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Study on Active Control of Stick-slip Vibration of Drill String System

Jie Luo, Jun Liu, Qiang Li

School of Mechatronic Engineering, Southwest Petroleum University, Chengdu, 610500, China

Abstract: *The stick-slip vibration phenomenon of drill string widely exists in the drilling process, which has a great negative impact on drilling equipment, efficiency, cost and safety. In order to effectively suppress the stick-slip vibration of the drill string system, a two-degree-of-freedom torsional vibration model of the drill string system is established considering the nonlinear friction between the drill bit and rock. Based on this model, an optimal LQR controller is designed to suppress the stick-slip vibration of drill string, and the driving torque is fed back in real time through the state error. The effectiveness of the controller under parameter perturbation is verified by simulation analysis. The results show that the controller can effectively suppress the stick-slip vibration of drill string system under the desired speed, feedforward torque, start time and bit weight parameter perturbation, achieve real-time feedback of driving torque, and ensure that the angular velocity of rotary table and BHA can be tracked and maintained at the desired speed in a short time. The control effect shows good steady-state error and dynamic performance. The research results have a certain reference significance for suppressing stick-slip vibration of drill string system and improving drilling efficiency.*

Keywords: *drill string system; stick-slip vibration; lqr; parameter perturbation; active control*

1. Introduction

With the increasing demand for oil and gas resources, oil and gas development is gradually developing to deep and ultra-deep Wells[1], and the formation structure becomes complex and diversified. High temperature, high pressure, uneven distribution of stress and other factors lead to longitudinal, transverse and torsional vibration of drill string system[2]. Stick-slip vibration caused by torsional vibration has the most serious harm, accounting for more than 50% of the total drilling time[3]. Stick-slip vibration will intermittently cause high-speed sliding and viscous static periodic oscillating motion of the drill string. The sudden release of its stored energy can make the rotation speed of the BHA change from zero to six times of the top rotating mechanism[4]. This greatly speeds up the failure of drilling equipment, causes underground accidents and increases the cost of drilling. Therefore, effective active control system is very important, and it is necessary to carry out further research on this problem.

In order to suppress the stick-slip vibration of drill string, scholars at home and abroad have carried out a large number of studies on this issue. Xiao[5] introduced the soft torque control system into the rotary mechanism at the top of the drill string, which can compensate the system motor torque according to the predicted trend. Many documents have applied PID controller to suppress stick-slip vibration of drill string[6], including PI controllers[7], PD controllers[8] and enhanced control schemes based on fractional controllers (FOPI, FOPD, FOPID)[9]. In view of the parameters uncertainty and nonlinear characteristic in the drill string system, researchers proposed control[10], robust control[11], sliding mode control (SMC)[12] and backstepping control[13]. These control methods make the closed-loop feedback system have robust stability by optimizing the performance of multiple indicators.

Therefore, based on the established torsional vibration model of the drill string system with two degrees of freedom, a linear quadratic regulator (LQR) is designed to suppress the stick-slip vibration of the drill string system. Under the perturbation of the desired speed, feedforward torque, start time and bit weight parameters, the real-time tracking of the angular velocity of the rotary table and the BHA to the desired speed is realized. The closed-loop control system is simulated and verified by Matlab/Simulink.

2. Establishment of dynamic model of drill string system

Under the condition that the system characteristics are basically unchanged, the drill string system is

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simplified into a two-degree-of-freedom concentrated mass torsional pendulum model of the rotary table and the BHA. Each part is connected by a linear spring with torsional stiffness and torsional damping, as shown in Figure 1. The actual downhole conditions are considered, and the following assumptions are made for the model: The research object is the drill string system in a vertical well; The effect of drilling fluid on drill string system is equivalent to viscous damping. The lateral and axial vibration of the bit is ignored and the weight on bit is constant. Ignoring the effect of friction on the connection between the drill pipe and the wall.

