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Application of Spline Interpolation Function in Back Analysis of Displacement of Pile Support in Foundation Pit

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ABSTRACT. *with the great development of globalization, more and more high-rise buildings have been built up, accompanied by more and more foundation pit support and excavation. The forms of foundation pit support are more and more diverse, among which row pile support is widely used in engineering because of its mature construction technology and good supporting effect. However, it is difficult to directly measure the internal force of pile in engineering practice. In practice, the displacement of pile body is often monitored and the results are more accurate. In this paper, the method of spline interpolation function is used to measure the displacement of pile. The back analysis of the pile stress provides a reference for the application of pile structure in foundation pit.*

KEYWORDS: *pile arrangement, spline function, inversion*

1. Introduction

There are many complex methods to measure the earth pressure behind the pile, but the displacement of the pile is easier to obtain. The displacement of the pile can be replaced by the displacement of the inclinometer tube by binding the inclinometer tube and the reinforcement of the pile together. This method is more mature and easy to operate in engineering practice [1]. We consider the displacement of the pile to reverse the external force of the soil behind the pile, and then we can get the earth pressure.

The displacement of the pile in the soil is caused by many factors, such as the displacement of the soil behind the pile to the pile, the variable load on the top of the pile, and the displacement of the pile at different depths is different. It can be seen from the analysis of the load that the displacement of the pile is related to the concentrated load, the bending moment, the horizontal displacement and the rotation angle of the pile bottom, which is the result of the joint action of various loads [2]. However, in order to facilitate the analysis of the load we assumed after the pile,

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only the earth pressure of the soil on the pile is ignored in the analysis process. In this way, we can deduce the magnitude of the pile load through the displacement value of the actual monitoring pile and the stress analysis of the material mechanics.

2. Methodology

2.1 Basic mechanics analysis in support structure

In the process of analyzing the force on the pile, because there is no supporting structure on the top of the pile, it can be assumed that the top of the pile is a free end, while the bottom of the pile is inserted into the soil to a certain depth[3]. To simplify the analysis, the bottom of the pile is regarded as a fixed rod constraint, and the whole pile can be regarded as a cantilever structure with one end fixed and one end free.As shown in Figure 1

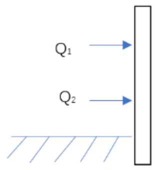

Figure 1 Stress diagram of cantilever pile

According to the force analysis of the object in the pure bending state in the online elastic range, the relationship between the bending moment of the pile and the corresponding radius of curvature under the model is that $\frac{1}{\rho}=\frac{M}{EI}$, the plane curvature can be obtained from the plane geometric relationship $\frac{1}{\rho}=\pm\frac{\omega''}{\left(1+\omega'^2\right)^{2/3}}$ and $\frac{M}{EI}=-\frac{\omega''}{\left(1+\omega'^2\right)^{2/3}}$ can be obtained by combining the above two equations. Considering that the length of the pile is generally much larger than the section size of the pile, the deflection curve is a relatively gentle curve, ω' Compared with 1, it can be ignored, so $EI\omega''=-M\left(x\right)$ 5simplified formula can be obtained, EI is the bending rigidity of the pile and ω is the deflection curve equation of the pile. According to the knowledge of material mechanics, the first derivative of bending moment of a section is the shear force value of the section, namely $M\left(x\right)'=F\left(x\right)$. At the same time, the first derivative of shear force $F\left(x\right)$ is the set degree of load $q\left(x\right)$. To sum up, the second derivative of bending moment

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of a point is equal to the load concentration of the point $\mathrm{M}(x)'' = q(x)$. We can consider the displacement value of each monitoring point obtained through actual monitoring, by fitting a reasonable displacement deformation curve, and by using the differential and integral relationship of the function, the load of each point $q(x)$ can be obtained.

