

Low Complexity Hybrid Beamforming Methods for Single-User Millimeter Wave MIMO Systems

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ABSTRACT. Millimeter wave communication is an attractive new research area due to specialization for high symbol rate wireless system and increasingly serious spectrum congestion. The benefits of mmWave communication system include realizing high speed, low latency data transmission, offering higher bandwidth communication channels and so on. However, the signal processing is so different compared with low-frequency propagation, such as larger penetration losses, limited transmission range and lower diffraction in front of obstacles. To solve these problems mentioned, beamforming has been proposed. In this paper, multiple precoding implementations are introduced including full-digital as well as the hybrid one under the single-user condition. First, a digital system is constructed while the signal is processed using zero-forcing-Serial Interference Cancellation criterion (ZF-SIC) and Minimum Mean Square Error (MMSE)-Serial Interference Cancellation criterion (MMSE-SIC) respectively. Then, two hybrid beamforming (HBF) algorithms based on different criteria are introduced. We utilize the spatial structure of the millimeter wave channel to formulate the precoding/combining problem as a sparse re-construction problem and this hybrid precoding algorithm is applied to approach optimal unconstrained precoders and combiners. Subsequently, an energy efficient HBF method with sub-connected architecture is investigated, which is able to reduce the energy consumption and computation complexity. The results of all the algorithms are compared and the performance of different precoding methods is presented in terms of spectral efficiency and Signal to Noise Ratio (SNR).

KEYWORDS: Hybrid beamforming, Millimeter waves, MIMO system, Spatially sparse precoding, Energy efficiency

1. Introduction

With the rapid development in the field of wireless communication, the capacity requirements of the wireless transmission is increasing the data traffic and area spectral efficiency. Such high requirements challenge to conventional MIMO systems. Even with the help of a great number of hardware enhancement in the physical layer. The use of Millimeter Waves and massive multiple-input and multiple-output method and design can significantly increase the bandwidth, spectral efficiency and transmission quality as well as data transmission rate. It can overcome the frequency shortage. Such properties make it essential for the future wireless communication systems. However, there are still a few problems in terms of using millimeter wave systems such as having a greater signal attenuation, the deficiency in the distance of transmission, the lack of ability of diffraction and so on. As a result, beamforming technique is implemented in this paper to reduce the path loss problem. In the full digital part, the beamforming method is used in different precoding criterion to achieve the optimal precoding matrix. The data streams will be combined to get the output signals. However, in conventional MIMO architecture, each antenna requires a dedicated radio frequency (RF) chain (including DAC/ADCs, etc.), which leads to limitation both in space and power consumption. In order to reduce the complexity as well as the number of RF chains and increase the efficiency of the system, a single-user, fully connected hybrid precoding architecture with both digital and analog beamforming system model is then introduced. On the transmitter side, this paper plan to implement the optimization by using the spectral efficiency equation. This research managed to turn the optimization problem into a spectral efficiency equation based minimum mean square error (MMSE) problem and so achieved an optimal product of the baseband precoder and the RF precoder. While at the receiver side, the idea is to make the received signal as approximate as possible to the transmit signal. So this research used the method of calculating the minimum mean square error (MMSE) between the two side signal. The

algorithm will be illustrated in detail in the hybrid precoding investigation part.

This paper focuses its investigation on both the fully digital precoding methods and the hybrid precoding algorithm method, together with their simulation as well as comparison. The paper can be divided into the following structure:

The paper is constructed by several parts: Introduction Part, where the back ground information and the problem formulation are included in. System and Channel Model part introduce the basic models applied in this paper. For the third part, the algorithm performance between fully digital and hybrid beamforming are compared for single user system in terms of spectral efficiency with the variation of SNR (Signal to Noise Ratio). In the Simulation part, the paper constructs the graphs to show the property of each algorithm. Finally, the achievement obtained is summarizes in the Conclusion part.

Notation: This research use following notation throughout this paper: $\|\cdot\|_1$, $\|\cdot\|_2$ and $\|\cdot\|_F$ respectively denote the l_1 , l_2 norm of a vector and the Frobenius form of a matrix. Lower-case and upper-case boldface letters denote vectors and matrices, respectively. $(\cdot)^*$, $(\cdot)^{-1}$ and $|\cdot|$ denote the conjugate transpose, inversion and a determinant of a matrix, respectively. $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real and imaginary parts of a complex number, respectively. \mathbf{I} is the $N \times N$ unit matrix. Last, i.i.d denotes independent identically distribution.

2. System Model

The architecture of the hybrid system includes digital precoder, analog precoder implemented by phase shifters and RF chains. This research first considers a point to point single-user based narrow band millimeter wave MIMO system. In this system model, N_s number of data streams pass through the baseband precoder \mathbf{F}_{BB} . Transmitted by N_{RF} RF chains, the signal is then processed by the $N_t \times N_{RF}$ analog precoder \mathbf{F}_{RF} . After passing through the wireless channel \mathbf{H} with size of $N_r \times N_t$. The data stream are collected by N_t transmit antennas and N_r receive antennas at the transmit and receive side respectively. The signal arrives at the mobile station devices which is connected to the analog combiner and N_{RF} number of RF chains and a digital combined \mathbf{W}_{BB} .

This work denotes \mathbf{s} as the $N_s \times 1$ symbol vector, then the transmitted signal is given by:

$$\mathbf{x} = \mathbf{F}_{BB} \mathbf{F}_{RF} \mathbf{s} \quad (1)$$

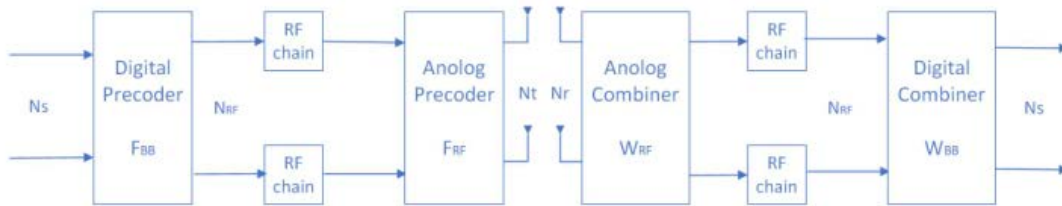


Fig.1 Point to Point Narrow Band Mm Wave Mimo System Diagram[1]

With channel matrix presented as \mathbf{H} , the received signal at the receiver antenna can be written in the equation of:

$$\mathbf{r} = \mathbf{H} \mathbf{F}_{BB} \mathbf{F}_{RF} \mathbf{s} + \mathbf{n} \quad (2)$$

where \mathbf{n} represents the $N_r \times 1$ additive Gaussian noise vector.