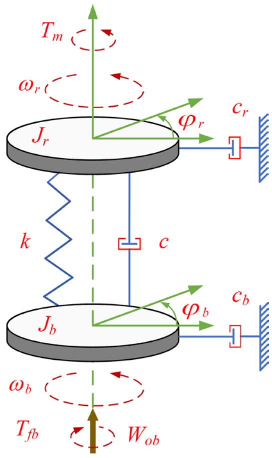

Figure 1: Simplified model of torsional vibration of drill string

In Figure 1, the equation of motion of the drill string rotation system can be expressed as:

$$\begin{cases} J_r\ddot{\varphi}_r + c\left(\dot{\varphi}_r - \dot{\varphi}_b\right) + k\left(\varphi_r - \varphi_b\right) + c_r\dot{\varphi}_r = T_m \\ J_b\ddot{\varphi}_b - c\left(\dot{\varphi}_r - \dot{\varphi}_b\right) - k\left(\varphi_r - \varphi_b\right) + c_b\dot{\varphi}_b = -T_{fb}(x) \end{cases} \tag{1}$$

Where J_r and J_b are the equivalent rotational inertia of the rotary table and the BHA respectively, $kg\cdot m^2$; k is the equivalent torsional stiffness of the drill string system, $(N\cdot m)/rad$; c is the equivalent torsional damping of the drill string system, $(N\cdot m\cdot s)/rad$; c_r and c_b are the viscous damping of the rotary table and the BHA, $(N\cdot m\cdot s)/rad$; φ_r and φ_r are the angular displacement of rotary table and the BHA, rad; $\dot{\varphi}_r$ and $\dot{\varphi}_b$ are the angular velocity of rotary table and the BHA, rad/s; $\ddot{\varphi}_r$ and $\ddot{\varphi}_b$ are the angular acceleration of rotary table and the BHA, rad/s^2; T_m is the driving torque of rotary table; T_{fb} is the nonlinear friction torque on the bit, $N\cdot m$.

In order to accurately describe the continuity of bit speed in the zero-speed interval, Karnopp friction model in Figure 2 is adopted to simulate the nonlinear torque between bit and rock interaction[14]. The expression is as follows:

$$T_{fb}(x) = \begin{cases} T_{eb}(x) & if\ |\dot{\varphi}_b| < D_v,\ |T_{eb}| \le T_{sb} \\ T_{sb}sign(T_{eb}(x)) & if\ |\dot{\varphi}_b| < D_v,\ |T_{eb}| > T_{sb} \\ T_{cb}(x)sign(\dot{\varphi}_b) & if\ |\dot{\varphi}_b| \ge D_v \end{cases} \tag{2}$$

$$\begin{cases} T_{eb}(x) = c\left(\dot{\varphi}_r - \dot{\varphi}_b\right) + k\left(\varphi_r - \varphi_b\right) - c_b\dot{\varphi}_b \\ T_{sb} = W_{ob}R_b u_{sb} \\ T_{cb} = W_{ob}R_b u_b\left(\dot{\varphi}_b\right) \\ \mu_b\left(\dot{\varphi}_b\right) = \mu_{cb} + \left(\mu_{sb} - \mu_{cb}\right)e^{-(\gamma_b/v_f)|\dot{\varphi}_b|} \end{cases} \tag{3}$$

Where T_{fb}, T_{eb}, T_{sb} and T_{cb} are nonlinear frictional torque at the joint of bit face, static equilibrium torque, maximum static frictional torque moment between bit and rock, coulomb frictional torque respectively, $N\cdot m$; μ_{sb} and μ_{cb} are static friction coefficient and coulomb friction coefficient related to moment of inertia J_b respectively, μ_{sb}, $\mu_{cb} \in (0,1)\ and\ \mu_{sb} > \mu_{cb}$; W_{ob} is weight on bit, N;

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R_b is the radius of drill bit, m ; $D_v > 0$ is the boundary layer thickness; γ_b and ν_f are the empirical constant.

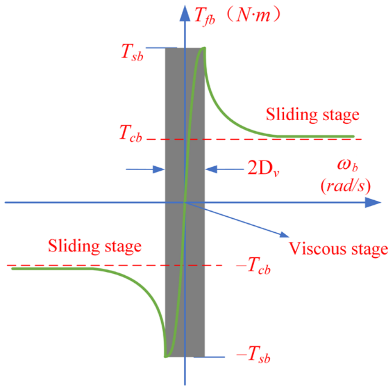

Figure 2: Karnopp's model for bit-rock interaction

3. LQR controller design

In the original state, the speed of the main motor driving the turntable will change with the load, and the constant speed output cannot be achieved. Therefore, it is hoped that the controller can provide real-time feedback to the driving torque based on the externally set feedforward torque, so that the angular velocity of the rotary table and the BHA can be tracked and maintained at the desired speed, and the stick-slip vibration of the drill string system can be suppressed. The feedforward torque is a parameter related to the stick-slip motion characteristics of the open-loop system. It is set to avoid the bit getting stuck when the controller is initially opened. According to the analysis in Section 2, its value is the minimum torque of the bit breaking the sticky state:

