. Repeat the method of one or two more questions, the final answer will converge to a limit value.

Traversal method: since it is known that the minimum pitch must be less than 0.55m, it sets the search range of the minimum pitch to start from 0.55m, i.e., $[50: -0.5: 42] \times 10^{-2}$. A traversal calculation is performed for each pitch value.

Through the iterative loop collision detection algorithm, the minimum angle that can be reached by the front handle of the faucet is recorded at $\theta_{\min}$, and the theoretical minimum angle that can be reached in the turnaround space under the corresponding pitch is calculated by the formula $\theta_{\min}$, and the pitch-angle relationship curve is plotted and analyzed to find that the intersection point is the minimum pitch required.

First set the search range, 0.42 m to 0.55 m. Calculate the value of the objective function at the midpoint of the search range, and then, based on the sign of the value, decide whether to conduct the next round of search to the left or right of the midpoint. Repeat this process until the search range is smaller than the preset tolerance value.

The search range of the minimum pitch is set to start from 0.50m, while the size of the pitch determines the denseness of the screw threads in the whole screw equation, as the pitch decreases one by one the theoretical minimum angle $\theta_{\min}$ will increase, but the possibility of collision of the dragon dance team will also increase. While traversing the method, it is constantly optimizing the iteration [8] to find a balance point.

Still, construct the solenoidal equation first:

$$r = k\theta = \frac{l}{2\pi}\theta \quad (14)$$

where l is The screw pitch.

A differential equation is established so that the relationship between the angle θ and the time t can be derived, i.e., the frontal equation of motion of the faucet:

$$\frac{d\theta}{dt} = -\frac{1}{k\sqrt{1+\theta^2}} \quad (15)$$

$D1$ is the distance between the handles of the dragon head, $D2$ is the distance between the handles of the dragon body and the dragon tail, the coordinates of the front handles (x_1, y_1) and the position of the rear handles can be obtained by solving the following non-linear equation:

$$(k\theta \cos \theta - x_1)^2 + (k\theta \sin \theta - y_1)^2 = d^2 \quad (16)$$

Then, according to the established collision detection model, consider detecting whether the neighboring benches and the neighboring dangerous benches in the outer circle overlap and require that no collision is allowed to occur before entering the turnaround space. When the faucet enters the turnaround space, no collision occurs in the dragon dance team, and the pitch at this time meets the compliance conditions. Record the current minimum angle that the dragon head can reach $\theta_{\min}$, and draw the curve between the actual minimum angle $\theta_{\min}$ and the theoretical arrival at the boundary of the turnaround space $\theta_{\min}$. Determine the optimal minimum pitch.

3. Results

3.1 Collision detection results

According to its established iterative loop collision detection algorithm, combined with the solenoidal equations derived, as shown in Figure 2. It starts from 300s and set the step size to 0.2s for more accurate detection. Entering the iterative loop, it solves the differential equations of motion of the front handle of the headboard bench by the Lungkuta method:

$$\frac{d\theta}{dt} = -\frac{1}{k\sqrt{1+\theta^2}} \quad (17)$$

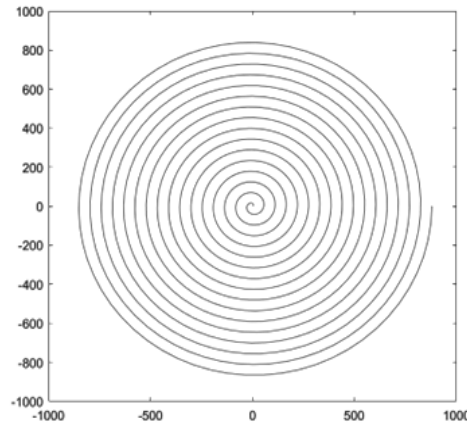


Figure 2 Spiral path

From this, it can derive the angle of the next time point θ , and the coordinates of the new time point, combined with the first question to derive the position of each part of the dragon dance team. Then use the above approach for collision detection, call the sub-function to find if intersects, and determine whether the two benches' thresholds overlap, if there is overlap it will set the flag variable to flag=1, jump out of the loop, end the run; if there is no overlap flag=0, continue to iterate the loop until it stops running. The Trajectory at termination is shown in Figure 3. The calculated termination moment is around 412.6s.

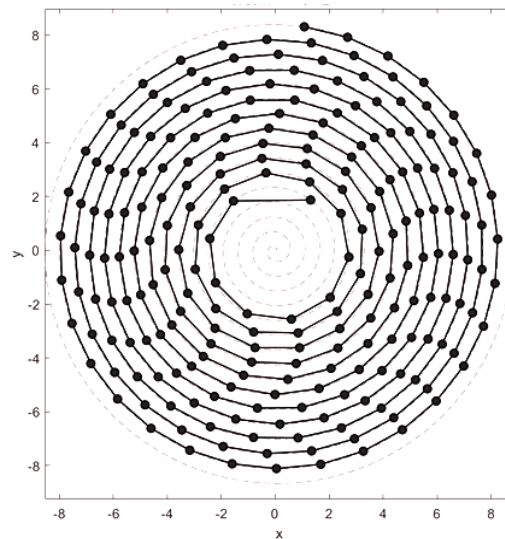


Figure 3 Trajectory at termination

Finally, the median difference method was performed to calculate the speed of the dragon dance team at each point:

$$v_i = -k\sqrt{1 + \theta_i^2} \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta t} \quad (18)$$

For the first point of the dragon's head and the last point of the dragon's tail still, the forward difference method and the backward difference method are performed to solve. Finally, the data obtained, the coordinates, and the speed of each point are summarized and processed to retain 6 as a decimal, as shown in Table 1.

