ISSN 2616-5902 Vol. 7, Issue 6: 15-22, DOI: 10.25236/AJBM.2025.070603

# Price Volatility and Risk Measurement of EU Carbon Futures under Structural Mutation

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***Abstract:*** *The EU's carbon emissions trading system, including carbon futures, has experienced significant price swings since its launch, affecting emission goals and economic activities. Existing research on this volatility has primarily focused on the first two phases, overlooking the third phase and new patterns emerging in the fourth phase (since 2021). This study utilizes phase 3–4 EUA futures data, applying the Bai-Perron test to identify structural breaks and incorporating them into a GARCH model to analyze volatility and market risks. Results indicate that the price series has breakpoints; the adjusted GARCH model improves its fit and reveals price dynamics, although risk measurement may overfit.*

***Keywords:*** *Carbon Futures; Structural Mutations; Price Fluctuations; Risk Measurement*

## 1. Introduction

In March 1994, the United Nations Framework Convention on Climate Change (UNFCCC/FCCC), the first international convention to address GHG emissions and the impacts of global warming, entered into force. Subsequently, agreements such as the Kyoto Protocol, Copenhagen Accord, and Paris Agreement were adopted to clarify emission reduction responsibilities and introduce carbon quota trading mechanisms for market-driven emission reductions and optimal carbon resource allocation. The EU established the EU Emissions Trading System (EU ETS) to achieve emission targets and introduced carbon futures. The rapid development of the EU carbon futures market has attracted extensive academic attention regarding price volatility and risk measurement.

Scholarly research on carbon prices focuses on three key areas: (1) Carbon futures prices display significant time-varying volatility, clustering, thick-tailed distributions, and jumps. Zhang et al. (2011) [1]used GED-GARCH models to demonstrate that EU carbon futures returns in Phases I–II exhibited time-varying, clustering, and fat-tailed characteristics, while Huang (2020)[2] confirmed volatility clustering in the EUA/CER series through GARCH. Studies by Guo et al. (2011)[3], Gao et al. (2018)[4], and Di Pan et al. (2022)[5] further indicate that major economic events induce structural breaks, leading to nonlinear price dynamics. (2) Carbon spot price volatility is characterized by leptokurtosis, thick tails, strong clustering, long memory, asymmetric effects, and structural breaks. Wu et al. (2012)[6] and Yuan et al. (2015)[7] employed CARCH models to find that EU spot prices exhibited leptokurtic-thick-tailed distributions with significant leverage effects (negative news impacts exceeding positive ones). Daskalakis et al. (2009)[8] observed similar jumps and non-stationary features in EU carbon spot and futures prices. (3) Carbon futures and spot prices are highly interdependent, driven by common market fundamentals (supply and demand for emission rights) and macroeconomic/policy factors. Jiang et al. (2015)[9] found that EU Phase II futures and spot markets both demonstrated leptokurtic-thick-tailed, autocorrelated, and volatility-clustered characteristics using GARCH-EVT-VaR models, while Liu et al. (2020) [10]confirmed their high interdependence. Therefore, studying the characteristics of carbon futures prices is crucial for predicting market trends and managing risks. In research methodology, scholars commonly utilize GARCH models to analyze carbon price volatility and VaR for risk measurement[11]. Early studies employed GARCH-EVT-VaR frameworks but overlooked price jump characteristics. Subsequent research integrated structural breaks and jump information into models like GARCH-MIDAS-JUMP, demonstrating improved predictive accuracy[5][12]. Studies utilizing the Bai-Perron test further showed that GARCH models with breakpoint integration effectively reduce volatility persistence.

Existing studies on EU carbon price volatility focus predominantly on Phases I–II, overlooking Phase III and Phase IV (2021–present) dynamics. Using the Bai-Perron test, this study identifies structural breaks in EU carbon futures prices (Phases III–IV) and incorporates them as dummy variables into a

ISSN 2616-5902 Vol. 7, Issue 6: 15-22, DOI: 10.25236/AJBM.2025.070603

GARCH model with a TARCH term for asymmetry. The model analyzes volatility and VaR-based risk under structural breaks, revealing new-phase patterns to inform China's carbon futures market development.

## 2. Analysis of the Volatility of Carbon Futures Prices

### *2.1 Factors Influencing Carbon Prices*

Carbon prices are driven by carbon allowance supply-demand dynamics. Long-term factors (e.g., energy prices, macroeconomics) influence prices gradually via supply and demand without altering volatility patterns. In contrast, major events (e.g., EU ETS reforms, crises) cause abrupt supply-demand shifts, triggering price jumps, volatility changes, and nonlinearity. Thus, incorporating structural breakpoints is essential for identifying true price fluctuation patterns.

