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Permeability Model of Unsaturated Fractured Porous Media Based on Tree-like Network and Finite Element Method

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Abstract: *In order to characterize the multi-scale structures of fractures and understand the multiphase fluid flow mechanisms in unconventional reservoirs, a physical conceptual model is developed for the gas-water flow through fractured porous media based on tree-like networks. The finite element method is carried out to explore the local flow field properties based on the level set method. The results show that the successive branching ratio and branching angle of the network take significant effect on the absolute permeability and relative permeability of fractured porous media. The proposed model for fractured porous media may provide theoretical basis for the development of unconventional oil and gas resources, carbon dioxide geological sequestration, ground water mining etc.*

Keywords: *fractured porous media; tree-like network; relative permeability; finite element method*

1. Introduction

Research on unconventional resources has increased significantly over the last two decades due to technological breakthroughs in horizontal drilling and hydraulic fracturing [1, 2]. The fractures play a major channel role in the fluid flow through low-permeability unconventional reservoirs [3]. However, the flow back of fracturing water and long period of water production in fractured horizontal well indicate significant influences on the transport mechanism and production prediction of unconventional reservoirs. Thus, the gas-water flow through fractured porous media has attracted increasing interests from multi disciplines.

The effective permeability and relative permeability are the key parameters in the gas-water flow though fractured porous media, which have been studied via laboratory experiment and theoretical model as well as numerical simulation. Diego [4] used a visual simulation device to study the phenomenon of gas-water displacement in the fracture network, revealing that when a large fracture was connected, gas could quickly flow through the entire permeability zone. Yang et al. [5] used the horizontal simulation device to study and analyze the permeability mechanism, and they showed that the fluid saturation was a key factor affecting the macroscopic relative permeability of gas-water two-phase fluid. Zhang et al. [6] conducted unsteady state experiment on the fractured coal core, and found that the stress change was negatively correlated with the relative permeability of the two-phase fluid. Wang et al. [7] measured the gas-water two-phase permeability in the fracture with the triaxial compression experiment, and they found that each fluid phase had a separate flow channel due to the phase interference.

A few researchers proposed theoretical models to predict the relative permeability of unsaturated fractured porous media. Table 1 summarizes the theoretical analysis and conclusions of the mainstream gas-water two-phase permeability models for fractured porous media. Romm [8] assumed that there was no phase interference between each phase of fluid, and proposed a linear relationship between the relative permeability and fluid saturation. However, it was confirmed by later experiments [9] and numerical studies [10] that this assumption was out of reality. Then, Corey [11] incorporated the phase interference effect of fluids into the model parameters of relative permeability. Persoff [12] pointed out that Corey model could not fit the experimental data well. Brooks and Corey [13] established a gas-water two-phase permeability model using the porous medium method, where the pore size distribution was taken into account. Fourar [14] argued that the gas-water relative permeability was a function of fluid saturation and viscosity. Chima [15] adopted the cubic law to derive the gas-water two-phase permeability model, where the fracture was assumed to be horizontally oriented. Based on Brooks-Corey model, Li et al. [16] improved the relative permeability model by considering the influence of bending degree.

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Table 1: The gas-water two-phase permeability models

Reference	Time	Theoretical model
Brooks and Corey [13]	1966	$K_{rg}=\left(1-S_w\right)^2\left[1-\left(S_w\right)^{(1+2/\zeta)}\right]$ $K_{rw}=\left(S_w\right)^{(3+2/\zeta)}$
Fourar and Lenormand [14]	2001	$K_{rg}=S_g^3+\frac{3\mu_g}{2\mu_w}S_gS_w\left(2-S_w\right)$ $K_{rw}=\frac{S_w^2\left(3-S_w\right)}{2}$
Chima and Geiger [15]	2012	$K_{rg}=S_g^2\left(\frac{2\mu_wS_g^2+3\mu_gS_w^2+6S_gS_w\mu_g}{12\mu_w}\right)$ $K_{rw}=S_w^2\left(\frac{4S_w^2+6S_wS_g}{12}\right)$
Li et al [16]	2014	$K_{rw}=\frac{\left(S_w-S_{wc}\right)^2}{\left(1-S_{wc}\right)^3}\left[\frac{2\left(S_w-S_{wc}\right)+3S_g}{2}\right]$ $K_{rg}=\frac{S_g}{\left(1-S_{wc}\right)^3}\left[S_g^2+\frac{3\mu_g}{2\mu_w}\left(S_w-S_{wc}\right)^2+\frac{3\mu_g}{\mu_w}S_g\left(S_w-S_{wc}\right)\right]$
Notes: K_{rg}-gas phase relative permeability, K_{rw}-water phase relative permeability, S_g-gas saturation, S_w-water saturation, S_{wc}-irreducible water saturation, μ_g-gas viscosity, μ_w-water viscosity, ζ-pore-size distribution index.		

