

Resource Availability and Sex Ratios in Lampreys

Wang Hanfei, Yun Jiabao, Zheng Tianyi

School of Electrical and Information Engineering, Beijing University of Civil Engineering and Architecture, Beijing, 102616, China

Abstract: As a critical consumer layer and key regulator of ecosystem function, the sex ratio of lampreys can be influenced by environmental resource conditions. To investigate the impact of this biological adaptive strategy on both their population and the broader ecosystem, we have developed multiple models to address four primary issues. Firstly, we utilized the Lotka-Volterra model as a foundation. Given the significant differences in environmental factor requirements and competitive dynamics between genders, we extended the model to incorporate gender-specific influences. This allows for a more precise exploration of changes in the food chain and the interaction mechanisms between interspecific competition and sex ratio. Secondly, we designed an innovative population dynamics model that incorporates nest competition. Unlike traditional single-population dynamic models, this approach considers numerous related factors, such as external environmental conditions and population competition, making it more suitable for analyzing lamprey populations and their ecological impacts. This enhancement significantly improves the accuracy of the results. Thirdly, the TOPSIS model was employed to evaluate and predict the optimal solution for lampreys within their ecological context. By proportionally dividing and scoring various datasets, the model identifies the balance point of the optimal solution, enabling analysis of how lamprey size affects the advantages and disadvantages of their population dynamics. Finally, to more accurately assess whether changes in the sex ratio of lampreys benefit other species, we utilized a cellular automaton model. This simulation accounts for key environmental factors, including food availability and competition intensity, providing a comprehensive understanding of potential cross-species interactions.

Keywords: Multispecies Lotka-Volterra; Population Dynamics Model; TOPSIS; Cellular Automata

1. Introduction

1.1 Background

In nature, the sex ratio of many species is not fixed, but is influenced by environmental conditions. This change in adaptive sex ratio plays an important role in ecosystems, especially for some important species such as lampreys. Lamprey is a species that lives in lake and marine habitats, and its sex ratio may change depending on the external environment.

The sex ratio of lampreys is affected by the growth rate of the larval stage, which in turn is affected by the food supply. In environments with a low food availability, the proportion of males can reach about 78% of the population, while in environments where food is more readily available, males account for about 56% of the population. This suggests that the sex ratio of lamprey populations is somewhat flexible and can be changed depending on the availability of resources.

Lampreys play a complex role in the ecosystem. In some lake habitats, they are seen as parasites that have a significant impact on the ecosystem, while in other areas, lampreys are also an important food source. Therefore, understanding the impact of lamprey abundance on sex ratio and ecosystems is important for ecosystem management and conservation.

1.2 Restatement of the Problem

In order to gain insight into the resulting interactions in the ecosystem. We are concerned about the sex ratio of marine lampreys and their dependence on local conditions. This type of eel lives in the habitat of lakes or oceans and spawns in rivers. One of the key tasks is to study the ability of a species to adjust its sex ratio in the face of changes in resource availability, and to gain insight into the impact of this phenomenon on ecosystems.

2. Predict changes in lamprey populations

2.1 A predator model that does not take gender into account

Limiting resource is a angular stone of the interactions between species in ecosystems such as competition, prey-predators and foodchain systems^[1]. As shown in Figure 1. A model modeled in MATLAB that simulates predator-bait dynamics uses the Lotka-Volterra equations to describe the interaction between predator and bait. In the second half of the code, the Lotka Volterra equation is redefined and different initial conditions are set, and then the phase trajectories between the number of bait and predators are plotted for each set of initial conditions using a for loop. Finally, the plot function is used to plot the phase trajectory plot. As shown in Figure 2.