2.2 Basic spline analysis method in pile support

Spline interpolation is a common numerical simulation method in Engineering. Generally, cubic spline interpolation function is a piecewise function. On the interval $[a,b]$, the whole interval is divided by a given interpolation node, $a = x_0 \langle x_1 \langle \cdots \langle x_{n-1} \langle x_n = b$ .It can be seen that the function is a cubic polynomial in every cell divided by node x_i . If $S(x)$ is the cubic spline interpolation function of $f(x)$, the following four points need to be met: (1) The interpolation condition is $S(x_j) = f(x_j),\ j = 0,1,\cdots n.$ (2)The continuity condition of a function is $\lim_{x\to x_j} S(x) = S(x_j) = y_j,\ j = 1,\cdots,n-1.$ (3) The continuous condition of the first derivative function is $\lim_{x\to x_j} S'(x) = S'(x_j),\ j = 1,\cdots,n-1.$ (4) The continuous condition of the second derivative function is $\lim_{x\to x_j} S''(x) = S''(x_j),\ j = 1,\cdots,n-1.$ When s satisfies the above conditions, the cubic polynomials on a cell $[x_j, x_{j+1}]$ can be set to $S(x) = A_j x^3 + B_j x^2 + C_j x + D_j,\ j = 0,1,2\cdots,n-1.$ A_j, B_j, C_j, D_j is a pending parameter. It can be seen that there are 4n unknown parameters in the whole interval. Since condition 1 can be obtained, the function needs to go through every interpolation point so that n+1 equations can be determined. According to condition 2, each point of the function is continuous, so n-1 equations can be determined. Similarly, n-1 equations can be determined by condition 3 and condition 4 respectively. The 4n-2 equations can be determined by the above four conditions. To solve all 4N unknowns, we need two more equations. In this case, we give the remaining equations by considering the boundary conditions. There are three common boundary conditions: (1) Given by the first derivative at the end of the interval is $S'(x_0) = f'(x_0), S'(x_n) = S'(x_n)$ (2) From the second derivative at the end of the interval, we can find that $S''(x_0) = f''(x_0), S''(x_n) = f''(x_n)$ In particular, the natural boundary is

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$S''(x_0)=0, S''(x_n)=0$. (3) From the function value or derivative value at the end of the interval satisfying the periodic condition, the result is. $S(x_0+0)=S(x_n-0), S'(x_0+0)=S'(x_n-0), S''(x_0+0)=S''(x_n-0)$ By choosing the appropriate boundary conditions, all the undetermined parameters can be uniquely determined, so the cubic spline function can be uniquely determined.

2.3 Back analysis of foundation pit stress

In the solution method of cubic spline interpolation function, if 4N undetermined parameters are used, the calculation amount is large. Generally, we determine the interpolation function by undetermined some parameters, reduce the undetermined parameters to the minimum by using known conditions, such as undetermined first derivative, undetermined second derivative, and determine the interpolation function by using the basis function method, which can greatly simplify the operation process.

In the calculation of cubic spline interpolation, because the physical meaning of the second derivative function of $S(x)$ is clear, the value of $S(x)$ represents the bending moment value of a point, and at the same time, the calculation amount can be reduced in the numerical calculation, so it is also a commonly used solution. Suppose $S''(x_i)=\mathrm{M}_i$ of $S(x)$ on $[x_i, x_{i+1}]$ is a cubic polynomial. It is known that the second derivative of the two end points between cells is $S''(x_i)=\mathrm{M}_i, S''(x_{i+1})=M_{i+1}$. Since $S(x)$ is a third-order polynomial, then the second-order derivative function is a first-order polynomial, then we know:

$$S_i''(x)=\frac{\mathrm{M}_{i+1}-\mathrm{M}_i}{h_i}(x-x_i)+\mathrm{M}_i, x\in[x_i, x_{i+1}],$$