With the development of modern computer technology, numerical simulation methods have been developed rapidly [17-19]. Therefore, the finite element method is used to study the gas-water flow behavior in fractured porous media. And the tree-like network is adopted to characterize the complex fracture structures. The absolute and relative permeability of unsaturated fractured porous media are calculated accordingly.

2. Fracture network model

A reprehensive element consisted of parent matrix and fracture is shown in Figure 1. The tree-like model is used to characterize the fracture network, which can be generated by an iterative algorithm. Firstly, a parent fracture bifurcates into two daughter fractures at a certain bifurcation angle θ. Then each daughter fracture bifurcates into more branches with the similar rules. This bifurcation process is repeated until the branching level reaches its maximum value. For simplification, it is assumed that the branching fractures are straight round pipes with negligible wall thickness.

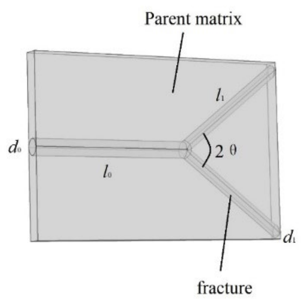

Figure 1: The tree-like model for fractured porous media

To characterize the branching structures, two scale factors were introduced:

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$$\alpha = l_{k+1} / l_k \tag{1}$$

$$\beta = d_{k+1} / d_k \tag{2}$$

where α and β are the successive length and diameter ratios, respectively. Based on fractal theory [20, 21], the optimal values of $\alpha = 2^{-1/3} \approx 0.8$ and $\beta = 2^{-1/2} \approx 0.7$ as well as $\theta = 37.45^{\circ}$ were used in the following simulation. The diameter and length of the parent fracture were $d_0 = 1\text{mm}$ and $l_0 = 10\text{mm}$, respectively. The Navier-Stokes equation combined with the level set method was used to study the gas-water flow through fractures.

$$\rho(\mathbf{u} \cdot \nabla \mathbf{u}) = \nabla \cdot \left[-p\mathbf{I} + \mu\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) \right] + \mathbf{F} \tag{3}$$

$$\rho \nabla \cdot \mathbf{u} = 0 \tag{4}$$

where ρ is the fluid density, p is the velocity vector, u is the velocity vector, I represents the unit diagonal matrix, μ is the dynamic viscosity, and F is the volume force vector.

The evolution and iteration process of level set can be used to describe the dynamic change characteristics of the shape of the gas-water two-phase interface. The level set equation describing the change of the phase interface of a two-phase fluid can be written as:

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = \gamma \nabla \cdot \left(\varepsilon_{ls} \nabla \varphi - \varphi(1-\varphi) \frac{\nabla \varphi}{|\nabla \varphi|} \right) \tag{5}$$

The expression on the right side of the equation is used to describe the movement of the phase interface, $0 \le \varphi \le 1$ is the level set variable, ε_{ls} is the interface thickness control parameter, γ is the level set function that reinitializes the parameters. The following boundary and initial conditions were adopted.

(1) The fractures were filled with gaseous methane under the initial conditions, the left side was displaced by water and the fluid flows out from the right;

(2) The pressure difference between the inlet and outlet of the bifurcated fracture was set to be 10Pa, and the symmetrical boundary conduction was applied on the upper and lower sides.