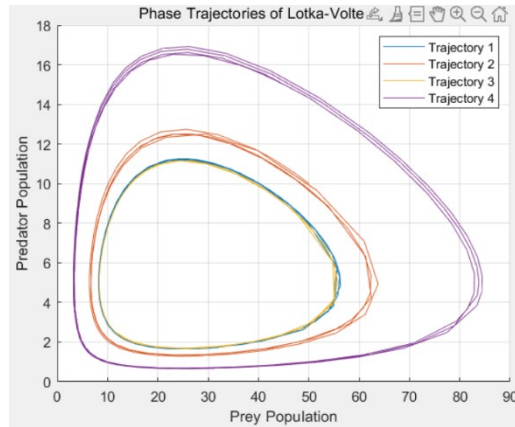


Figure 1 Lotka-Volterra Predator-Prey

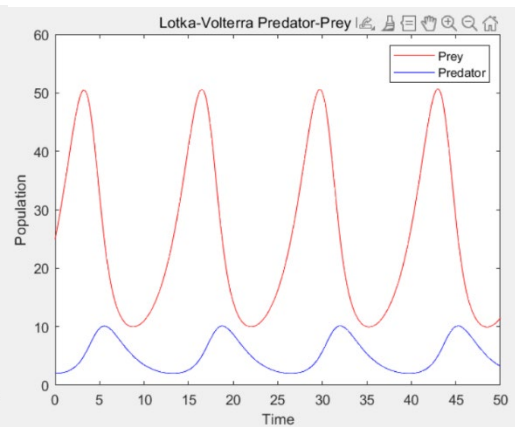


Figure 2 Phase Trajectories of Lotka-Volte

2.2 A predator model that take gender into account

The growth of each population is restricted by the Logistic law. Two populations depend on the same resource to survive, and the greater the number of these two populations, the less resources are available, thus reducing the population growth rate of the species. In addition, with the increase of the number of population groups, the increment of each population will have a certain limiting effect on the change of the other population^[2]. The above sex-neutral predator model shows the interaction between predator and prey through graphs. On this basis, we optimize the model and build a predator model considering sex ratio. Firstly, the following model equation is established:

$$\begin{aligned} \frac{dprey}{dt} &= \alpha_{prey} \times prey - \beta_{prey} \times prey \times predator \\ \frac{dlamprey}{dx} &= \beta_{prey} \times prey \times predator - \gamma_{predator} \times lamprey - \beta_{lamprey_lamprey} \times lamprey \times predator \\ \frac{dlamprey}{dx} &= \delta_{lamprey} \times lamprey \times predator - \gamma_{lamprey_predator} \times predator \end{aligned} \quad (1)$$

Table 1: Symbols used by the Predator model

Symbol	Description
α_{prey}	The natural growth rate of the bait
β_{prey}	The probability that the bait will be preyed
$\gamma_{predator}$	Natural mortality of predators
$\delta_{predator}$	Predation success rate
$\alpha_{lamprey}$	The natural growth rate of lampreys
$\beta_{lamprey_prey}$	Efficiency of preying on lampreys
$\gamma_{lamprey}$	Natural mortality of lampreys
$\delta_{lamprey}$	Increased survival rates of lampreys due to predation
$\beta_{lamprey_lamprey}$	Lamprey predator's efficiency in preying on lampreys
$\gamma_{lamprey_predator}$	Natural mortality of lamprey predators
$\delta_{predator}$	The survival rate of lamprey predators increased by eating lampreys

In the cycle, the parameters of the model are first initialized according to the different male ratios of

lampreys (i). As shown in Table 1. Subsequently, the model equations were numerically solved using ode45 solver, and the variation of prey, lampreys and lampreys prey populations over time was obtained.

By constructing the time-varying parameter differential equation model which includes the dynamic changes of lampreys and their prey populations, the numerical simulation analysis was carried out in MATLAB environment. As shown in Figure 3. In particular, we focused on the ecosystem effects of a variable called the proportion of male lampreys, and explored its long-term effects by adjusting the time-dependent parameters representing this proportion in the model. In the simulation process, with the increase of time in the model, the number of lampreys males increased, which led to the enhancement of predation pressure on the bottom feeding population, resulting in a gradual decrease in its number. The resulting simulation results graphically show how changes in the lamprey population interact with the feeding population over time, revealing that the lower structure of the lamprey chain tends to be unstable. This imbalance in predator-prey relationships can reduce the stability of the entire ecosystem and may be accompanied by a decline in biodiversity, especially for other species that are closely related to prey populations.