The signal is then received at the receiver side and are further processed by the combiner. After the process of the combining, with an $N_r \times N_{RF}$ analog combiner \mathbf{W}_{RF} followed by an $N_{RF} \times N_s$ baseband combiner \mathbf{W}_{BB} , the processed signal is yielded as:

$$\mathbf{y} = \mathbf{W}_{BB}^* \mathbf{W}_{RF}^* \mathbf{H} \mathbf{F}_{BB} \mathbf{F}_{RF} \mathbf{s} + \mathbf{W}_{BB}^* \mathbf{W}_{RF}^* \mathbf{n} \quad (3)$$

3. Channel Model

The millimeter wave propagation environment is characterized by a geographical clustered channel model with N_C clusters and N_R rays within each cluster. This paper configures the half-wave based uniform linear array (ULA) at both transmitting and receiving terminals. As a result, the $N_r \times N_t$ channel matrix \mathbf{H} [2] can be written in the equation of:

$$\mathbf{H} = \sqrt{\frac{N_r N_t}{N_C N_R}} \sum_{i=1}^{N_C} \sum_{j=1}^{N_R} \alpha_{ij} \mathbf{a}_r(\theta_{ij}^r) \mathbf{a}_t(\theta_{ij}^t)^* \quad (4)$$

in which case, the α_{ij} denotes the complexity gain of the j -th ray of the i -th propagation cluster. While $\mathbf{a}_r(\theta_{ij}^r)$ and the $\mathbf{a}_t(\theta_{ij}^t)$ represent the normalized steering vector of the transmitter and receiver to the j -th ray in i -th cluster, respectively, where the θ_{ij}^r and the θ_{ij}^t represent the azimuths of arrival and departure.[1]

4. Problem Formulation

In order to solve the shortage mentioned before about the mmWave wireless propagation, different kinds of beamforming and precoding methods are introduced in this section. This paper is going to discuss the different kinds of algorithm and criterion in terms of fully digital as well as hybrid precoding methods. In the full digital part, different criteria of precoding methods-Zero forcing (ZF) criterion and Minimum Mean Square Error (MMSE) criterion are introduced and compared. While the fully digital architecture can provide the most rather optimal outcome, it is considered by our group that in real-life practical situation, it is unlikely to implement such method due to the properties of complexity in the fully digital architecture because it requires the same amount of RF chains with the number of antennas which is both not practical and costly in the physical layer. As a result, the research introduces a low complexity beamforming method for single-user millimeter wave MIMO system. This work intends to compare our hybrid beamforming method with the full digital one with different precoding criteria.

4.1 Full-Digital Scenario

In full digital scenario, only the precoder, the channel matrix, the Gaussian white noise is taken into consideration in our system model.

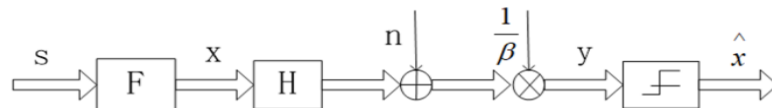


Fig.2 Full-Digital Signal Processing Flow Chart

the \mathbf{F} represents the precoding matrix, the \mathbf{H} is the channel matrix, \mathbf{n} is the Gaussian white noise, β is a

scaling factor which is determined by the total transmitted power: $\beta = \sqrt{\frac{N_t}{T_r \{\mathbf{F}\mathbf{F}^*\}}} \quad [1-3]$

Then our processed signal at \mathbf{y} would be of the expression:

$$\mathbf{y} = \frac{1}{\beta} \mathbf{H}\mathbf{F}\mathbf{s} + \frac{1}{\beta} \mathbf{n} \quad (5)$$

In the following context, the paper would introduce two full digital precoding criteria based on the model

above.

4.1.1 Zf Criterion

The idea of zero forcing algorithm is to modify the point to point model:

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{n} \quad (6)$$

In the zero-forcing method, the paper want to make the expression of HF most approximate to the identity matrix I. By doing so, we can transform the receiving model into:

$$\mathbf{y} = \mathbf{s} + \mathbf{n} \quad (7)$$

the designing of the precoding matrix F is to let the results of HF most approximate to the identity matrix I. Which is a least square problem of:

$$\min_{\mathbf{F}} \|\mathbf{I} - \mathbf{H}\mathbf{F}\|_F^2 \quad (8)$$

So, when the number of transmitting antennas is larger than the one of the receiving antennas, our precoding matrix is designed and so can the processed signal x be derived:

$$\mathbf{F} = \mathbf{H}^* (\mathbf{H}\mathbf{H}^*)^{-1} \quad (9)$$

$$\mathbf{x} = \mathbf{H}^* (\mathbf{H}\mathbf{H}^*)^{-1} \mathbf{s} \quad (10)$$

4.1.2 Mmse Criterion

Different from the ZF algorithm, the MMSE criterion consider the influence of the Gaussian white noise into consideration. The main idea of the MMSE algorithm is to find the rather most optimal method to minimize the variance between the transmitted and the received signal. In order to do that, in MMSE, the paper try to design a precoding matrix F so that such requirements can be achieved.