(3) The boundary meets the requirements that the pressure boundary condition complies with the Dirichlet boundary condition in fluid mechanics, the fluid reflux is forbidden, and there is no tangential velocity;

(4) The fracture walls of the model were closed boundaries, and there was no fluid exchange with the peripheral matrix.

3. Results and discussion

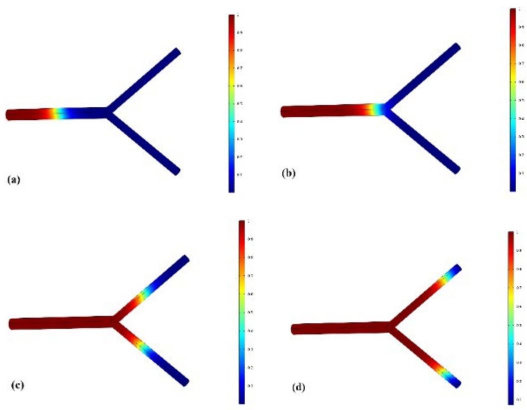

Figure 2: Fluid volume fraction at different time in fractures: (a) t=0.2s; (b) t=0.6s; (c) t=1.2s; (d) t=2.0s.

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It can be found in figure 2 that the water enters the tree-like fracture from the left inlet, and the gas phase is displaced along the pipe. The simulation results show that during the entire gas-water displacement process, the water has obvious fingering phenomenon.

The Darcy velocity can be calculated by the average velocity on the exit interface. Then the permeability can be estimated with Darcy's law.

$$K = \frac{\mu L Q_O}{\Delta P A} = \frac{\mu L}{\Delta P} v_D \tag{6}$$

where Q_o is the flow rate in outlet, v_D is Darcy velocity. The relative permeability for the gas and water phases are:

$$K_{rg} = \frac{K_g}{K} \tag{7}$$

$$K_{rw} = \frac{K_w}{K} \tag{8}$$

where, K_g and K_w are the effective permeability for gas and water phases, K is the absolute permeability of fractured porous media.

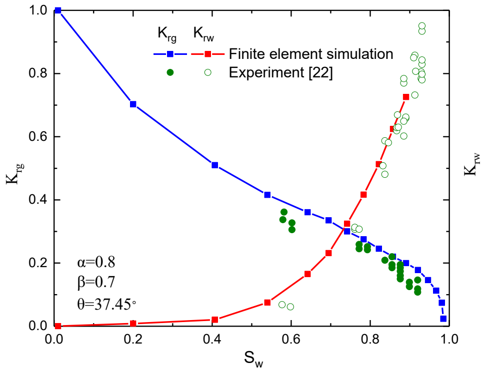

Figure 3: Comparison of finite element simulation results with experimental data

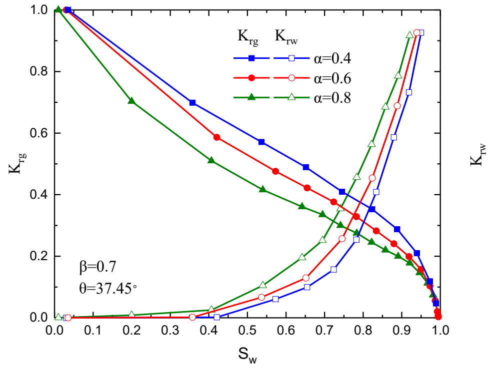

Figure 4: Effect of successive length ratio on relative permeability of fractured porous media

As shown in Fig. 3, the reliability of the finite element simulation results is verified by comparing with the experimental data of gas-water two-phase permeability in fractured porous media [22]. The relative permeability for gas phase decreases with the increase of water saturation, while the relative permeability for water phase increases as the water saturation increases. As shown in Fig. 4, the relative permeability for gas and water phases decreases and increases with the increment of successive length ratio, respectively. The increased successive length ratio reduces the permeability by increasing flow resistance. However, the reduction rate of gas-phase permeability is larger than that of absolute permeability. While the reduction rate of water-phase permeability is smaller than that of absolute permeability. For example (Table 2), the maximum decline rate for absolute, gas-phase and water-phase permeability are 20.72%, 33.67% and 5.47% at water saturation of S_w=0.7, respectively.