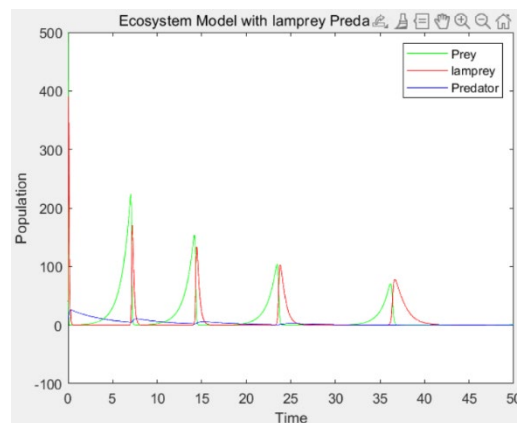


Figure 3 Ecosystem Model with lamprey Preda

2.3 Competitor model considering gender

To study female and male competition in lampreys, we developed a peer competitor model that takes into account sex. The competitor model is a mathematical model used to describe and predict the competitive relationship between biological populations. In ecology, the competitor model is often used to study the competition between different species in the same ecological niche and their impact on resource utilization and population size. We used the odeint function from the SciPy library in Python to solve the differential equation model, and the NumPy and Matplotlib libraries to plot the results of the model. First, we set up the following equation:

$$\frac{dN_1}{dt} = r_1 \times N_1 \times \left[1 - \frac{N_1 + \alpha_{12} \times (b \times a \times N_{2-male} + c \times (1-a) \times N_{2-female})}{K_1} \right] \quad (2)$$

$$\frac{dN_{2-male}}{dt} = r_{2-male} \times a \times N_2 \times \left[1 - \frac{a \times N_2 + \alpha_{21} \times N_1}{K_{2-male}} \right] \quad (3)$$

$$\frac{dN_{2-female}}{dt} = r_{2-female} \times [(1-a) \times N_2] \times \left\{ 1 - \frac{[(1-a) \times N_2] + \alpha_{21} \times N_1}{K_{2-female}} \right\} \quad (4)$$

$$\frac{dN_2}{dt} = \frac{dN_{2-male}}{dt} + \frac{dN_{2-female}}{dt} \quad (5)$$

The model uses np.linspace to generate time point t, which is used to simulate a time range from 0 to 100. Then odeint function is used to solve the differential equation numerically, and the variation of

the number of three species with time in a given time range is obtained. The results of the model were drawn using the Matplotlib library, and the variation curves of the number of three species over time were plotted respectively through the plt.plot function. As shown in Figure 4.

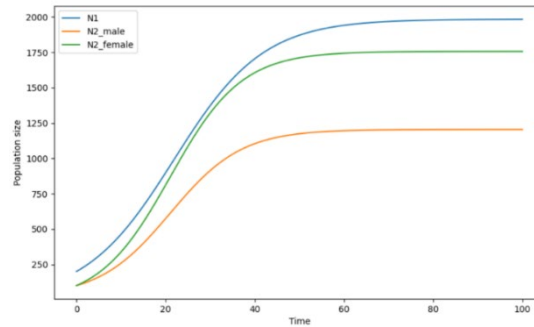


Figure 4 Curve of the number of species over time

3. Population dynamics model

In order to study the effect of the sex ratio of lampreys on offspring reproduction, we established a population dynamic model. We created two arrays to store the number of people each day in low food supply and high food supply environments. The number of people in low and high food supply environments for each day was calculated using cycles, based on difference equations and environmental conditions. The following model equations are established:

$$\begin{aligned}\frac{dN}{dt} &= 0.3 \left(1 + \frac{S_1}{100}\right) \left(1 - \frac{N}{S_2}\right) \frac{N}{0.02t} \\ \frac{dN}{dt} &= 0.1 \left(1 + \frac{S_1}{100}\right) \left(1 - \frac{N}{S_2}\right) \frac{N}{0.02t}\end{aligned}\quad (6)$$

3.1 High and low food supply and number of species

A list deduction was used to calculate the sex ratio for each day in both low-food and high-food environments. matplotlib was used to draw a graph of species numbers over time, as shown in Figure 5. showing species growth in low and high food supply environments respectively, and the resulting graph is as follows. As can be seen from the figure above, the number of species under high food supply is higher than that under low food supply.