According to the equation which represents the variance e between transmitted and received signal:

$$e = \mathbf{y} - \mathbf{s} = \left(\frac{1}{\beta} \mathbf{H}\mathbf{F} - \mathbf{I} \right) \mathbf{s} + \frac{1}{\beta} \mathbf{n} \quad (11)$$

since the transmitted signal and the Gaussian white noise is uncorrelated, the mean square error (MSE) can then be represented by:

$$\epsilon(\mathbf{F}, \beta) = T_r \left(\mathbb{E}[\mathbf{e}\mathbf{e}^*] \right)$$

which also equals to:

$$T_r \left\{ \sigma_n^2 \left(\frac{1}{\beta} \mathbf{H}\mathbf{F} - \mathbf{I} \right) \left(\frac{1}{\beta} \mathbf{H}\mathbf{F} - \mathbf{I} \right)^* \right\} + \sigma_n^2 \frac{1}{\beta^2} \mathbf{I} \quad (13)$$

The research then tries to minimize the mean square error under the constraint of our transmitter power, transmitter generally has the limitation of transmitting power, so it is considered to minimize the MSE under the condition that the total transmitting power of the transmitter is constraint:

$$P_T = N_t \sigma_s^2 \quad (14)$$

here the σ_n^2 and the σ_s^2 denotes the variance of the noise and the variance of the signal respectively. Then the minimizing problem can be transformed into:

$$(\mathbf{F}, \beta) = \operatorname{argmin}_{\mathbf{F}, \beta} \{ \epsilon(\mathbf{F}, \beta) \} \quad (15)$$

$$T_r \{ \mathbf{F}\mathbf{s}\mathbf{s}^* \mathbf{F}^* \} = N_t \sigma_s^2 \quad (16)$$

This is a conditional Lagrangian extreme problem, which can be solved by Lagrangian multiplier method, and our optimal precoding matrix F can be denoted by:

$$\mathbf{F} = \mathbf{H}^* (\mathbf{H}\mathbf{H}^* + \alpha\mathbf{I})^{-1} \quad (17)$$

where α denotes the equation of: $\alpha = \sigma_n^2 (\sigma_s^2)^{-1}$

4.2 Hybrid Scena Rio

In conventional MIMO architecture at frequencies below 6GHz, both precoding and combining are done digitally at baseband.[3,4] However, the high power consumption and space limitations make it hard to arrange a RF chain for each antenna at mmWave. Taking these factors into consideration, then generalize the full digital circumstance into a hybrid one, which is a promising solution for the practical massive MIMO system to overcome path loss and reduce complexity.[5]

4.2.1 Spatially Sparse Algorithm

In this part, the paper makes use of the spatial structure of the channel model[2] and convert the hybrid beamforming problem as a problem of the sparse reconstruction[2] and convert the hybrid beamforming problem as a sparse reconstruction problem.[6] Then, the paper decouples the joint optimization problem into the transmitter part and the receiver part effectively. This research provides an algorithmic precoding solution and a combiner design based on the concept of orthogonal matching pursuit and compare the performance between the full-digital and hybrid algorithm according to the spectral efficiency. Assume that the transmitted signal sis a

$N_s \times 1$ symbol vector, such that $\mathbb{E}[\mathbf{s}\mathbf{s}^*] = \frac{1}{N_s} \mathbf{I}_{N_s}$

With the proposed channel model, the processed received signal can be denoted as:

$$\tilde{\mathbf{y}} = \sqrt{\rho} \mathbf{W}_{BB}^* \mathbf{W}_{RF}^* \mathbf{H} \mathbf{F}_{RF} \mathbf{F}_{BB} \mathbf{S} + \mathbf{W}_{BB}^* \mathbf{W}_{RF}^* \mathbf{n} \quad (18)$$

where ρ denotes the average power of the received signal, n is the Gaussian vector of i.i.d $CN(0, \sigma_n^2)$, thus the achievable spectral efficiency is given by:

$$R = \log_2 \left(\left(\mathbf{I}_{N_s} + \frac{\rho}{N_s} \mathbf{R}_n^{-1} \mathbf{W}_{BB}^* \mathbf{W}_{RF}^* \mathbf{H} \mathbf{F}_{RF} \mathbf{F}_{BB} \times \mathbf{F}_{BB}^* \mathbf{F}_{RF}^* \mathbf{H}^* \mathbf{W}_{RF} \mathbf{W}_{BB} \right) \right) \quad (19)$$

where equation $\mathbf{R}_n = \sigma_n^2 \mathbf{W}_{BB}^* \mathbf{W}_{RF}^* \mathbf{W}_{RF} \mathbf{W}_{BB}$

The paper tries to optimize hybrid mmWave precoders $(\mathbf{F}_{RF}, \mathbf{F}_{BB})$ to achieve the maximum efficiency rate expression in (19). This is a non-convex problem due to the transmitter's total power constraint, namely $\|\mathbf{F}_{RF} \mathbf{F}_{BB}\|_F^2 = N_s$. To simplify the transmitter design, the paper substitute the mutual information for spectral efficiency and construct the function in terms of the “distance” between $\mathbf{F}_{RF} \mathbf{F}_{BB}$ and the digital unconstrained precoder \mathbf{F}_{opt} , which derives from the singular value decomposition of channel matrix H.

$$\mathbf{H} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^* \quad (20)$$

where U and V is unitary matrix, $\mathbf{\Sigma}$ is a diagonal matrix of singular values. By extracting the first Ns columns of V, we can form the optimal digital precoder \mathbf{F}_{opt} [10].

Subsequently, by using some equivalent approximation and mathematical deformation, the precoder design problem can be rewritten as:

$$(\mathbf{F}_{RF}^{opt}, \mathbf{F}_{BB}^{opt}) = \arg \min_{\mathbf{F}_{RF}, \mathbf{F}_{BB}} \|\mathbf{F}_{opt} - \mathbf{F}_{RF} \mathbf{F}_{BB}\|_F \quad (21)$$

$$s.t. \mathbf{F}_{RF} \in \mathcal{F}_{RF}$$

$$\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 = N_s$$

By leveraging the sparse structure of millimeter wave channel, we note that the integral optimal precoder matrix can be written as linear combination of $\mathbf{a}_i(\theta_{ij}^t)$, which is the normalized responses of transmit arrays to j -th ray in i -th cluster[19]. Define \mathbf{A}_i as a $N_t \times N_C N_R$ codebook matrix comprised of transmit array response vectors, which can be denoted as $\mathbf{A}_i = [\mathbf{a}_i(\theta_{i1}^t), \dots, \mathbf{a}_i(\theta_{iN_C, N_R}^t)]$. The paper can derive that the optimal RF precoder \mathbf{F}_{RF}^{opt} will be given by NRF columns of \mathbf{A}_i and optimal baseband precoder \mathbf{F}_{RF}^{opt} will be given by the combination of \mathbf{F}_{RF} and \mathbf{F}_{opt} . The detailed computation of \mathbf{F}_{BB} and \mathbf{F}_{RF} is given in Algorithm 1.