### *2.2 The Impact of EU Carbon Futures Price Volatility on China's Carbon Market*

In recent years, China has actively engaged in global environmental governance and climate mitigation by committing to carbon peaking before 2030 and carbon neutrality before 2060. Since launching nationwide carbon trading in July 2021, the market has attracted 2,257 key enterprises by late 2023, covering 5.1 billion tons of annual $CO_2$ emissions, with a trading volume of 440 million tons and a turnover of 24.9 billion yuan. The January 2024 issuance of the State Council's Interim Regulations on Carbon Emission Trading Administration formalized China's carbon market legal framework, although the market remains in its infancy, with efficiency lagging behind mature markets[13]. Drawing on international experience, introducing carbon financial derivatives like carbon futures is critical to building a robust carbon trading system.

China's carbon market exhibits price volatility characterized by sharp peaks, thick tails, strong clustering, and asymmetric effects, as confirmed by Fan et al. (2023)[14] using an ARMA-GARCH model. These features mirror those of the EU carbon market, although the EU shows more pronounced clustering and volatility. Sun (2017) [15]finds a long-term equilibrium and mutual guidance between the two markets, with the EU exerting a stronger spillover effect. As a mature market, the EU's policies and dynamics offer critical insights for China's carbon market development, enabling a better understanding of market mechanisms, risk management, and enhanced Sino-EU collaboration to address global climate challenges and advance carbon market integration.

Therefore, studying EU carbon futures volatility is of paramount importance for China's carbon market development, offering three primary advantages: (1) Enhanced market transparency and understanding of volatility mechanisms; (2) Improved risk management through volatility-driven hedging strategies; (3) Policy optimization and market regulation informed by the EU's regulatory precedents, particularly in optimizing allocation mechanisms and stabilization policies. Collectively, this research offers actionable insights for China's carbon market development, focusing on derivative design, risk tool innovation, and regulatory refinement.

## 3. GARCH Model Construction and Risk Measurement under Sudden Structural Change

The Bai-Perron structural break test is a multiple structural break test that identifies and quantifies regime shifts in regression models via stepwise or global optimization, eliminating the need for predefined breakpoint locations. This approach systematically detects structural changes in data, outperforming traditional methods like the Chow test, which requires predefined breakpoints and detects only single breaks in flexibility and versatility. While the GARCH model is a standard tool for analyzing time-series volatility, its assumption of symmetric volatility limits its ability to capture real-world dynamics. Given the significant impact of structural breaks on carbon futures volatility[12], this study employs the Bai-Perron test to identify breakpoints in pricing dynamics and incorporates them as dummy variables into a GARCH model. A TARCH term is added to capture asymmetric leverage effects, forming a modified GARCH framework. Finally, VaR is used to measure market risk, with detailed modeling steps as follows:

### *3.1 Bai-Perron structural mutation point test*

The Multiple Unknown Structural Break Test proposed by Bai and Perron (1998, 2003)[16] extends

ISSN 2616-5902 Vol. 7, Issue 6: 15-22, DOI: 10.25236/AJBM.2025.070603

traditional tests such as Chow and Quandt-Andrews to detect multiple unobserved structural breaks in time-series data without prespecifying breakpoint locations. It employs UDmax/WDmax statistics to identify breakpoints by comparing goodness-of-fit across candidate partitions, rejecting the null hypothesis of no breaks when statistics exceed critical values, and utilizes BIC/LWZ information criteria to balance model fit and complexity for selecting the optimal number of breaks via criterion minimization.This framework systematically integrates statistical inference and model selection to characterize regime transitions in time-series data, offering a robust approach for detecting endogenous structural changes.

Consider a multiple linear regression model with m mutation points:

$$y_t = x_t'\beta + z_t'\delta_j + u_t \tag{1}$$

Where $t=T_{j-1}+1,\ldots,T_j$, $j=1,2,\ldots,m+1$, $y_t$ is the dependent variable observed at time t, $x_t(p\times 1)$ and $z_t(q\times 1)$ are the vectors of covariates, $\beta$ and $\delta_j$ are the corresponding vectors of the coefficients, $u_t$ is the residual term, and the structural mutation point ( $T_1, T_2,. .T_m$ ) are unknown (where $T_0=0$ and $T_{m+1}=T$).