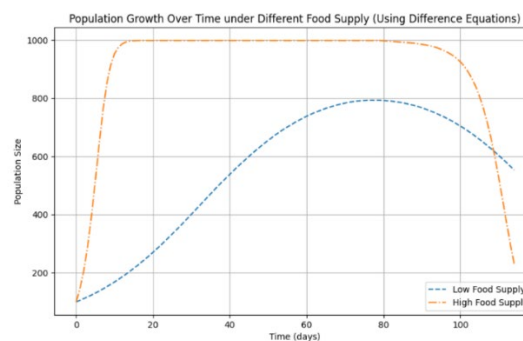


Figure 5 Population Growth Over Time under Different Food Supply

3.2 Sex ratio and food availability

These models take into account the effects of food supply, carrying capacity and time, and simulate population growth under different food supply conditions through different growth rates^[3]. Next, we initialized the environment for two different food supply conditions and set the time frame and initial population size. Finally, we calculated the sex ratio of populations under low and high food availability conditions, and plotted to show the changes in population size and sex ratio over time under different food availability conditions. As shown in Figure 6.

It can be seen from the above figure that the curve of high food supply is always above the curve of low food supply, which means that the male ratio under low food supply conditions is always lower than that under high food supply conditions. The red line represents 56% male proportion, and the green line represents 78% male proportion. The model fits the curve in line with the problem conditions.

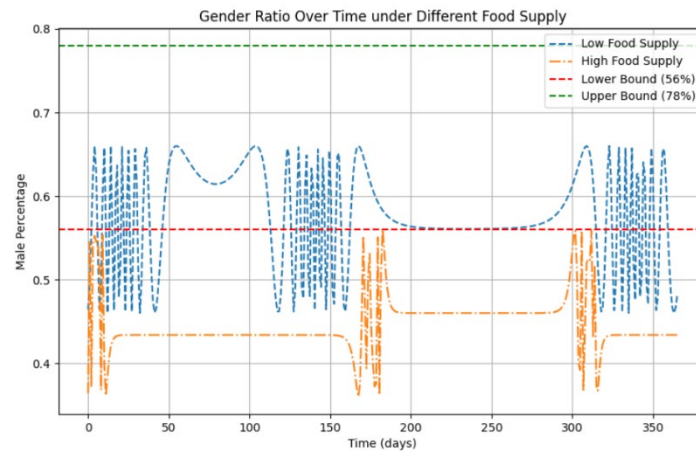


Figure 6 Gender Ratio Over Time under Different Food Supply

4. Technique for Order of Preference by Similarity to Ideal Solution

4.1 Model structure and establishment

In the multi-attribute, we respectively considered the effects of reproductive rate, adaptability to environment (temperature, humidity, food quantity, geographical location and other multiple factors) and drug resistance [4] on lampreys, and conducted comprehensive measurement by randomly establishing four groups of lampreys indexes, A, B, C and D, and finally found the relative proximity in the data.

Four groups of lampreys A, B, C and D were randomly established with the data:

1) Lampreys: When the temperature is 10, the average survival rate is 95.29%; With the lowest food quantity 10, the proportion of males can reach the highest value 78%; When the dose rate was 10%, 6 lived together.

2) Lampreys: When the temperature was 14, the average survival rate was 95.93%; The amount of food is 40, the proportion of males can reach 70%; When the dose rate was 25%, 4 lived together.

3) Lampreys: When the temperature was 18, the average survival rate was 96.49%; When the amount of food is 70, the proportion of males can reach 64%; When the dose rate was 50%, 4 lived together.

4) Lampreys: When the temperature was 22, the average survival rate was 54.14%; When the food quantity is 100, the proportion of males can reach the lowest value of 56%. When the dose rate was 100%, 10 coexisted.

$$\begin{bmatrix} 78,95.29,6 \\ 70,95.93,4 \\ 64,96.49,4 \\ 56,54.14,10 \end{bmatrix}$$

Analyze the data content, analyze the four groups of data and establish the data matrix. We first normalize the data, calculate the norm for each indicator, and for each column (j), calculate its Euclidean norm. A corresponding indicator norm is determined, and each indicator is divided by this norm to obtain a value that is relatively conducive to comparison, which is conducive to making a fair and reasonable analysis of data of different dimensions, and is also conducive to the results reflected in the optimal solution because lampreys are affected by various factors.