$h_i=x_{i+1}-x_i \ (i=0,1,\cdots,n-1)$. By integrating the above formula once, we can get $S_i'(x)=\frac{\mathrm{M}_{i+1}-\mathrm{M}_i}{2h_i}(x-x_i)^2+\mathrm{M}_i(x-x_i)+C_i$, One more integration to get $S_i(x)=\frac{\mathrm{M}_{i+1}-\mathrm{M}_i}{6h_i}(x-x_i)^3+\mathrm{M}_i(x-x_i)^2+C_i(x-x_i)+D_i$. Where Ci and Di are integral constants, which can be solved by interpolation condition $S(x_i)=f(x_i)=y_i, S(x_{i+1})=f(x_{i+1})=y_{i+1}$. At the same time, from the continuity of interpolation function, we can know that $S_i(x_{i+1})=S_{i+1}(x_{i+1})=y_{i+1}$, From the above formula, we can get $C_i=f[x_i,x_{i+1}]-\frac{2\mathrm{M}_i+\mathrm{M}_{i+1}}{6}h_i$. $D_i=y_i$ can be derived from $S(x_i)=y_i$. From the above, we can get that the expression of cubic spline $S(x)$ on the i cell $[x_i, x_{i+1}]$ is $S(x) = S_{i1}(x-x_i)^3 + S_{i2}(x-x_i)^2 + S_{i3}(x-x_i) + S_{i4}$. i =0,1,2,…, n-1

$$S_{i1} = \frac{M_{i+1} - M_i}{6h_i}, S_{i2} = \frac{M_i}{2}, S_{i3} = C_i = f[x_i, x_{i+1}] - \frac{2M_i + M_{i+1}}{6} h_i, S_{i4} = D_i = y_i,$$

It can be seen that the coefficient of interpolation function is an expression containing second-order derivative function and function value [4]. In order to obtain the coefficient value, the continuity condition of the first derivative function at the left and right intervals of the interpolation point can be used

$S'_{i-1}(x_i) = S'_i(x_i), i = 1, 2, \cdots, n-1$. It can be seen from the definition that $S'_{i-1}(x_i)$ is defined on interval $[x_{i-1}, x_i]$ and $S'_i(x_i)$ is defined on interval $[x_i, x_{i+1}]$. Through analysis, we can know that $\varepsilon_i M_{i-1} + 2M_i + \gamma_i M_{i+1} = d, i = 1, 2, \cdots, n-1$. Where $\varepsilon_i = \frac{h_{i-1}}{h_{i-1} + h_i}, \gamma_i = 1 - \varepsilon_i, d_i = 6f[x_{i-1}, x_i, x_{i+1}], i = 1, 2, \cdots, n-1. d_0 = f[x_0, x_1], d_n = f[x_{n-1}, x_n]$.

From the analysis of the above formula, it can be seen that this basic equation is a system of equations that has not yet been determined, and two equations need to be determined according to the boundary conditions. Because it is a fixed spline, the boundary condition of the spline is $m_0 = f'(x_0), m_n = f'(x_n)$. According to the expression of the first derivative of the spline interpolation function s(x) obtained in the previous derivation process, we can get $S'_0(x_0) = m_0, S'_{n-1}(x_n) = m_n$ further, $2M_0 + M_1 = \frac{6(d_0 - m_0)}{h_0}, M_{n-1} + 2M_n = 6\left(\frac{m_n - d_n}{h_{n-1}}\right)$.

According to the above equations, the cubic spline interpolation function can be obtained by solving the equations simultaneously. The matrix expression is as follows;

$$\begin{bmatrix} 2 & 1 & & & & \\ \varepsilon_1 & 2 & \gamma_2 & & & \\ & \varepsilon_2 & 2 & \gamma_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \varepsilon_{n-1} & 2 & \gamma_{n-1} \\ & & & & 1 & 2 \end{bmatrix} \begin{pmatrix} M_0 \\ M_2 \\ M_3 \\ \vdots \\ M_{n-1} \\ M_n \end{pmatrix} = \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_{n-1} \\ d_n \end{pmatrix}, \quad d_0 = \frac{6}{h_0}\left(f[x_0, x_1] - m_0\right), \quad d_n = \frac{6}{h_{n-1}}\left(m_n - f[x_{n-1}, x_n]\right)$$

The equations are tridiagonal and can be solved by Gauss elimination. After the moment is calculated, the earth pressure of the structure can be inversed by using similar methods.