Table 1 Position coordinates and speed

	Horizontal coordinate x (m)	Vertical coordinate y (m)	Speed (m/s)
Faucet Front Handle	1.312391	1.869198	0.999164
Section 1: Handle in front of the dragon's body	-1.547381	1.833081	0.990755

Section 51: Handle in front of the dragon's body	1.398137	4.287713	0.976558
Section 101: Handle before the Dragon's Body	-0.658356	-5.865887	0.974334
Section 201: Handle in front of the dragon's body	-7.909810	-1.109142	0.972933
Dragon's Tail Rear Handle	1.077883	8.306562	0.972781

It comprehensively considered the movement law and geometric relationship of the dragon dance team, made several parameter adjustments to the simulation, and were able to find the termination moment more accurately. The error was minimized as much as possible by applying the median difference method [9] and the Runge Kutta method [10].

3.2 Minimum spacing prediction

With the test results, it takes 0.42m~0.50m as the general range, as shown in Figure 4.

Collision occurred at screw distance 0.410204!
Collision occurred at screw distance 0.414286!
Collision occurred at screw distance 0.418367!
No collision occurred at screw distance 0.422449.
No collision occurred at screw distance 0.426531.
No collision occurred at screw distance 0.430612.
No collision occurred at screw distance 0.434694.
No collision occurred at screw distance 0.438776.
No collision occurred at screw distance 0.442857.

Figure 4 Intersection detection results

The traversal method [11] starts from 0.50m: spiral pitch range = [50: -0.5: 40] * 1e-2; the initial angle is set to $\theta_0 = 2\pi N_0$

$$N_0 = \text{ceil}(4.5/l) + 3 \quad (19)$$

The above equation ensures that the dragon dance team is dished in from the outside of the header space. The boundary conditions of the head-turning space Eq:

$$x^2 + y^2 \leq (4.5)^2 \quad (20)$$

Theoretical minimum angle $\theta_{\min}$ curve:

$$\theta = \frac{9\pi}{l} \quad (21)$$

Solve the differential equations of motion of the dragon dance team by the Lungkuta method, to derive the angle and the corresponding position coordinates of the next time point. The coordinates of each point after that are found by the solve function. Then the collision test sub-function finds if intersect is called to determine that there is no collision when the dragon's head enters the turnaround space. Instead, if a collision occurs, the flag variable flag=1 will jump out of the loop to increase the pitch iteration and record the minimum actual angle for each pitch. The theoretical arrival of the minimum angle of the head space editor $\theta_{\min}$ and the actual minimum angle $\theta_{\min}$ of curve image merger analysis, the minimum value of the pitch of the intersection, is the minimum pitch the study require, probably in 0.454m. The Pitch vs. minimum angle is shown in Figure 5.

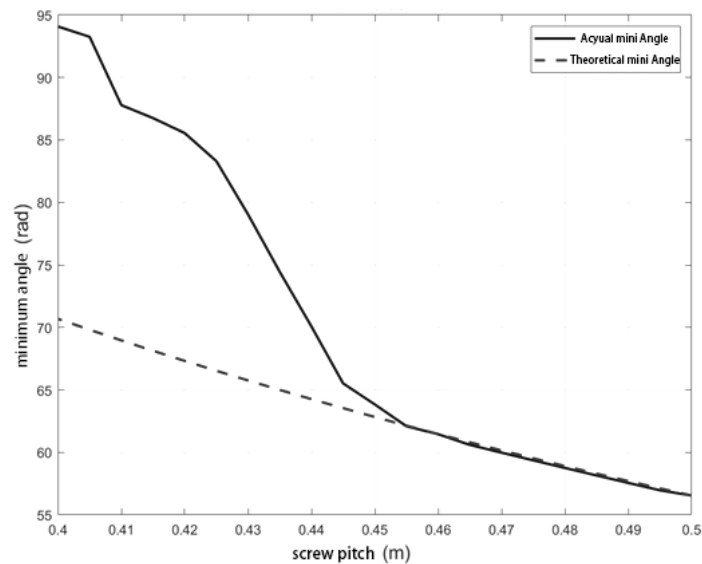


Figure 5 Pitch vs. minimum angle

4. Conclusions

In this study, a collision detection model is developed to consider the collisionality of a bench with multiple adjacent benches around it, which is used to detect whether a dragon dance team will collide or not during the performance. It is determined by iteratively determining whether there is any overlap in the function threshold between the benches at each time point. From there, the coordinate position and speed of each point of the dancing dragon at that angle are calculated. And on the basis of the established model, continuous iterative cycle detection, through the traversal method and bisection method of the two methods can be iterated to find the pitch minimum and will not collide with the equilibrium point, you can find the minimum pitch. The minimum pitch can be found by iteratively finding the equilibrium point with the minimum pitch without collision through the two methods of iteration and bifurcation, and then the minimum pitch can be found.

The iterative cyclic collision detection model uniformly discretizes the points on the dragon's head and the dragon's body bench, transforms the complex collision problem into a simple function overlapping problem with the help of the rotation matrix and accurately determines whether the discretized points are on the danger zone by using the iterative algorithm. The model improves the computational efficiency, which is important for improving the dragon dance performance.

In the future, the model can be used to solve engineering production problems. For example, the grinding and polishing treatment of optical lenses, and controlling the pitch of threads to ensure finish. It can also be used for complex dynamic system analysis, such as ray collision detection, collision detection in games and other issues. It can greatly improve the accuracy and efficiency of the calculation, and is of great significance for the collision detection of spiral motion and the determination of the minimum pitch.

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