Multiple linear regression models can be represented in matrix form:

$$Y = X\beta + \bar{Z}\delta + U \tag{2}$$

Where $Y=(y_1,\ldots,y_T)'$, $X=(x_1,\ldots,x_T)'$, $U=(u_1,\ldots,u_T)'$, $\delta=(\delta'_1,\delta'_2\ldots,\delta'_{m+1})'$, and $\bar{Z}$ is a diagonal matrix , $\bar{Z} = \mathrm{diag}(Z_1,\ldots,Z_{m+1})$,$Z_i = (z_{T_{i-1}+1},\ldots,z_{T_i})'$ .

Bai-Perron (1998, 2003) based on the principle of least squares, for each partition ($T_1,T_2, . .T_m$), estimates of $\beta$ and $\delta_j$ are obtained by minimizing the residual sum of squares:

$$S_T(T_1,T_2,..T_m) = \min\sum_{i=1}^{m+1}\sum_{T_{i-1}+1}^{T_i}[y_t - x_t'\beta + z_t'\delta_i]^2 \tag{3}$$

The partition in which the sum of squares of the residuals $S_T(T_1, T_2,\ldots, T_m)$ is minimized is the segmentation resulting from the estimation:

$$\left(\widehat{T_1},\widehat{T_2},\ldots\widehat{T_m}\right) = \mathrm{argmin}S_T(T_1,T_2,..T_m) \tag{4}$$

Finally, a significance test is conducted to determine whether structural breaks exist in the data-generating process.

### *3.2 Modified GARCH-VaR models*

#### *3.2.1 GARCH Model under Structural Breaks*

The identified structural mutation points are treated as dummy variables and incorporated into the GARCH model with a TARCH term:

$$y_t = \mathbf{x_t'\beta} + \varepsilon_t \tag{5}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 + \sum_{k=1}^{p}\lambda_k\varepsilon_{t-k}^2\cdot\mathbf{1}(\varepsilon_{t-k}>0) + \gamma_1 D_1 + \cdots + \gamma_n D_n \tag{6}$$

$$\varepsilon_t = v_t\sqrt{\sigma_t^2} \tag{7}$$

Where $x_t'$ is the carbon price series, $\sigma^2_t$ is the conditional variance, $v_t$ is an independently and identically distributed random variable, $v_t$ and $\sigma_t$ are independent of each other, $\mathbf{1}(\cdot)$ is an indicative function, i.e., it takes the value of 1 when $\varepsilon_{(t-k)} > 0$ and 0 otherwise, $\varepsilon^2_{(t-k)}\cdot\mathbf{1}(\varepsilon_{(t-k)} > 0)$ is the TARCH term, $\lambda_k$ is the coefficient of the TARCH term that captures the effect of positive past returns on current volatility, $D_l=1,\ldots, D_n=1$ are dummy variables, $t_l$ denotes the first structural mutation point, $D_l=1$ when $t>t_l$ and 0 otherwise, and $t_n$ denotes the nth structural mutation point, $D_n = 1$ when $t > t_{(n)}$ and 0 otherwise.

#### *3.2.2 Calculation of VaR value*

VaR (Value at Risk) is the maximum possible loss of a financial asset or portfolio of securities under normal market fluctuations[17], which is given by the formula:

$$Prob(\Delta P > -VaR) = 1 - \alpha \tag{8}$$

Where Prob denotes the probability; $\Delta P$ denotes the value of the change in the value of the asset portfolio; $\alpha$ is the confidence level, which is usually taken as 90%, 95%, and 99% confidence level. In this paper, the VaR value is calculated based on the conditional heteroskedasticity calculated by the GARCH model with the following formula:

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$$VaR = V_0 Z_\alpha \sigma \quad (9)$$

Where $V_0$ is the initial value of the asset, σ is the time-varying standard deviation obtained from the estimation of the GARCH model, and $Z_\alpha$ is the quantile of the generalized error distribution at the confidence level α.