Algorithm 1 Spatially Sparse Precoding via Orthogonal Matching Pursuit

Input: \mathbf{F}_{opt}

1: $\mathbf{F}_{RF} = \text{Empty Matrix}$

2: $\mathbf{F}_{res} = \mathbf{F}_{opt}$

3: **for** $i \leq N_{RF}$ **do**

4: $\Psi = \mathbf{A}_i^* \mathbf{F}_{res}$

5: $k = \text{argmax}_{l=1, \dots, N_C N_R} (\Psi \Psi^*)_{l,l}$

6: $\mathbf{F}_{RF} = [\mathbf{F}_{RF} \mid \mathbf{A}_i^{(k)}]$

7: $\mathbf{F}_{BB} = (\mathbf{F}_{RF}^* \mathbf{F}_{RF})^{-1} \mathbf{F}_{RF}^* \mathbf{F}_{opt}$

8: $\mathbf{F}_{res} = \frac{\mathbf{F}_{opt} - \mathbf{F}_{RF} \mathbf{F}_{BB}}{\|\mathbf{F}_{opt} - \mathbf{F}_{RF} \mathbf{F}_{BB}\|_F}$

9: **end for**

10: $\mathbf{F}_{BB} = \sqrt{N_s} \frac{\mathbf{F}_{BB}}{\|\mathbf{F}_{RF} \mathbf{F}_{BB}\|_F}$

11: **return** $\mathbf{F}_{RF}, \mathbf{F}_{BB}$

At the receiver side, the widely used linear MMSE combiner is pretty feasible. Assuming that the hybrid precoder information is perfectly known at the receiver, we design the hybrid combiner in terms of minimizing the mean-squared-error (MSE) between the transmitted and received signal.

This problem can be formulated as:

$$(\mathbf{W}_{RF}^{opt}, \mathbf{W}_{BB}^{opt}) = \arg \min_{\mathbf{W}_{RF}, \mathbf{W}_{BB}} \mathbb{E} \left[\|\mathbf{s} - \mathbf{W}_{BB}^* \mathbf{W}_{RF}^* \mathbf{y}\|_2^2 \right] \quad (22)$$

s.t. $\mathbf{W}_{RF} \in \mathcal{W}_{RF}$

According to[7], the full-digital unconstrained combiner \mathbf{W}_{MMSE} can be written as:

$$\mathbf{W}_{MMSE}^* = \frac{1}{\sqrt{\rho}} \left(\mathbf{F}_{BB}^* \mathbf{F}_{RF}^* \mathbf{H}^* \mathbf{H} \mathbf{F}_{RF} \mathbf{F}_{BB} + \frac{\sigma_n^2 N_s}{\rho} \mathbf{I}_{N_s} \right)^{-1} \mathbf{F}_{BB}^* \mathbf{F}_{RF}^* \mathbf{H}^* \quad (23)$$

Just like the precoding case, the combiner optimization problem can be rewritten as:

$$(\mathbf{W}_{RF}^{opt}, \mathbf{W}_{BB}^{opt}) = \arg \min_{\mathbf{W}_{RF}, \mathbf{W}_{BB}} \left\| \mathbb{E} [\mathbf{y} \mathbf{y}^*]^{-\frac{1}{2}} (\mathbf{W}_{MMSE} - \mathbf{W}_{RF} \mathbf{W}_{BB}) \right\|_F \quad (24)$$

s.t. $\mathbf{W}_{RF} \in \mathcal{W}_{RF}$

This allow us to solve the combing problem by exploiting the sparse structure of the channel. Define

$\mathbf{A}_r = \left[\mathbf{a}_r(\theta_{11}^r), \dots, \mathbf{a}_r(\theta_{N_C, N_R}^r) \right]$ as a $N_r \times N_C N_R$ matrix of array response vectors and can then optimize the hybrid combiner through the method of orthogonal matching pursuit like the transmitter side. The detailed computation of \mathbf{W}_{BB} and \mathbf{W}_{RF} is designed as follow:

Algorithm 2 Spatially Sparse MMSE Combining via Orthogonal Matching Pursuit

Input: \mathbf{W}_{MMSE}

1: $\mathbf{W}_{MMSE} = \text{Empty Matrix}$

2: $\mathbf{W}_{res} = \mathbf{W}_{MMSE}$

3: **for** $i \leq N_{RF}$ **do**

$$4: \mathbf{\Psi} = \mathbf{A}_r^* \mathbb{E}[\mathbf{y}\mathbf{y}^*] \mathbf{W}_{res}$$

$$5: k = \operatorname{argmax}_{l=1, \dots, N_C N_R} (\mathbf{\Psi} \mathbf{\Psi}^*)_{l,l}$$

$$6: \mathbf{F}_{RF} = \left[\mathbf{F}_{RF} \mid \mathbf{A}_r^{(k)} \right]$$

$$7: \mathbf{W}_{BB} = \left(\mathbf{W}_{RF}^* \mathbb{E}[\mathbf{y}\mathbf{y}^*] \mathbf{W}_{RF} \right)^{-1} \mathbf{W}_{RF}^* \mathbb{E}[\mathbf{y}\mathbf{y}^*] \mathbf{W}_{MMSE}$$

$$8: \mathbf{W}_{res} = \frac{\mathbf{W}_{MMSE} - \mathbf{W}_{RF} \mathbf{W}_{BB}}{\|\mathbf{W}_{MMSE} - \mathbf{W}_{RF} \mathbf{W}_{BB}\|_F}$$

