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A review of the application of neural networks in finite element methods in the field of solid mechanics

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Abstract: *In this paper, the application of artificial neural networks in finite element methods in the field of solid mechanics is reviewed, including element stiffness matrix operation, node displacement operation based on the global stiffness matrix, and optimization of finite element calculation result correction. Several studies have shown that artificial neural networks have certain advantages in solving difficult computation problems and parameter corrections in finite element methods in solid mechanics; compared with traditional methods, the method incorporating artificial neural networks is efficient and easy to compute with powerful parallel computing capabilities, and has excellent prospects.*

Keywords: *Artificial neural network; Finite element method; Model correction; Stiffness matrix; Solid mechanics*

1. Introduction

In the design of various engineering cases, it has become customary to use multiple calculation tools to analyze mechanical properties. Among them, the finite element [1], as a numerical method for analyzing structures and continuum, is represented by the Computer Aided Engineering (CAE) software, which can accurately give the numerical solutions of structural deformation, stress, vibration, and other stress problems under external load. Therefore, it can greatly improve the efficiency and quality of engineering design. However, in recent years, with the increasing complexity of the problem, engineers will spend a lot of time in modeling, pre-processing, solving, post-processing and other processes. In addition, when engineers encounter similar engineering cases in the past, they often have to repeat this process several times, even if it is only a small change in geometric shape or set parameters, and can not make better use of knowledge such as previously completed simulation results.

In recent years, with the continuous development of computer science, machine learning and data science have become the research focus of scholars. Machine learning and data science, through training many samples, can find the high-dimensional functional relationship between the input and output of data samples. At present, they have been successfully applied to the research fields of natural sciences such as chemistry and biology. Among them, deep learning is one of the methods with multiple representation levels in machine learning, and it is an extension of neural network algorithm in machine learning. It combines simple but nonlinear modules, each of which can convert a simple level representation to a higher, more abstract level representation [2]. Compared with traditional strict rules, deep learning can handle more complex mapping relationships between many parameters. It is used to classify, transform and generate images and audio. With the development of new learning architecture and the improvement of computer ability, recent progress shows that machine learning is increasingly used in the field of computational engineering. In recent years, these technologies have also attracted the attention of relevant scholars in the field of computational mechanics, such as fluid simulation [3, 4], design and topology optimization [5, 6], nonlinear dynamics analysis [7], autonomous vehicle [8, 9], molecular dynamics simulation [10-12], quantum learning [13-15], etc. With the maturity of artificial intelligence technologies such as data science and machine learning, it is possible to integrate mechanical analysis and numerical simulation methods with deep learning.

Finite element methods used in solid mechanics is one of the most common methods to solve practical mechanical problems. Firstly, the problem area is divided into several small parts. After that, the unsolved field function is approximated by the values of each element node and the corresponding interpolation function, and then global algebraic equation is assembled to solve the unknown quantity by substituting the element interpolation into the variational principle.[16] However, finite element methods still have some shortcomings in practical applications—they require a long time to calculate the element matrix, which makes it difficult to meet the requirements of real-time calculations; for complex models, the final solution operation is difficult; and there is a lack of efficient and stable correction methods for the final model correction. ANN has a powerful mapping capability, which can be applied to solve mechanical models by building suitable models, and its ability to compute in parallel and in real time can effectively improve the computational speed of finite element algorithms. ANN has been widely used in the field of finite element calculations in solid mechanics and there is still great potential for development.[17]

An Artificial Neural Network (ANN) is an information processing system that simulates the neurons of the human brain, with functions such as memory, classification, and recognition.[18] ANN generally consists of an input layer, an output layer, and several hidden layers composed of neurons. After the data of the input layer is processed by neurons, the output that has a causal relationship with the input data can be simulated according to certain rules.

The finite element method has excellent universality for the problem. As long as there are certain analysis models and conditions, no matter how complex the geometry of the original problem is, it can be solved. But for complex problems, the number of units will be very large, the solution time will be correspondingly long, and the requirements for computing equipment will exceed the affordable range of personal computers. This has become an obstacle for engineers in non-large enterprises or colleges to solve project problems. Even for simple structural problems, if you want to optimize the structure, every time you change the structure, you need to redo the whole process from modeling to result analysis, which greatly reduces the calculation efficiency.