## 4. Empirical Analysis of Data and Models

### *4.1 Sample and Data*

The EU Emissions Trading System (EU-ETS), operational since 2005, has advanced through four developmental phases: Phase I (2005–2007) pioneered carbon pricing with high volatility and allowance oversupply; Phase II (2008–2012) stabilized pricing amid market maturation; Phase III (2013–2020) introduced institutional reforms (e.g., Market Stability Reserve, MSR) to curtail supply and drive steady price growth; and Phase IV (2021–2030) has seen normalized trading with record-high carbon prices. While prior studies primarily focus on Phases I–II, Phase III remains under-researched, and Phase IV's emerging volatility patterns are scarcely analyzed. To address this gap, this study utilizes Phase III–IV EU Allowance (EUA) futures settlement prices from the WIND database (2013–01–01 to 2024–07–01, excluding holidays/missing data, N = 2,968) to examine price dynamics and quantify risks in the latest market stages. Descriptive statistics of EUA futures settlement prices were first performed, as shown in Table 1:

*Table 1: Descriptive Statistics*

| price series | average value | standard deviation | skewness | kurtosis | J-B test (p-value) |
|---|---|---|---|---|---|
| EUAF | 30.006 | 29.622 | 0.912 | 2.272 | 0.0000 |

The EUA futures price series exhibits a mean of 30.006 and a standard deviation of 29.622, reflecting substantial volatility. With a skewness of 0.912 (right-skewed) and kurtosis of 2.272 (thicker tails than normal distribution), the Jarque-Bera (J-B) test statistic of 477.1 ($p < 0.01$) rejects the null hypothesis of normality. These results confirm a non-normal, right-skewed distribution with pronounced thick tails, suggesting a higher likelihood of large positive price deviations and elevated tail risks of extreme price movements.

### *4.2 Discussion and Analysis of Empirical Results*

#### *4.2.1 Bai-Perron structural mutation point test*

Using Eviews 13.0, structural breakpoints in the EUA futures price series were identified via the Bai-Perron test. UDmax and WDmax statistics exceeded 5% significance critical values, rejecting the null hypothesis of no breaks and confirming at least one breakpoint. While an F-test initially suggested five breaks, UDmax/WDmax methods indicated three. The LWZ information criterion, minimizing model complexity, validated three optimal breakpoints occurring on March 20, 2018; March 21, 2020; and December 1, 2021. These divide the sample period into four subintervals with distinct volatility regimes, the specific test results are shown in Table 2 and Table 3.

*Table 2: B-P structural mutation site test*

| Number of mutation points | F-statistic | Scaled F-statistics | Weighted F-statistic | Critical Value |
|---|---|---|---|---|
| 1* | 3418.281 | 6836.562 | 6836.562 | 11.47 |
| 2* | 3911.789 | 7823.577 | 9203.737 | 9.75 |
| 3* | 4214.701 | 8429.402 | 11565.22 | 8.36 |
| 4* | 3247.434 | 6494.869 | 10361.08 | 7.19 |
| 5* | 2623.968 | 5247.937 | 10289.54 | 5.85 |
| UDMax* | | 8429.402 | UDMax Critical Value** | 11.70 |
| WDMax* | | 11565.22 | WDMax Critical Value** | 12.81 |

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*Table 3: B-P structural mutation point test*

| Number of mutation points | Number of coefficients | sum of squares of the residuals | log-likelihood function | BIC* Criterion | LWZ* Criterion |
|---|---|---|---|---|---|
| 0 | 2 | 594839.1 | -12077.21 | 5.305790 | 5.316934 |
| 1 | 5 | 179898.2 | -10302.49 | 4.117972 | 4.145832 |
| 2 | 8 | 94679.93 | -9349.928 | 3.484165 | 3.528742 |
| 3 | 11 | 62330.46 | -8729.539 | 3.074195 | 3.135490 |
| 4 | 14 | 60804.68 | -8692.761 | 3.057494 | 3.135508 |
| 5 | 17 | 60226.17 | -8678.574 | 3.056016 | 3.150750 |

To corroborate the association between structural breakpoints and real-world events, economic events spanning January 1, 2013, to July 1, 2024, were collated. Analysis revealed that each breakpoint coincided with major economic events, systematically summarized in Table 4. Event data were sourced from platforms including the Shanghai Environmental Energy Trading Network and the EU Carbon Emission Exchange.