$$u^* = T_{sb} = W_{ob} R_b u_{sb} \tag{4}$$

LQR is a kind of mature controller, it can make the system through less cost to obtain better comprehensive performance, simple and effective, has been widely used in engineering practice. Firstly, the torsional vibratio n equation of drill string in Equation (1) is linearized as follows:

$$\begin{cases} J_r \ddot{\varphi}_r = T_m - c(\dot{\varphi}_r - \dot{\varphi}_b) - k(\varphi_r - \varphi_b) - c_r \dot{\varphi}_r \\ J_b \ddot{\varphi}_b = c(\dot{\varphi}_r - \dot{\varphi}_b) + k(\varphi_r - \varphi_b) - c_b \dot{\varphi}_b \end{cases} \tag{5}$$

Fetch state variable $x = \begin{bmatrix} \dot{\varphi}_r & \varphi_r & \dot{\varphi}_b & \varphi_b \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ Then $\dot{x}$ can be expressed as:

$$\begin{cases} \dot{x}_1 = \dfrac{1}{J_r}\left[T_m - c(x_1 - x_3) - k(x_2 - x_4) - c_r x_1\right] \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = \dfrac{1}{J_b}\left[c(x_1 - x_3) + k(x_2 - x_4) - c_b x_3\right] \\ \dot{x}_4 = x_3 \end{cases} \tag{6}$$

With control torque T_m input, the equation of state of drill string system can be expressed as:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{7}$$

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Where $A=\begin{bmatrix} -\dfrac{c+c_r}{J_r} & -\dfrac{k}{J_r} & \dfrac{c}{J_r} & \dfrac{k}{J_r} \\ 1 & 0 & 0 & 0 \\ \dfrac{c}{J_b} & \dfrac{k}{J_b} & -\dfrac{c+c_b}{J_b} & -\dfrac{k}{J_b} \\ 0 & 0 & 1 & 0 \end{bmatrix}$; $B=\begin{bmatrix} \dfrac{1}{J_r} \\ 0 \\ 0 \\ 0 \end{bmatrix}$

According to Matlab controllability discriminant program, the state matrix (A, B) is full rank. Therefore, the system is completely controllable and can be designed with LQR controller. By referring to the method in reference [15], the LQR controller is designed to suppress drill string stick-slip vibration. Its goal is to obtain the optimal control law based on state feedback, in the sense of minimum performance index J, so that the state vector is closer to the desired value. According to the above analysis, the performance functional of the active control system of drill string stick-slip vibration is established as follows:

$$J(u)=\frac{1}{2}\int_0^\infty \left(x^T(t)Qx(t)+u^T(t)Ru(t)\right)dt \tag{8}$$

Where Q is the weighting coefficient matrix of the state variable and a semi-positive definite matrix, which is used to reflect the relative importance of each controlled state; R is the weighting coefficient matrix of the input variables and is a positive definite matrix.

According to expert experience, the relevant parameters of Q and R matrices are determined by trial and error method as follows:

$$\begin{cases} Q=\begin{bmatrix} 1e13 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 2.1005e12 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \\ R=1e5 \end{cases} \tag{9}$$

The task of the state regulator is to keep each component of the system state close to the desired state without consuming too much energy when the system state deviates from the equilibrium state for any reason. Therefore, the control of the stick-slip vibration of the drill string system is mainly to enable the rotary table and the BHA to track the desired rotational speed in real time, and maintain a small torsion Angle of the drill string, which is reflected in the performance index is to reduce the angular velocity and angular displacement deviation of the rotary table as much as possible, and the required active control torque should be within the maximum power range of the rotary table driver. Therefore, let the deviation between the state vector x and the desired state vector x_d be:

$$x_e=x-x_d=\begin{bmatrix} x_{e1} & x_{e2} & x_{e3} & x_{e4} \end{bmatrix}^T=\begin{bmatrix} \dot{\varphi}_r-\omega_d & \varphi_r-\omega_d\cdot t & \dot{\varphi}_b-\dot{\varphi}_r & \varphi_b-\varphi_r \end{bmatrix}^T \tag{10}$$

According to the extreme value principle, the optimal control rate of LQR controller is:

$$u(t)=-Kx_e(t) \tag{11}$$

Where, K is the optimal feedback gain matrix, and $x_e(t)$ is the real-time deviation of the state vector.