The emergence of machine learning may become a shortcut to solving such problems. After parameter optimization, the neural network can quickly give the output results after the input changes, which can reduce the difficulty and time of solving complex problems, and improve the efficiency of solving optimization problems. At present, machine learning has made remarkable achievements in the field of image recognition and image processing. Its principle is to learn the deep features or change modes of a class of data on the basis of a large number of known samples. The samples provided by the finite element method can be completely generated by the computer through independent calculation. Compared with the image recognition, it is much easier to collect samples in reality. In addition, the finite element results can be expressed in matrix form, which is very consistent with the input requirements of machine learning.

2. Applications of ANN algorithm to optimize finite element calculations

2.1. Optimizations of element stiffness matrix operations

ln practical finite element calculations, the element stiffness matrix is calculated based on the shape, size, orientation, and modulus of elasticity of the elements. The stiffness matrix represents the correspondence between nodal displacements and nodal forces and is an important parameter in the finite element calculation. In engineering problems such as elastoplasticity, crack expansion, and structural optimization, the stiffness matrixes are not constant matrixes and need to be constantly adjusted according to the situation.

The neural network model trained based on a deep learning algorithm can calculate the element stiffness matrix in real time based on the relative coordinates of the elements. This calculation method is time-consuming in the preliminary training process, however, after the training has reached the required accuracy, the trained model can be used for efficient real-time computing.

Sun et al. used BP networks to calculate the element stiffness matrix of triangular-shaped elements and used the relative coordinates of the nodes and the corresponding calculated element stiffness matrix as the training data.[19] The trained neural network model had an error of less than 2% in calculating the element stiffness matrix, confirming the feasibility of BP networks in calculating the element stiffness matrix. Jia et al. proposed the use of convolutional neural networks to solve the element stiffness matrix.[20] By convolving the relative coordinates of nodes of quadrilateral planar elements and extracting their feature relations, a neural network model with an error of less than 0.001 in the calculation result was successfully trained.

This method of using a neural network to calculate the element stiffness matrix uses the element stiffness matrix of a conventional finite element calculation as the training data, so there are inevitably errors that make it difficult to achieve the accuracy of conventional finite element algorithms. In addition, when calculating the element stiffness matrix of a spatial three-dimensional element by means of a neural

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network, the difficulty of the simulation increases significantly. Therefore, this method has not yet been applied on a large scale.

2.2. Optimization of nodal displacement operations based on the global stiffness matrix

According to the principle of minimum potential energy and the variational principle, the solution of the nodal displacement can be transformed into the quadratic optimization problem described by the following equation.[21]

$$\min_{x\in\Omega}\Pi=\frac{1}{2}\boldsymbol{\delta}^{\boldsymbol{T}}\boldsymbol{K}\boldsymbol{\delta}-\boldsymbol{q}^{\boldsymbol{T}}\boldsymbol{\delta} \tag{1}$$

$$\text{s. t. } \mathbf{A}\boldsymbol{\delta}=\overline{\boldsymbol{\delta}} \quad (\text{in } S_u) \tag{2}$$

In which

$$\boldsymbol{K}=\sum_{e=1}^{n}\int_{\Omega}(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{D}\boldsymbol{B})\ d\Omega \tag{3}$$

$$\boldsymbol{q}=\sum_{e=1}^{n}\int_{\Omega}(\boldsymbol{N}^{\boldsymbol{T}}b)\ d\Omega+\sum_{e=1}^{n}\int_{\Omega}(\boldsymbol{N}^{\boldsymbol{T}}\overline{q})\ d\Omega \tag{4}$$

δ is the nodal displacement array, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{B}$ is the strain matrix, $\boldsymbol{D}$ is the principal structure matrix, $\boldsymbol{N}$ is the form function, $\boldsymbol{q}$ is the nodal force array, and $\mathbf{A}$ is the constraint matrix.