9: **end for**

10: **return** $\mathbf{W}_{RF}, \mathbf{W}_{BB}$

4.2.2 Hybrid Sic Algorithm

This work also provides another hybrid precoding algorithm called successive interference cancellation (SIC)-based hybrid beamforming algorithm which has nearly optimized performance, low computation complexity and low energy consumption.[8] This algorithm tries to maximize the spectral efficiency R. Compared with Spatially Sparse Precoding algorithm, the SIC hybrid precoding algorithm rarely use the inverse and singular value decomposition of matrices. Since lots of matrices in the hybrid precoding are nearly singular, the complexity of the algorithm is reduced with the decrease of the inverse times of the matrix. Also in order to decrease the complexity of this algorithm, the number of RF chains is equal to the number of data streams, namely $N_s = N_{RF} = N$. Different from the Spatially Sparse Precoding algorithm, the SIC hybrid precoding algorithm considers sub-connected architecture. Thus, SIC Algorithm consume less energy than Spatially Sparse Precoding algorithm dose. The same number of RF chains and data streams also greatly simplifies the complexity of the algorithm.[8] This research also assume that the transmitted signal is a $N_s \times 1$ symbol vector. The received signal is denoted as:

$$\tilde{\mathbf{y}} = \sqrt{\rho} \mathbf{H} \mathbf{F}_{BB} \mathbf{F}_{RF} \mathbf{S} + \mathbf{n} = \sqrt{\rho} \mathbf{H} \mathbf{P} \mathbf{s} + \mathbf{n} \quad (25)$$

where ρ represents the average power of the received signal and \mathbf{n} is the Gaussian vector of i.i.d $CN(0, \sigma_n^2)$. In this algorithm, the paper does not take the combiner matrices into account at the receiver side. $\mathbf{P} = \mathbf{F}_{BB} \mathbf{F}_{RF}$ is a precode matrix. And \mathbf{F}_{BB} can be simplified as a $N_s \times N_{RF}$ diagonal matrix and the \mathbf{F}_{RF} is a $N_r \times N_{RF}$ block diagonal matrix. Since this work do not take the combiner matrices into account, the spectral efficiency can be expressed as:

$$\mathbf{R} = \log_2 \left| \mathbf{I}_{N_r} + \frac{\rho}{N_s \sigma^2} \mathbf{H} \mathbf{P} \mathbf{P}^* \mathbf{H}^* \right| \quad (26)$$

Unfortunately, the optimizing problem is non-convex.[8] According to the structures of matrix \mathbf{F}_{BB} and \mathbf{F}_{RF} , it is clear that the P is also a block diagonal matrix(diagonal elements are \mathbf{p}_n^{opt}) as \mathbf{F}_{RF} and the signal processing is conducted independently on different sub-antennas. Thus, the optimizing problem can be converted

into a battery of sub array optimizing problems. Then the total spectral efficiency can be denoted as:

$$R = \sum_{n=1}^{N_{RF}} \log_2 \left(1 + \frac{\rho}{N_{RF} \sigma^2} \mathbf{p}_n \mathbf{H} \mathbf{T}_{n-1}^{-1} \mathbf{H}^* \mathbf{p}_n^* \right) \quad (27)$$

where n denotes sub-antenna array n. Then the optimizing problem can be expressed as:

$$\mathbf{p}_n^{opt} = \arg \max_{\mathbf{p}_n \in \mathcal{F}} \log_2 \left(1 + \frac{\rho}{N_{RF} \sigma^2} \mathbf{p}_n^* \mathbf{G}_{n-1} \mathbf{p}_n \right) \quad (28)$$

where

$$\mathbf{G}_{n-1} = \mathbf{H}^* \mathbf{T}_{n-1}^{-1} \mathbf{H} \quad (29)$$

According to the properties of the P[8], it is clear that \mathbf{p}_n has $\frac{N_t}{N_{RF}}$ non-zero entries from the $(N_t(n-1)+1)$ th one to the $(N_t n)$ th one.

$$\bar{\mathbf{p}}_n^{opt} = \arg \max_{\bar{\mathbf{p}}_n \in \bar{\mathcal{F}}} \log_2 \left(1 + \frac{\rho}{N_{RF} \sigma^2} \bar{\mathbf{p}}_n^* \bar{\mathbf{G}}_{n-1} \bar{\mathbf{p}}_n \right) \quad (30)$$

where

$$\bar{\mathbf{G}}_{n-1} = \mathbf{R} \mathbf{H}^* \mathbf{T}_{n-1}^{-1} \mathbf{H} \mathbf{R}^* \quad (31)$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{0}_{\frac{N_t}{N_{RF}} \times \frac{N_t}{N_{RF}} (n-1)} & \mathbf{I}_{\frac{N_t}{N_{RF}}} & \mathbf{0}_{\frac{N_t}{N_{RF}} \times \frac{N_t}{N_{RF}} (N-n)} \\ \mathbf{0}_{\frac{N_t}{N_{RF}} \times \frac{N_t}{N_{RF}} (n-1)} & \mathbf{I}_{\frac{N_t}{N_{RF}}} & \mathbf{0}_{\frac{N_t}{N_{RF}} \times \frac{N_t}{N_{RF}} (N-n)} \end{bmatrix} \quad (32)$$

Then apply the SVD (singular value decomposition) at the \mathbf{G}_n matrix, we can rewrite the optimizing problem as:

$$\bar{\mathbf{p}}_n^{opt} = \arg \min_{\bar{\mathbf{p}}_n \in \bar{\mathcal{F}}} \left\| \mathbf{v}_1 - \bar{\mathbf{p}}_n \right\|_2 \quad (33)$$

where \mathbf{v}_1 is the first right singular vector of matrix \mathbf{G}_n . The details of the algorithm are as follows:

Algorithm 3 Power iteration algorithm based on SVD

Input: $\bar{\mathbf{G}}_{n-1}$

1: **for** $1 \leq s \leq S$ **do**

2: $\mathbf{z}^{(s)} = \bar{\mathbf{G}}_{n-1} \mathbf{u}^{(s-1)}$

3: $m^{(s)} = \arg \max_i |z_i^{(s)}|$

4: **if** $1 \leq s \leq 2$ **then**

5: $\mathbf{n}^{(s)} = m^{(s)}$

6: **else**

$$n^{(s)} = \frac{m^{(s)} m^{(s-2)} - (m^{(s-1)})^2}{m^{(s)} - 2m^{(s-1)} + m^{(s-2)}}$$

7: **end if**

$$8: \mathbf{u}^{(s)} = \frac{\mathbf{z}^{(s)}}{\mathbf{n}^{(s)}}$$

9: **end for**

10: $\Sigma_1 = \mathbf{n}^{(S)}$

$$11. \mathbf{v}_1 = \frac{\mathbf{u}^{(S)}}{\left\| \mathbf{u}^{(S)} \right\|_2}$$

12. return $\Sigma_1, \mathbf{v1}$

The $\mathbf{u}_0 \in \mathbb{C}^{\frac{N_r}{N_{RF}} \times 1}$ is always initialized as $[1, 1, 1, \dots, 1, 1]^T$. S is the maximum number of iteration.

Algorithm 4 SIC-based hybrid precoding

Input: $\bar{\mathbf{G}}_0$

1: **for** $1 \leq n \leq N_{RF}$ **do**

2: Compute Σ_1 and $\mathbf{v1}$ by using Power iteration algorithm

$$3: \mathbf{a}_n^{-opt} = \frac{1}{\sqrt{N_{RF}}} \exp \text{jangle}(\mathbf{v1}) \text{ and } \mathbf{d}_n^{opt} = \frac{\|\mathbf{v1}\|_2}{\sqrt{N_{RF}}}$$

$$4: \bar{\mathbf{G}}_n = \bar{\mathbf{G}}_{n-1} - \frac{\frac{\rho}{N_{RF}\sigma^2} \Sigma_1^2 \mathbf{v1} \mathbf{v1}^*}{1 + \frac{\rho}{N_{RF}\sigma^2} \Sigma_1}$$

5: **end for**

$$6: \mathbf{F}_{BB} = \text{diag}[\mathbf{d}_1^{opt}, \dots, \mathbf{d}_N^{opt}]$$

$$7: \mathbf{F}_{RF} = \text{diag}[\mathbf{a}_1^{-opt}, \dots, \mathbf{a}_n^{-opt}]$$

8: return $\mathbf{F}_{BB}, \mathbf{F}_{RF}$

5. Simulation

5.1 Full-Digital Scena Rio

In this part, the simulation results of the full digital scenario after 2 different full digital precoding methods of zero forcing and minimum mean square error criterion are presented. According to the characteristics of the ZF and the MMSE based algorithm, the ZF method is not as good as the MMSE method because the ZF algorithm does not consider the Gaussian white noise. The ZF method can only obtain a rather good performance when the signal to noise ratio (SNR) is really high. In our simulation, we set the number of transmitting and receiving antennas N_t, N_r , to be both 4 and use the signal to noise ratio (SNR) in dB and the Bit error rate (BER) as our performance measure standard the simulation result is illustrated in the following figure

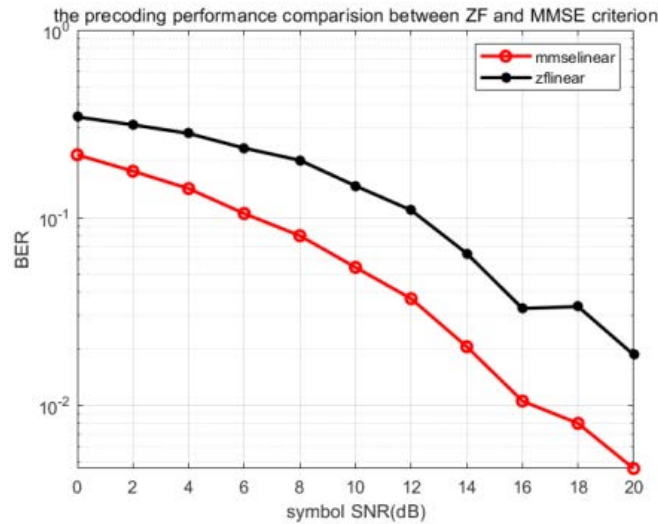


Fig.3 Comparison of the Zf and Mmse

Figure 3 shows that the minimum mean square error message can provide a lower BER under the same SNR. The paper then investigate and extend our full digital scenario precoding with the method of serial interference cancellation (SIC), the MMSE-SIC and ZF-SIC criterion. The traditional linear detection algorithms such as ZF and MMSE are simple but have poor performance. Because of the layered space-time structure of the receiving message the signal vector is the superposition of all the trans-mitting antenna signals, so each receiving antenna receives useful signals and interference signals Therefore, the nonlinear algorithm with serial interference cancellation can be used to improve the detection performance. The purpose of (SIC) is to remove the interference of other antennas to each antenna, and then demodulate the decision to obtain the antenna's transmission. Firstly, the signal sent from a certain layer is detected, and then the interference caused by this layer is offset from other layers, and the whole signal vector is finally detected through successive iterations. It requires a two-step operation: first advanced line linear processing, and then SIC detection. Linear processing is a partial decomposition, and then according to the order of signal energy from large to small, SIC is used for interference cancellation. The simulation of the result is shown by the following graph, we compare the linear ZF and MMSE precoding and the ons with SIC:

As can be shown in the graph, ZF detection ignores the interference of channel noise, so even though SIC-ZF can eliminate the interference between antennas, it is still not the optimal choice. The MMSE detection algorithm takes the influence of antenna interference and noise interference into consideration, making its performance better than that of ZF detection.