4.2 Results and analysis

Through the search and integration of parameters, the most influential factors on lampreys population and the corresponding ecosystem were found. In the process of mathematical modeling, the normalized and weighted data are first output to Figure 7 to ensure the correctness of the data before calculation, and also reduce the possible errors in a wide range of optimal solutions. Through addition and subtraction of different data types, the three groups of data that can best reflect the relative changes of lampreys are found, and the accuracy and correctness of the optimal solutions are also guaranteed. The optimization process time and data search time are greatly improved. It can be found that lampreys have a good effect on some chemical drugs currently used by people. Drugs are very harmful to lampreys population, and to a certain extent, they help human beings in species invasion and maintaining ecosystem balance. ^[5]

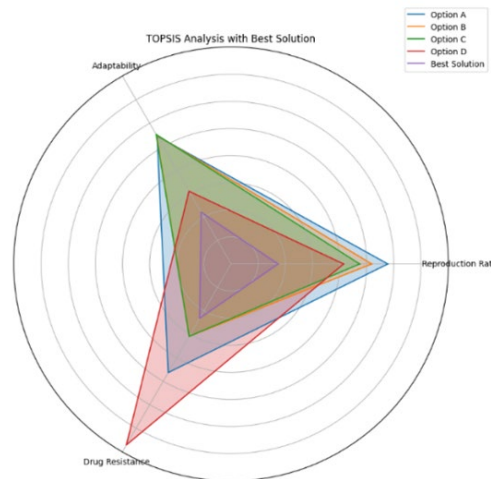


Figure 7 Radar chart

5. Cellular automata model

5.1 Model structure and establishment

In the cellular automata model, the changes of cells in the time dimension are discrete, and the data is not convenient to be counted within the set boundary^[6]. Therefore, random variables and the demand for food supply are used to set the initial value of population number. Then, the model obtains the state of each cell node at the next time according to the set conversion rules through the initial state of cells. In determining the cellular status, we considered the influence of gender (in terms of competition and food supply demand) and different species, and finally determined that the cellular states in the space, State = 0,1,2,3, respectively represent empty space (not occupied), occupied by female lampreys, occupied by male lampreys, and occupied by other races (for example, parasites or preying on lampreys populations). As shown in Figure 8.

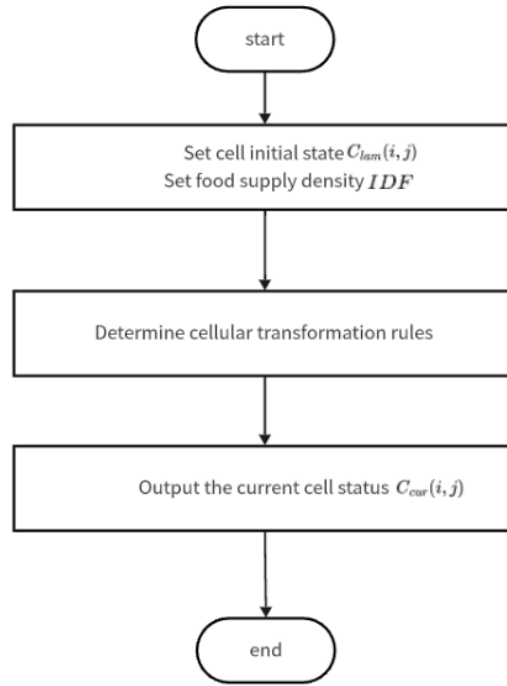


Figure 8 Cellular automaton model flowchart

By analyzing the population diffusion model of female, male and other competitive races of lampreys, the calculation formula of model variables can be listed through the definition of parameters and variables. Given a uniformly distributed random variable U on the interval $[0,1]$, its probability density function is $f_u(u)=1$, When u belongs to $[0,1]$ the initial food density range is defined as

$$F_{min}=IDF*(1-FDR) \text{ and } F_{max}=IDF*(1+FDR) \quad (7)$$

$$\begin{aligned}
 C_{lam}(i, j) &= 0, u < 0.1 \\
 1, 0.1 &\leq u < F_{min} \\
 2, F_{min} &\leq u < F_{max} \\
 3, F_{max} &\leq u < 1
 \end{aligned} \quad (8)$$

In cellular automata, the state of each cell at the next moment depends on its own state and the state of its neighbors. On the grid, the neighbors of each cell are the same size. Here we consider that in a two-dimensional grid, there are several possibilities: in addition to the cell itself, there are four neighboring cells in the east, south, west, and north (VonNeumann neighbors), or five cells above and four diagonal cells in the northeast, southeast, southwest, and northwest^[7] (Mare neighbors).