The Hopfield neural network, with the given neuron weights, can make the network energy function gradually decrease in the operation, and finally reach the stability point of the system, that is, the minimum value of the energy function.[22] By setting a suitable energy function that corresponds to this quadratic optimization problem, the minimum value of the total potential energy can be solved through the Hopfield neural network, which, by the principle of minimum potential energy, corresponds to the node displacement as the unique solution. Compared to the traditional finite element method, this method uses a finite element hardware circuit to ensure accuracy and the solving process is a hardware operation, which can be done within the circuit time constants and is much faster.

Since the global stiffness matrix is non-positive, to avoid convergence to local minima, Sun et al. used the "simulated annealing method" to help achieve global convergence while using the Hopfield neural network to solve the problem.[23] The simulated annealing method searches in the direction of the deteriorating solution with a certain probability while performing the optimization operation, which extending the search range. However, the simulated annealing method has certain problems, as it cannot complete the global search with full probability, and is less universal, requiring the reformulation of relevant parameters for different problems. Therefore, Huang et al. proposed an improved solution by referring to the manual correction of the stiffness matrix, firstly excluding the effects of displacement constraints and corresponding constrained counterforce and transforming the global stiffness matrix into a positive definite matrix.[24] In this way, the convergence solution and the global minimum value can be obtained. This method not only eliminates the shortcomings of the simulated annealing method but also avoids the mismatch between the energy function of the neural network and the function to be optimized in the old method, reducing the circuit requirements and improving calculation accuracy.

2.3. Application of corrections to optimized finite element calculation results

In practical engineering, based on known material or design parameters, the model features that are calculated using the finite element method often have errors when compared with the actual measured data, because the material or design data cannot be completely consistent with the actual data. This is where the finite element model needs to be corrected.

The use of deep learning neural networks is a good approach to the correction problem. As the design parameters are mapped to the structural features, the structural features are used as the input to the neural network and the design parameters as the output, and the neural network is trained to simulate the corresponding design parameters based on the actual features measured. This method significantly reduces the computational difficulty of the design parameter correction method, while retaining the advantages of the design parameter method.

Fei et al. used the traditional design parameter type method and the neural network method to correct the design parameters of the GARTEUR aircraft model—the mean square error of the design parameters before correction was 17.88%, and the mean square error after correction was 2.89% for the traditional design parameter type method and 1.00% for the neural network method.[25] It is evident that the neural network method is better than the traditional design parameter-based method in terms of accuracy. Du et al. modified the simplified finite element model of the bridge tower using this method.[26] Before the correction, the first to fourth order vibration frequencies of the simplified model had a large error with the measured data, which exceeded that of the solid cell model. After training with the data calculated using the simplified model, the correction of the neural network caused the error between the calculated and measured frequencies of the simplified model to be significantly reduced, even much lower than that of the uncorrected solid cell model. The results show that with the help of neural network correction, the simplified model constructed by using spatial rod units can also simulate the structural feature quantities better when calculating complex large structures, and the process is greatly simplified and can be widely used, and it has good application prospects.

3. Conclusion and Outlook

Artificial neural networks are widely used in finite element methods in the field of solid mechanics, and cover aspects such as the calculation of the element stiffness matrix and nodal displacement based on the global stiffness matrix in the calculation process. They also have mature applications in the field of parameter correction in post-processing. The traditional finite element calculation methods are optimized in terms of computational speed or accuracy by combining with artificial neural networks. Future research on the application of artificial neural networks to finite element calculations in solid mechanics has the following aspects:

(1) Stable online application. Although the artificial intelligence neural networks have high accuracy and efficiency, the neural network needs to be trained in advance, they lack effective online learning methods, and are still generally used for offline preview and for theoretical research.

(2) Practical computational feasibility. Most applications of neural networks in finite element methods in the field of solid mechanics are in the theoretical stage—they have only proven theoretical feasibility, and no large-scale applications have been made yet. Whether these theoretical models can remain stable in practical calculations and become a sufficient alternative to traditional finite element algorithms for better solutions requires further research and practical improvement.

(3) Software tools and intelligent hardware development. Finite element algorithms have become one of the most commonly used algorithms in solid mechanics calculations due to the availability of a large number of mature and powerful software such as Ansys. The use of ANN technology to improve existing computational software, or to develop new software and hardware, can enable a wide number of technicians and researchers to perform calculations quickly and facilitate data analysis and applications.

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