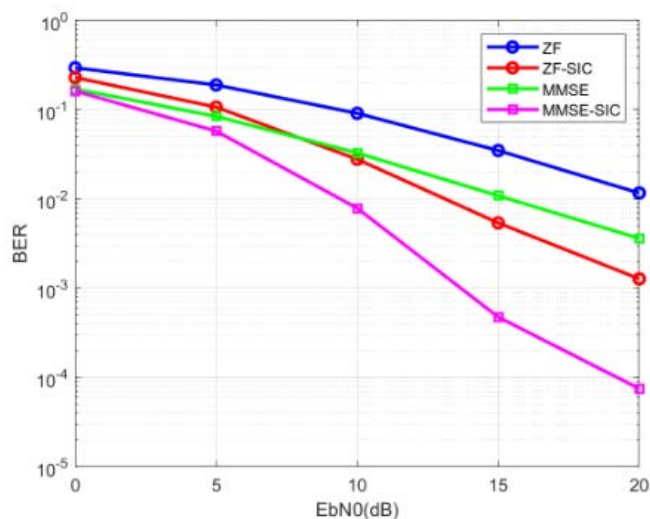


Fig.4 Zf and Mmse Methods Comparison with Sic

5.2 Hybrid Scena Rio

This part of the paper provides the results of the simulation to evaluate the full-digital and hybrid precoding algorithms performance in terms of their spectral efficiency. The paper considers the system described in the SYSTEM MODEL section employing uniform linear array (ULA) with 64 antennas at transmitter and 16 antennas at receiver. The number of cluster is set to $N_c = 5$ and the number of rays in each cluster is set to $N_r = 10$. The research assume that α_{ij} follows standard normal distribution, and the arrival and departure angles of the model correspond with the Laplacian distribution with uniformly distributed mean angles over $[0, 2\pi]$. The angular spread is 10 degrees with each cluster.

5.2.1 Simulation Scenario for Varying Number of Rf

Figure 5 represents the results of the simulations for our proposed different performance of precoding with varying number of RF. Here the research set $N_s = 2$. Number of RF is randomly taken in order to observe the variation in the spectral efficiency for increasing number of RF chains. It is observed, with the increasing of number of RF chains, the curve of hybrid precoding becomes closer and closer to the curve of full-digital.

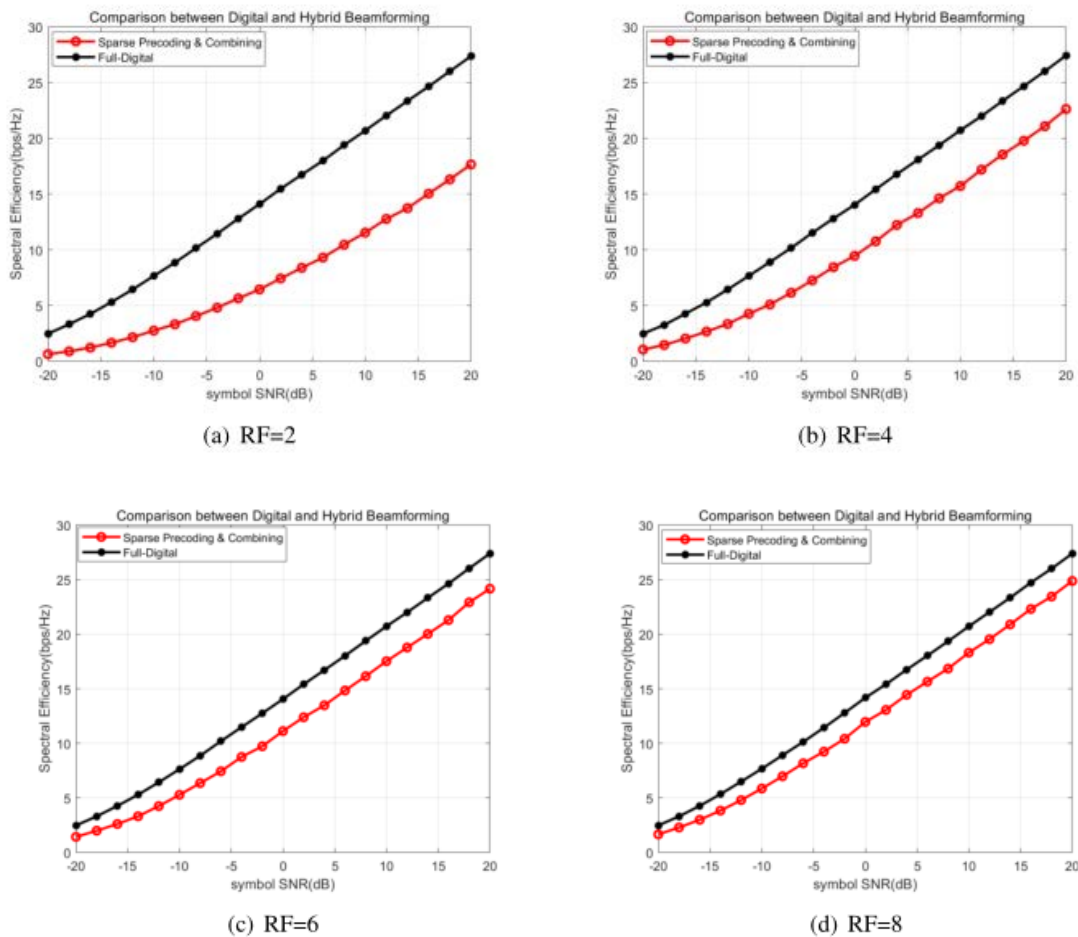


Fig.5 Impact on Varying Snr and Number of Rf.

5.2.2 Simulation Scenario for Varying Number of Ns

Figure 5 (a), (b), (c) and (d) show that we have known the impact of the number of RF chains on the performance of precoding. And in Figure 6 (a), (b), (c) and (d) we set the $RF=4$, then vary the number of N_s to investigate the variation in spectral efficiency for increasing number of N_s . It is observed that with the increasing of number of N_s , the performance of hybrid precoding becomes weaker and weaker and further and further away from the curve of full-digital.