*Table 4: Mutation Points Corresponding to Major Events*

| breakpoint time | major economic event |
|---|---|
| March 20, 2018 | To address unexpected demand shocks and allowance surpluses, the EU established the Carbon Market Stabilization Reserve (MSR) in 2018, which stabilizes carbon prices by withdrawing a percentage of annual auctioned allowances into the MSR or releasing them from the MSR based on market supply-demand dynamics. In November 2018, the European Commission unveiled the A Clean Planet for All strategy, committing to net-zero emissions by 2050—the first major economy to propose a mid-century carbon neutrality goal. |
| March 21, 2020 | The 2020 COVID-19 pandemic severely disrupted global economic activity, reducing energy demand and carbon emissions during lockdowns, which impacted carbon market prices and trading volumes. |
| December 1, 2021 | The EU ETS entered Phase IV in 2021, increasing the annual total discount factor from 1.74% (Phase III) to 2.20% and revising free allowance baselines for manufacturing. Legislation raised the 2030 emission reduction target from 40% to 55%, with the EC's carbon market reform proposing a 61% reduction target for covered sectors (up from 43%). The Fit for 55 legislative package (July 14, 2021) adjusted carbon trading, carbon leakage measures, energy efficiency, alternative fuels, and climate financing to align with stricter targets. |

The collection of relevant policies and economic events reveals that the neighborhoods of the breakpoints all correspond to relatively significant economic events, suggesting that significant economic events lead to structural changes in carbon prices.

#### *4.2.2 The pattern of carbon price fluctuations under sudden structural change*

Before conducting the GARCH operation, the carbon futures price series is tested for smoothness using the ADF test, a correlation test, and the ARCH effect, the stability test results are shown in Table 5.

*Table 5: Stability (ADF) Tests*

| Time series | t-statistic | Critical Value | | | P-value |
|---|---|---|---|---|---|
| | | 1% | 5% | 10% | |
| EUAF | -0.814 | -3.430 | -2.860 | -2.570 | 0.8150 |
| dEUAF | -56.796 | -3.430 | -2.860 | -2.570 | 0.0000 |

The ADF test for EUA futures prices yields a t-statistic of -0.814 ($p=0.8150$), exceeding the 10% critical value, confirming non-stationarity. First-order differencing transforms the series to stationarity, with a t-statistic of -56.796 ($p \to 0$). Correlation tests show significant autocorrelation (Q-statistics > critical values, $p<0.05$), and an ARCH-LM test ($p=5$) confirms ARCH effects. A GARCH(1,1) model is specified, incorporating a TARCH term for asymmetric news effects and structural breakpoints as dummy variables in the variance equation to analyze their impact on volatility patterns.

The parameter comparison between the original and modified GARCH models is shown in Table 6. The results indicate that introducing structural mutation points significantly improves the model fit.

*Table 6: Comparative analysis of GARCH models*

| parameters | GARCH(1,1) | Modified GARCH(1,1) |
|---|---|---|
| α | 0.0777117 | 0.0911477 |
| β | 0.8932788 | 0.8620849 |
| α+ β | 0.9709905 | 0.9532326 |
| Log-likelihood | -2029.261 | -2011.142 |

In the model, α measures ARCH effect strength (past residual impact on current volatility), and β represents GARCH effect strength (lagged volatility influence). The log likelihood quantifies model fit, with higher values indicating better alignment. Satisfying stationarity conditions ($\alpha, \beta > 0$; $\alpha + \beta < 1$), residual heteroskedasticity tests confirm no remaining effects, validating model specification. The modified model shows reduced volatility persistence ($\alpha + \beta = 0.9532$ vs. 0.9710) and improved fit (higher log likelihood), demonstrating that incorporating structural breakpoints enhances modeling accuracy.

### *4.2.3 Calculation of VaR value*

By substituting parameter estimates from both the GARCH(1,1) and modified GARCH(1,1) models into Eq. (9), VaR values for the EUA futures price series at varying confidence levels are calculated. Given the series' right-skewed, thick-tailed characteristics, VaR analysis employs the t-distribution to account for non-normal return dynamics, with results summarized in the following table 7:

*Table 7: Analysis of VaR values for the distribution of t*

| | GARCH(1,1) | | | Modified GARCH(1,1) | | |
|---|---|---|---|---|---|---|
| confidence level | 90% | 95% | 99% | 90% | 95% | 99% |
| quartile | 1.44 | 1.94 | 3.14 | 1.44 | 1.94 | 3.14 |
| average value | 1.18282 | 1.596405 | 2.581835 | 1.185986 | 1.600677 | 2.588745 |
| standard deviation | 1.226909 | 1.65591 | 2.678071 | 1.205184 | 1.626588 | 2.63065 |
| maximum values | 7.974971 | 10.7635 | 17.4076 | 7.665285 | 10.34553 | 16.73162 |
| minimum value | 0.116378 | 0.157070 | 0.254026 | 0.130479 | 0.176102 | 0.284807 |
| Failure days | 158 | 80 | 14 | 166 | 81 | 48 |
| failure rate | 5.32% | 2.70% | 0.47% | 5.59% | 2.73% | 0.54% |

At 90%, 95%, and 99% confidence levels under the t-distribution, the modified GARCH(1,1) model yields slightly larger mean VaR values but smaller standard deviations compared to the original GARCH(1,1) model. This indicates the modified model eliminates pseudo persistence in price fluctuations, providing a more accurate representation of carbon market risks and mitigating potential losses from underestimated risk. However, the modified model exhibits a higher failure rate than the original model.