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Table 2: The variation of permeability at different length ratios (Sw=0.7)

α	K ($\times10^{-9}m^2$)	K_g($\times10^{-10}m^2$)	K_w($\times10^{-10}m^2$)
0.4	1.3981	5.8721	1.8171
0.6	1.2349	4.9615	1.7654
0.8	1.1084	3.8949	1.7176

As shown in Fig. 5, when the length ratio and bifurcation angle are constant, the larger the successive diameter ratio, the larger the relative permeability for gas phase and the smaller the relative permeability for the water phase. This is because enhanced successive diameter ratio can lower the flow resistance in fractures. As shown in Table 3, the enhancement amplitude of gas-phased permeability (698.22%) is much larger than that of absolute (520.65%) and water-phase permeability (212.27%).

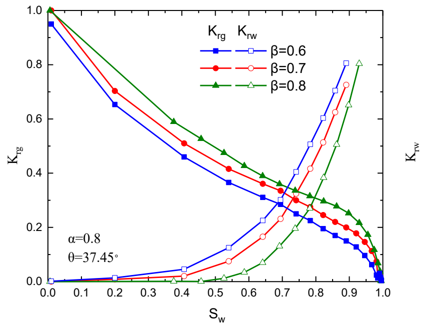

Figure 5: Effect of diameter ratio on relative permeability of gas-water two phases

Table 3: The variation of permeability at different diameter ratios (Sw=0.7)

β	K ($\times10^{-10}m^2$)	K_g($\times10^{-10}m^2$)	K_w($\times10^{-10}m^2$)
0.6	4.7925	1.3475	1.4465
0.7	1.1084	3.8949	1.7176
0.8	2.9745	1.0756	4.5173

It can be seen in Fig. 6 that the relative permeability for the gas phase decreases as the bifurcation angle increases, while the relative permeability for the water phase increases as the bifurcation angle increase. The reason is similar to that of successive length ratio. For example, when the bifurcation angle increase from 30° to 45°, the absolute permeability decrease by 15.99%, while the gas- and water-phase permeability decreases by 24.66% and 3.69%, respectively. That is the reduction of gas-phase permeability is larger than that of absolute permeability, while the water-phase permeability is smaller than that of absolute permeability.

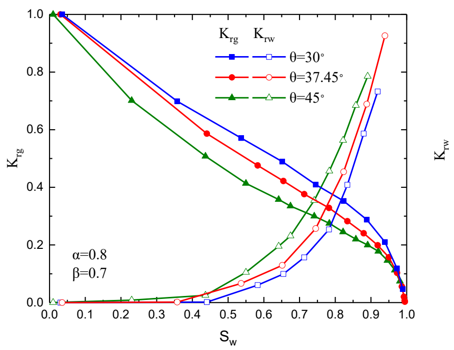

Figure 6: Effect of bifurcation angle on relative permeability of gas-water two phases

Table 4: The variation of permeability at different bifurcation angles (Sw=0.7)

θ	K($\times10^{-9}m^2$)	K_g($\times10^{-10}m^2$)	K_w($\times10^{-10}m^2$)
30°	1.3195	5.1698	1.7835
37.45°	1.2173	4.3258	1.7491
45°	1.1084	3.8949	1.7176

4. Conclusion

The tree-like network model was used to characterize the fracture networks in fractured porous media,

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and the laminar flow level set method was performed on the gas-water flow through fractures. The effect of water saturation, successive length and diameter ratios and bifurcation angle on the relative permeability were explored. It has been found that there is an obvious fingering phenomenon when gas displaces water in fractured porous media. The relative permeability for gas and water phases decreases and increases with the increment of water saturation, respectively. While the relative permeability for gas phase decreases with the increase of successive length ratio and bifurcation angle, it increases as the successive diameter ratio increases. On the contrary, the relative permeability for water phase increases as the successive length ratio and bifurcation angle increases, it decreases with the increase of successive diameter ratio. The present results may shed light on the multiphase fluid flow through fractured porous media, and provide theoretical basis for unconventional reservoir etc.

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