In the actual working condition, the stress of pile is very complex, and the distribution of earth pressure is very complex for most piles. Through a lot of research, it is shown that the distribution of earth pressure simulated by quadratic function is in good agreement with the actual distribution curve of earth pressure. Therefore, the deflection curve equation of pile needs to be simulated by quantic function at least. However, although the simulation by the quantic function of the case surface has a certain guiding role in the actual engineering, but the calculation is large, the fitting accuracy is sensitive to the number of times, and the fitting accuracy at the boundary of the pile is not ideal. However, it can be seen that the data fitting method is an important means for the inversion of internal forces, while the cubic spline interpolation method is a mature numerical analysis theory due to its small calculation amount, and the bending moment results obtained from the analysis are more reasonable[5]. At the same time, the interpolation method can be seen from the mechanical point of view that the first derivative of the fitting curve represents the rotation angle, and the second derivative represents the bending moment. By using the third derivative, the curve can be obtained by the method of the calculation of the bending moment. When the sub spline interpolation function is used to fit the continuity of each interpolation point, the first-order derivative function is also continuous, which means that the pile rotation angle is continuous, the second-order derivative function is also continuous, which means that the bending moment is also continuous. In addition, the reasonable selection of boundary conditions, the third-order spline function can effectively reflect the stress situation of the pile and the actual situation are more consistent.

Spline interpolation is a common numerical simulation method in engineering. The cubic spline interpolation function is a piecewise function. If a given interpolation node on $[a,b]$ the interval divides the entire interval $a = x_0 \langle x_1 \langle \cdots \langle x_{n-1} \langle x_n = b$, x_i is a cubic polynomial on each cell divided by the node. If it is a cubic spline interpolation function, the following four points need to be satisfied: (1) the interpolation element is $S(x_j) = f(x_j), j = 0,1,\cdots n.$ (2) the continuity condition of the function is $\lim_{x \to x_j} S(x) = S(x_j) = y_j, j = 1,\cdots,n-1.$ (3) the continuity condition of the first derivative function is $\lim_{x \to x_j} S'(x) = S'(x_j), j = 1,\cdots,n-1.$ (4) The continuous condition of the second-order derivative function is the cubic spline interpolation at this time $\lim_{x \to x_j} S''(x) = S''(x_j), j = 1,\cdots,n-1.$, and the above conditions are met. The cubic polynomial in $[x_j, x_{j+1}]$ can be set as $S(x) = A_j x^3 + B_j x^2 + C_j x + D_j, j = 0,1,2\cdots,n-1.$ among them A_j, B_j, C_j, D_j are a undetermined parameter. It can be seen that there are $4n$ unknown parameters in the whole interval, Because of condition 1, the function needs to go through every interpolation point, and 4N equations can be determined. From condition 2, each point of the function can be continuous to determine the N-1 equations and each equation can be determined by the same reason condition 3 and condition 4

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respectively. Two equations can be determined by the above four conditions. To solve all the unknowns, two equations are needed. At this time, we give three common boundary conditions by considering the boundary conditions: (1) given by the first derivative at the end of the interval, that is, (2)From the second derivative at the end of the interval, the special case is the natural boundary.(3) According to the periodic condition satisfied by the function value or derivative value at the end of the interval, all the undetermined parameters can be uniquely determined by selecting appropriate boundary conditions, so the cubic spline function can be uniquely determined

In the solution method of cubic spline interpolation function, if we use undetermined parameters to solve, the calculation amount is large, usually we determine the interpolation function by undetermined some parameters, use known conditions to reduce the undetermined parameters to the minimum, such as undetermined first derivative, undetermined second derivative, use the basis function method to determine the interpolation function, etc., which can greatly simplify the operation process.