### *4.3 Analysis of carbon futures price volatility by interval*

To further characterize the volatility dynamics of carbon futures prices, this study decomposes the series into four subintervals based on identified breakpoints: January 1, 2013–March 20, 2018; March 20, 2018–March 21, 2020; March 21, 2020–December 1, 2021; and December 1, 2021–July 1, 2024. Price data from each subinterval is incorporated into a GARCH model, and VaR values under the t-distribution are calculated separately, with results presented in the following table 8:

*Table 8: VaR results for different intervals of the EUA futures price series*

| | confidence level | Interval 1 | Interval 2 | Interval 3 | Interval 4 |
|---|---|---|---|---|---|
| VaR mean | 90% | 0.2438638 | 0.9108968 | 1.467177 | 3.060987 |
| | 95% | 0.3291331 | 1.229401 | 1.98019 | 4.131291 |
| | 99% | 0.5323007 | 1.988286 | 3.202524 | 6.681458 |
| failure rate | 90% | 5.20% | 5.61% | 4.33% | 6.30% |
| | 95% | 2.75% | 2.90% | 2.28% | 3.15% |
| | 99% | 0.45% | 0.58% | 0.46% | 0.60% |

Analysis of EUA futures VaR across four structural breakpoint-defined subintervals indicates time-evolving risk escalation at 90%, 95%, and 99% confidence levels, with failure rates rising in all intervals except the third. The first subinterval (2013–2018) exhibited low carbon prices (2.7–11.52 EUR/t$CO_2$eq,

ISSN 2616-5902 Vol. 7, Issue 6: 15-22, DOI: 10.25236/AJBM.2025.070603

$\mu$=6, $\sigma$=1.48) due to allowance oversupply, characterized by underdeveloped trading and stable dynamics.The second (2018–2020) experienced MSR-driven price growth (12.31–29.77 EUR/t$CO_2$eq, $\mu$=21.92, $\sigma$=4.31) disrupted by COVID-19-induced volatility. The third (2020–2021) showed post-pandemic recovery and EU ETS Phase IV policies, driving prices to 15.45–76.96 EUR/t$CO_2$eq ($\mu$=39.16, $\sigma$=15.22) through higher discount factors. The final subinterval (2021–2024) reflected market maturity (50.52–97.67 EUR/t$CO_2$eq, $\mu$=78.64, $\sigma$=9.97), with reduced volatility but increased tail risks from global uncertainties limiting traditional GARCH effectiveness.

## 5. Conclusions and Policy Recommendations

Incorporating structural breakpoints into EUA futures volatility analysis enhances model fit by eliminating pseudo-persistence from carbon market structural shifts. However, the modified GARCH model exhibits higher risk measurement failure rates due to parameter complexity, overfitting to historical data, and reduced robustness in stable markets. Interval analysis shows that the standard GARCH fits the first three stable periods but underperforms in the fourth, primarily due to heightened global macro uncertainties exacerbating tail risks. Despite these increased failure rates, the modified model adapts better to evolving market dynamics—especially as global carbon policies tighten—mitigating tail risks and offering superior explanatory power in complex environments. To address overfitting, the study suggests excluding underdeveloped early-period data (2013–2018), selecting GARCH models based on market characteristics, validating breakpoint-incorporated models through backtesting, and regularly optimizing parameters.

Based on these conclusions, the following suggestions are made to promote the establishment and improvement of China's carbon trading market: first, encourage financial innovation by motivating financial institutions to develop carbon financial derivatives, such as carbon futures, that provide diversified risk management tools to meet market participants' hedging needs; second, foster international collaboration by actively engaging in global carbon market cooperation, adopting international best practices, aligning with global standards, and advancing cross-border coordination—including joint emission reduction target-setting, low-carbon technology diffusion, and mutual recognition of trading mechanisms—to build an efficient global carbon market system; and third, advocate for the application of the modified GARCH model in risk management, particularly under conditions of high market uncertainty and complex macroenvironments, where it better captures dynamic risk compared to traditional models.

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