The optimal feedback gain matrix is solved as follows:

$$K=-R^{-1}B^TP \tag{12}$$

The constant positive definite matrix P is solved by $Riccati$ equation, specifically as follows:

$$PA+A^TP-PBR^{-1}B^TP+Q=0 \tag{13}$$

To sum up, the real-time input torque of the controlled drill string system is:

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$$
\begin{aligned}
U(t) &= u^* + u(t) \\
&= u^* + \left[-Kx_e(t)\right] \\
&= u^* + \left[-k_1\left(\dot{\varphi}_r - \omega_d\right) - k_2\left(\varphi_r - \omega_d \cdot t\right) - k_3\left(\dot{\varphi}_b - \dot{\varphi}_r\right) - k_4\left(\varphi_b - \varphi_r\right)\right]
\end{aligned}
\tag{14}
$$

Where $K = [k_1 \quad k_2 \quad k_3 \quad k_4]$ is the optimal feedback gain matrix; ω_d is the desired speed value set by the system, $\omega_d > 0$; $u*$ is the set feedforward torque used to overcome the viscous state at the initial stage of the bit; $U(t)$ is the real-time output of the total torque of the controller.

4. Numerical simulation results and analysis

4.1 Simulation Parameters

In order to verify the reliability and effectiveness of the above LQR controller, simulation analysis and verification were carried out in Simulink. The relevant simulation parameters are shown in Table 1. Relevant parameters of drill string system are selected from reference [10], and parameters of Karnopp friction model are selected from reference [15]. In order to prevent abnormal operation, the output torque of the top drive was limited to $0\sim25\ kN\cdot m$, and the initial desired angular velocity was $\omega_d = 12\ rad/s$.

Table 1: Drill string system model parameters

Parameter description	Numerical value	Parameter description	Numerical value
Equivalent rotational inertia of the rotary table J_r	$2122\ kg\cdot m^2$	Bit radius R_b	0.156 m
Equivalent rotational inertia of the BHA J_b	$374\ kg\cdot m^2$	static friction coefficient μ_{sb}	0.8
Equivalent torsional stiffness of the drill string system k	$473\ (N\cdot m)/rad$	Coulomb friction coefficient μ_{cb}	0.5
Equivalent torsional damping of the drill string system c	$23.2\ (N\cdot m\cdot s)/rad$	Boundary layer thickness $D_v > 0$	1e-6
Viscous damping of the rotary table c_r	$425\ (N\cdot m\cdot s)/rad$	Empirical constant γ_b	0.9
Viscous damping of the BHA c_b	$50\ (N\cdot m\cdot s)/rad$	Empirical constant v_f	1
Weight on bit W_{ob}	97347 N		

4.2 Analysis of stick-slip vibration characteristics of drilling string system in original state

Figure 3 shows the change curve of the angular velocity of the rotary table and the BHA with time under the original state of the drill string system. It can be found that in the original state, the angular velocity vibration of the BHA is large, the adjustment time is long, the angular velocity of the rotary table is poor, the angular velocity of the rotary table and the BHA presents periodic changes, and the BHA presents intermittent high-speed sliding and viscous static periodic oscillation motion. It is also proved that the nonlinear dynamic model established in this paper can accurately simulate the stick-slip vibration phenomenon of drill string system and be applied to the research of the stick-slip vibration control method of drill string.

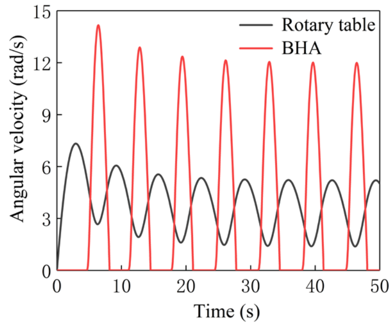

Figure 3: Dynamic characteristics of drill string system in its original state

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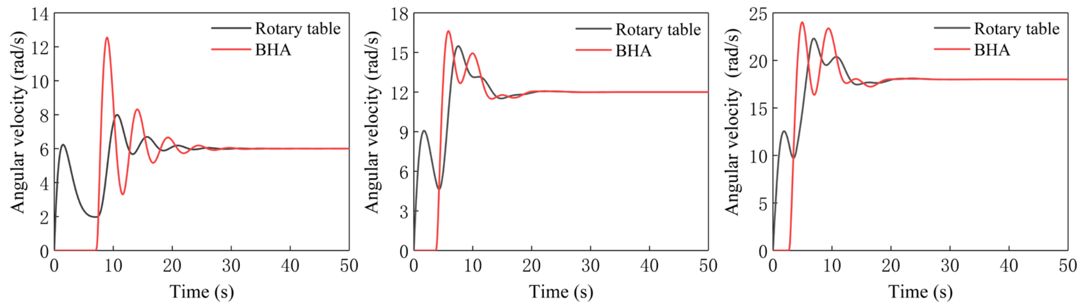
4.3 Controller Performance Analysis under parameter perturbation

In order to verify the effectiveness and robustness of the controller under parameter perturbation, whether the system instability caused by parameter changes can be regulated, the controller's performance of suppressing drill string stick-slip vibration under the perturbation of desired speed, feedforward torque, start time and bit weight parameters was analyzed.