In the calculation of cubic spline interpolation, because the physical meaning of the second derivative function is clear, the value of represents the moment value of a point, and at the same time, it can reduce the calculation amount in the numerical calculation, so it is also a commonly used solution. At $[x_i, x_{i+1}]$ The above is a cubic polynomial. It is known that the second derivative value of the two end points between cells is a polynomial of the third degree, so the second derivative function is a polynomial of the first degree, where $h_i = x_{i} + 1 - x_i$ (I = 0,1,...),n-1)。 The above formula is integrated once and then integrated once, where Ci and di are integral constants which can be solved by interpolation conditions. At the same time, it can be deduced from the continuity of interpolation function, and can be deduced from S. From the above derivation, we can get the expression of cubic spline in the first cell: where = 0,1,2, n-1. It can be seen that the coefficient of the interpolation function is an expression containing the second-order derivative function and the function value. In order to obtain the coefficient value, the first-order derivative function of the interpolation function at the left and right intervals of the interpolation point can be continuous, which can be defined in the interval; $S(x_i) = f(x_i) = y_i, S(x_{i+1}) = f(x_{i+1}) = y_{i+1}$ It's defined in the interval. You can get it through sortingεFrom the analysis of the above formula, it can be seen that this basic equation is a system of equations that has not yet been determined, and two equations need to be determined according to the boundary conditions[6]. Because it is a fixed spline, the boundary condition of the spline is the first derivative value of the known two ends. According to the expression of the first derivative of the spline interpolation function s (x) obtained in the previous derivation process, we can get the cubic spline interpolation function according to the above equation group. The matrix expression is as follows: Among them, the equations are tridiagonal and can be solved by Gauss elimination method. After the moment is calculated, the earth pressure of the structure can be inversed by using similar methods.

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3. Calculation example analysis

a retaining pile with a buried depth of 14m and a diameter of 1m is subjected to a unit uniform load on one side of the pile, in which the elastic modulus is e = 28gpa. The pile is considered as a cantilever beam structure in stress calculation, and the displacement of each point is calculated according to the knowledge of material mechanics. Then the calculated displacement of each point is disturbed by a random maximum value of 5%, and the obtained value is recorded as the measured displacement for inversion analysis. The analysis results are as follows

Table 1 Analysis of disturbance data from spline interpolation inversion

X(m)	0	2	4	6	8	10	12	14
Theoretical bending moment M(KN.m)	0	-5.8	-11.5	-23.1	-40.3	-57.5	-86.3	-115
Inverse moment Value M(KN.m)	0	-4.4	-9.8	-19.6	-39.2	-56.1	-87.9	-115.7
Relative error (%)	-	24.1	24.8	15.2	2.73	2.44	1.85	0.61
Theoretical earth Pressure Force value f (KN)	-1	-1	-1	-1	-1	-1	-1	-1
Inverted earth pressure Force value f (KN)	-1.28	-0.86	-1.34	-0.89	-1.21	-0.91	-1.17	-1.12
Relative error (%)	28	14	34	11	21	8	17	12

According to the analysis of the calculation results, the overall error of the results of the inverse performance of the cubic spline function is not too large, which meets certain requirements. In particular, it can be seen from the table that when the depth x = 4, the relative error is relatively large because the theoretical bending moment value is small and the calculation error of the inverse is expanded in the inversion process. At the same time, it can be found that the error of the inverted earth pressure is larger than that of the inverted bending moment, which may be caused by the continuous expansion and transmission of the error after multiple operations during the inversion of earth pressure. Generally speaking, the bending moment and earth pressure obtained by the cubic spline function inversion are within the acceptable range, which has certain reference and guidance for the actual stress analysis.

4. Conclusion

Through the analysis, we can see that the error of the moment value obtained by displacement inversion using cubic spline interpolation function is small, and the

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earth pressure on the structure is further calculated according to the moment value. Because of the expansion and transmission of the error in the inversion process, although the error is large at some points, the error is in a fixed range in general, which shows that the cubic spline interpolation. The value function is reasonable for inversion.

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