Considering competition intensity, sex transition, food supply and other factors. such as the rules of competition intensity: the number and competition intensity of male lampreys nearby, the data of female lampreys, and the competition intensity of female lampreys are numerically compared with the number of other races in the neighborhood, and the next state of the current cellular state of 0, 1, 2, 3 is analyzed. As shown in Table 2.

Table 2: Symbols used by the Cellular automata

Symbol	Description
neighbors	Current neighbors
empty_count	The number of vacant lots in the neighborhood
female_lamprey_count	Number of female lampreys in the neighborhood
male_lamprey_count	Number of male lampreys in neighbors
other_species_counts	Number of other races in the neighborhood
food_density	Food supply in the current neighboring area

5.2 Results and analysis

Since the parameters could not be searched and determined, different parameter combinations of each parameter were input into the model, and the trial-and-error method was used to simulate the distribution of *Spartina alterniflora* under these parameter combinations. The model parameters after calibration were analyzed as the reference values, and the model parameters were increased or decreased by a certain percentage one by one. By simulating the diffusion process of *Spartina alterniflora*, the simulated number of *Spartina alterniflora* was counted. Compared with the corresponding *Spartina alterniflora* quantity reference value, the parameters that had the greatest influence on the simulation results were found out. In the image drawn by the software, the proportion of female lampreys was higher than that of female lampreys in Figure 1 at the beginning, and then stabilized in Figure 9 at the end. Since the self-growth of food supply was not considered in the network, the food supply in the model would decrease, and it could also be seen that the number of male lampreys increased significantly compared with that of other races. Figure 10 shows a comparative case.



Figure 9 Initial spatio-temporal

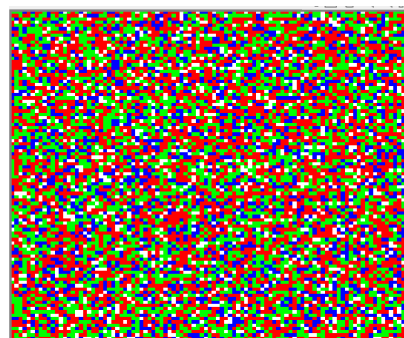


Figure 10 Spatiotemporal diagram diagram of stability

6. Strengths and Weaknesses

6.1 Strengths

1) In this ecological problem, the Lotka-Volterra model is applied to describe the interaction between species through simple mathematical equations, revealing the basic dynamic mechanism of population change, which has great advantages in predicting the change trend of the two interacting populations over time.

2) The cellular automata model is used to simulate the changing process of time and space in continuous space and analyze the evolution of ecology, which can be displayed intuitively through graphics.

6.2 Weaknesses

The established models still do not fully consider the environmental parameters, and rarely consider the effects of regional spatial structure, resource fluctuations, migration and adaptive evolution, and climate change such as extreme weather.

References

- [1] Ba I, Ndiaye P I, Ndao M, et al. An Extension of Two Species Lotka-Volterra Competition Model[J]. *Biomath Communications*, 2021, 8(2): 2112171-2112171.
- [2] Si Shoukui, Sun Xijing. *Mathematical Modeling Algorithm and Application* [M]. National Defense Industry Press, 2011.
- [3] Yin Haoran et al. "Preparation of life history Specimens of Japanese Lampreys." [J]. *Laboratory Science* 26.4(2023):35-39.
- [4] Hanson, Jr. Lee H. and Patrick J. Manion. "Chemosterilization of the sea lamprey (*Petromyzon marinus*)."[G]. *CTIT technical reports series* (1978): 0-15.
- [5] Meeuwig M H. Identification of Larval Pacific Lampreys (*Lampetra tridentata*), River Lampreys (*L. ayresii*) and Western Brook Lampreys (*L. richardsoni*) and Thermal Requirements of Early Life

History Stages of Lampreys : Annual Report 2001.[J].Office of Scientific & Technical Information Technical Reports, 2003.DOI:10.2172/821798.

[6] Zou Jie. *Research on the traffic flow model based on cellular automata [D]. Shandong University of Science and Technology, 2008.*

[7] Bo Xingye. *Construction of the microbial growth prediction model and the simulation study of the growth process [D]. Shanxi Agricultural University, 2022.DOI:10.27285/d.cnki.gsxnu. 2022.000849.*