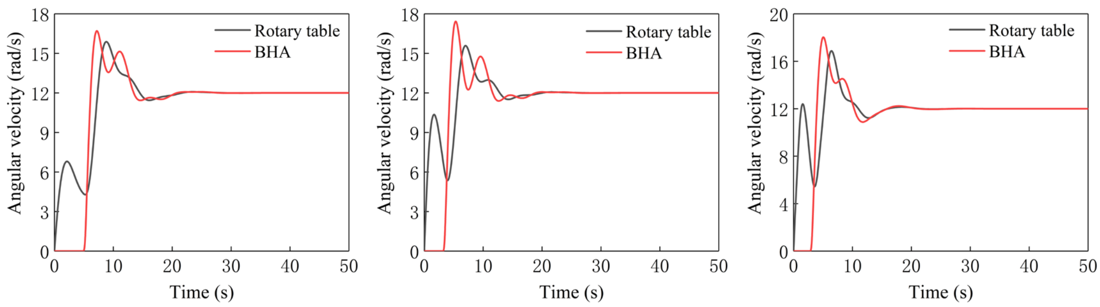
Figure 4 shows the change curve of the angular velocity of the drill head and the BHA with time at different desired speeds under the control condition. It can be found that the controller has good tracking performance and steady-state error for different desired velocities. The angular velocities of drill head and BHA can track and maintain the desired velocities well within 35s. At a smaller desired speed, the overshoot of the controller is larger. With the increase of the desired speed, the viscosity time of the BHA start-up stage shows a downward trend and the adjustment time is shorter.



(a) $6\ rad/s$ (b) $12\ rad/s$ (c) $18\ rad/s$

Figure 4: Viscosity reduction effect of controllers at different desired speeds

Figure 5 shows the change curve of the angular velocity of drillhead and BHA with time given different feedforward torques under the control state, and Figure 6 shows the change curve of the feedback torque of LQR controller with time. It can be found that with the increase of feedforward torque, the viscous time in the starting stage of the BHA is shorter, which also indicates that the increase of driving torque can suppress the stick-slip vibration of the BHA. The controller can effectively cope with the feedforward torque parameter perturbation. The angular velocity of both drill head and BHA is well tracked and maintained at the desired speed, but its control torque will change greatly. When the driving torque is not enough to drive the BHA, the controller will timely supplement the torque, and when the torque is large, the controller will timely reduce the torque. Moreover, the control torque tends to be flat with the suppression of stick-slip vibration. At the desired speed of $12\ rad/s$, the drive torque output by feedforward torque and feedback control torque 00 is about 13.272 $kN \cdot m$.



(a) $T_0 = 0\ kN \cdot m$ (b) $T_0 = 15\ kN \cdot m$ (c) $T_0 = 25\ kN \cdot m$

Figure 5: Viscosity reduction effect of controller under different feedforward torques

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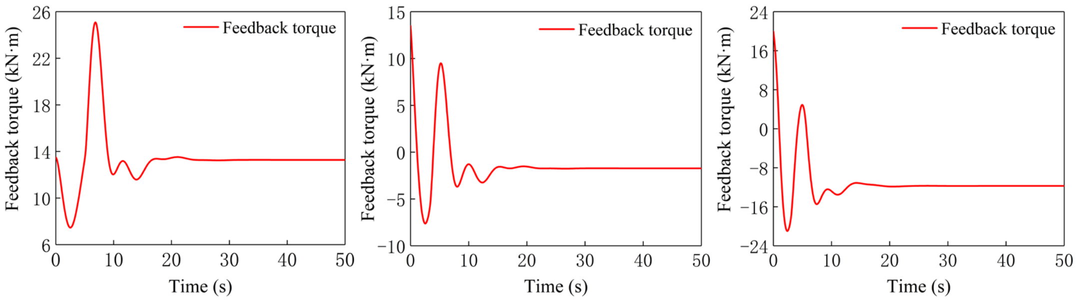
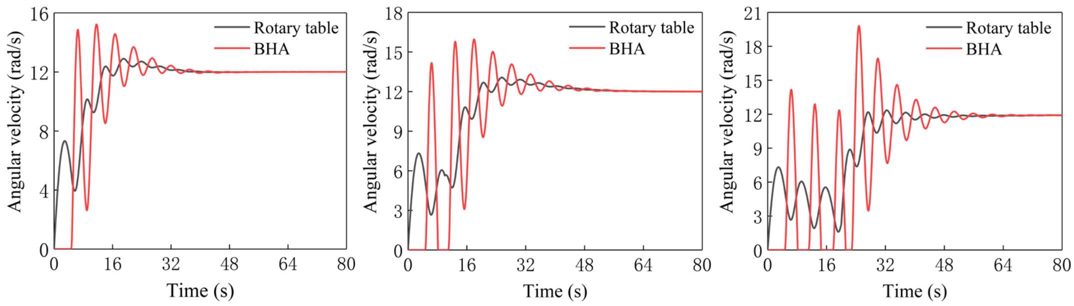