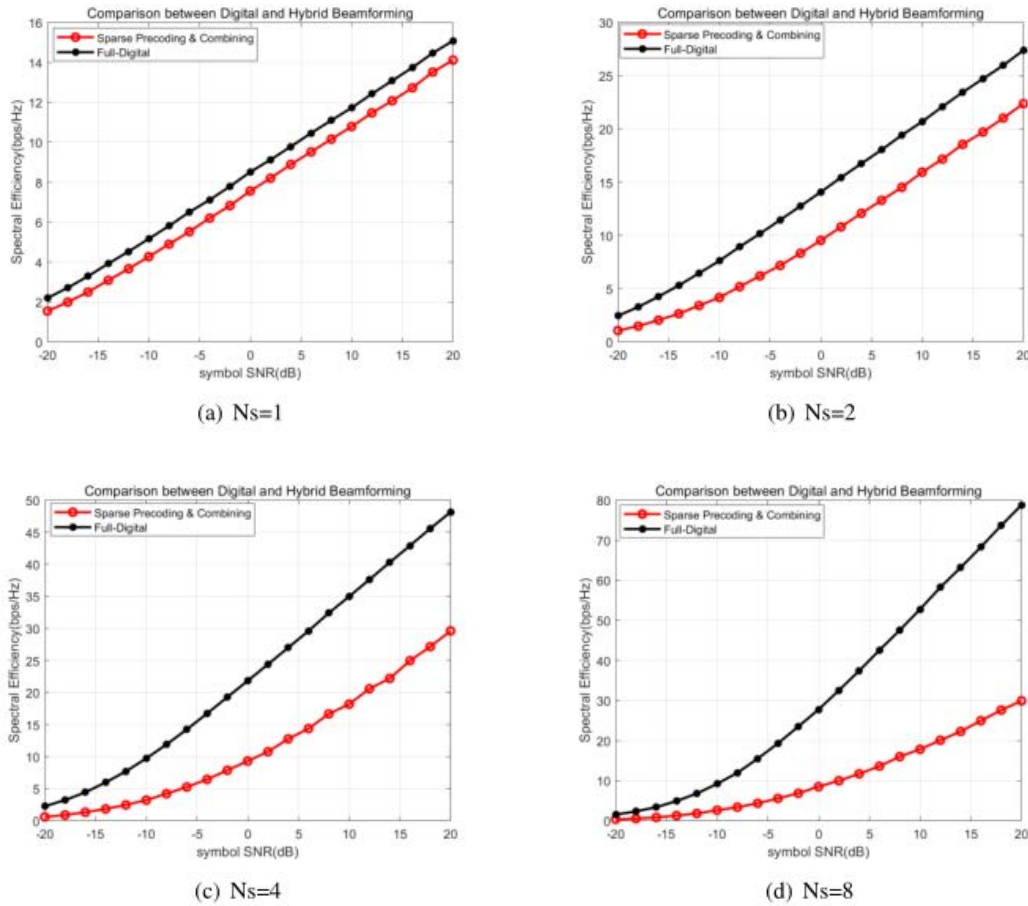


Fig. 6 Impact on varying SNR and number of N_s .

5.2.3 Comparison between Optimal Full-Digital, Spatially Sparse and Hybrid Sic Algorithm

In Figure 7, the research set the number of RF chain 4, transmitter antenna 64 and receiver antenna 16. We can see that in terms of spectrum efficiency, SIC algorithm is not as good as Spatially Sparse algorithm, because Spatially Sparse algorithm uses full-connected structure in RF precoder, and the complexity of the algorithm is higher than SIC algorithm. While the SIC algorithm uses sub-connected structure in RF precoder and the number of PSs used is far less than Spatially Sparse algorithm, so it is more energy-saving. Because the complexity of SCI algorithm is less than that of Spatially Sparse Algorithm, it is easier to implement than Spatially Sparse Algorithm.

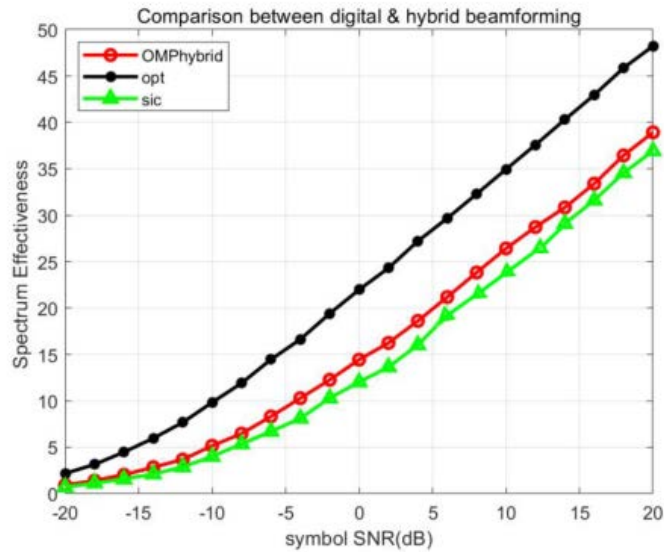


Fig.7 Comparison of Three Algorithms in Terms of Spectral Efficiency

6. Conclusion

The mmWave communication is qualified for high bandwidth transmission and enables massive antenna array to be placed in narrow space. And in order to deal with three mmWave problems: propagation distance, diffraction ability and limitation caused by many spatial factors, Beamforming was put forward. In this paper, this research begins with investigating full-digital scenario by comparing two beamforming methods respectively based on two common criterion: ZF and MMSE. And it is observed that MMSE has better performance because of taking noise in channel into consideration.

Then this paper combine SIC with ZF and MMSE to eliminate the interference and finally propose a optimal beamforming method in full-digital scenario: SIC-MMSE. To generalize the solution to practical communication system, Hybrid Beamforming (HBF) was put forward, which reduced the number of RF chains while guaranteeing a required beamforming gain with little loss in spectral efficiency. By exploiting the channel structure, this research can adopt a low complexity algorithm which convert the HBF into a sparse reconstruction problem. Then a SIC-based hybrid precoding problem was used to further simplify the complexity and reduce the energy consumption.

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