(a) $T_0 = 0\ kN \cdot m$ (b) $T_0 = 15\ kN \cdot m$ (c) $T_0 = 25\ kN \cdot m$

Figure 6: Feedback torques of the controller under different feedforward torques

Figure 7 shows the change curve of the angular velocity of the drill head and the BHA with time at different starting times under the control state. It can be found that starting the controller during operation can also effectively inhibit the stick-slip vibration of drill string, and the angular velocity of rotary table and BHA can be well tracked and maintained at the desired speed, but the speed regulation time, vibration frequency and vibration peak of rotary table and BHA will increase. In addition, if the control is started when the energy storage of the drill string is high, the overshoot of the angular velocity of the drill string assembly will be higher.



(a)Startup time $t_0 = 5\ s$ (b)Startup time $t_0 = 10\ s$ (c)Startup time $t_0 = 20\ s$

Figure 7: Viscosity reduction effect of controller under different startup time

Figure 8 shows the change curve of the angular velocity of the drillhead and the BHA over time with different bit weight input under the control condition, which is further consistent with the problems of sudden increase of bit weight and sudden drop of bit weight when the bit rebound occurs frequently in actual drilling. It can be found that: Under the perturbation of bit weight parameters, the controller can also effectively suppress the stick-slip vibration of the drill string, maintain good steady-state characteristics and overcome nonlinear friction. The angular velocities of both the rotary table and the BHA can be stabilized in a short time and track the desired velocity. However, with the increase of bit weight, the friction torque increase, resulting in a sudden drop in the angular velocities of the rotary table and the BHA. The vibration of the drillstring system becomes more intense and viscous behavior increases, and vice versa when weight on bit drops.

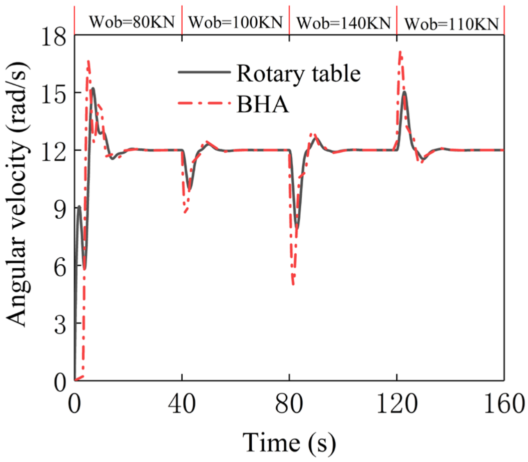


Figure 8: Viscosity reduction effects of controllers under different bit weights

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5. Conclusion

Based on the nonlinear two-degree-of-freedom drill string system dynamics model, an optimal LQR controller was designed, and its robustness and effectiveness under parameter perturbation were studied by simulation. It can be found that:

(1) Under different desired speeds or feedforward torques, the controller can effectively realize real-time feedback of the drive torque, showing good tracking perfor mance and steady-state error. The angular velocities of both the drill head and the BHA can track well and keep at the desired speed within 35s. However, when tracking a small desired speed, the overshoot of the controller is slightly larger.

(2) Start the control during operation, and the controller can effectively inhibit the stick-slip vibration of the drill string, but the speed regulation time, vibration frequency and vibration peak of the rotary table and the BHA will increase. Moreover, the controller can deal with the perturbed parameters of bit weight effectively, maintain good steady-state characteristics and overcome nonlinear friction.

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