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Hybrid quantum genetic algorithm based on spin and its performance analysis

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Abstract: *Quantum computing is a new interdisciplinary science combining information science and quantum mechanics. This paper presents a hybrid quantum genetic algorithm based on spin, which implements quantum crossover on quantum individuals, which is beneficial to retain relatively good gene segments. The strategy of updating quantum gate and adaptively adjusting search grid by using quantum bit phase method; In this paper, the critical properties of quantum Heisenberg model with mixed spins on simple cubic lattice are studied by using the mean field approximation of two spin groups, and the hybrid quantum genetic algorithm based on spins is applied to solve knapsack problem. At present, many problems in the fields of industry and financial investment can be transformed into backpack problems to solve. The effectiveness of spin-based hybrid quantum genetic algorithm in solving knapsack problem has been proved by several groups of experiments.*

Keywords: *Spin, Hybrid quantum genetic algorithm, Backpack problem, performance analysis*

1. Introduction

The wonderful properties of the quantum world have made quantum computing more and more widely used. Quantum computing mainly includes two aspects: quantum computers and quantum algorithms. Although quantum computers have been proposed in the 1980s [1], it was not possible until the 1990s. The rapid development has shown that it is significantly better than digital computers in solving specific problems. Evolutionary search is carried out by quantum gate action and quantum gate update, and the phase entanglement of quantum breaks through the concept that several individuals cross to produce offspring in traditional GA (genetic algorithm) method; Quantum crossover designed by using quantum entanglement can exchange information in the whole population, which makes it easy for the population to find excellent evolutionary individuals. Literature [2-4] respectively put forward quantum genetic algorithm, genetic quantum algorithm and parallel quantum genetic algorithm, which are used to solve optimization problems. The results show that QGA (Quantum Genetic Algorithm) has much better performance than traditional genetic algorithm.

In this paper, the first quantum genetic algorithm is introduced, and then the framework of spin-based hybrid quantum genetic algorithm is proposed. Then the hybrid quantum genetic algorithms based on binary coding and real coding are given respectively. Finally, the advantages of spin-based hybrid quantum genetic algorithm in optimization quality, parameter robustness and initial value robustness are verified by numerical simulation and comparison.

2. Introduction of quantum genetic algorithm

Quantum Genetic Algorithm (QGA) is a probabilistic algorithm, which is similar to CGA(Compact Genetic Algorithm). Therefore, for QGA, suitable termination conditions have a great impact on the performance of the algorithm. In the literature [5], the establishment of termination conditions does not consider the convergence of the whole population and the convergence degree of a single gene. Therefore, although different termination parameters γ are selected, the accuracy difference of the corresponding calculated objective function values is not obvious, but the running time corresponding to different γ values is quite different. This will cause unnecessary waste of time due to improper selection of γ; In addition, the revolving door operation that promotes evolution is highly targeted, and it is not suitable to change the gene value of the corresponding bit.

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2.1 Quantum bit coding

Quantum bit is a two-state quantum system acting as information storage unit, which is a unit vector defined in two-dimensional complex vector space. This space is composed of a pair of specific standard orthogonal bases $\{|0\rangle, |1\rangle\}$. Therefore, it can be in the superposition state of two quantum States at the same time, which is defined as:

$$|\Phi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

Among them, 0 and 1 represent spin-down state and spin-up state respectively in quantum mechanics, and α, β is two complex probability amplitude pairs; $|\alpha|^2 + |\beta|^2 = 1$, "$|\ \rangle$" is a representation of quantum state; $|\alpha|^2$ can be regarded as the probability that quantum is in spin-down state. $|\beta|^2$ can be regarded as the probability that quantum is in spin-up state, so a quantum bit can contain information of 0 and 1 at the same time, which is equivalent to binary symbol 0 or 1 in binary coding. In quantum genetic algorithm, a gene is expressed by using qubits.

When $|\alpha|^2 = 1$, the gene can have "0 state";

When $|\beta|^2 = 1$, the gene can have "1 state";

When $|\alpha|^2 \neq 0, |\beta|^2 \neq 0$, the gene can have any superposition state.

Therefore, the polymorphism problem can be coded by quantum bits as follows:

$$q_t^j = \begin{bmatrix} \alpha_{11}^t & \cdots\alpha_{1k}^t & \cdots\alpha_{ij}^t & \cdots\alpha_{n1}^t & \cdots\alpha_{nk}^t \\ \beta_{11}^t & \cdots\beta_{1k}^t & \cdots\beta_{ij}^t & \cdots\beta_{n1}^t & \cdots\beta_{nk}^t \end{bmatrix} \tag{2}$$

In which q_t^j is the j th quantum individual of the t generation, and k is the number of quantum bits used for coding each quantum gene. n is the number of quantum genes in chromosome, where $|\alpha_{ij}^t|^2$ is the probability that the j th gene takes 0 when the i th independent variable is encoded by binary system with length k, and $|\beta_{ij}^t|^2$ is the probability that the j th gene takes 1 when the i th independent variable is encoded by binary system with length k. There are 2^{nk} individuals of all possible binary strings produced by q_t^j, and binary strings with $n \times k$ length corresponding to quantum coding q_t^j can be determined by measurement.

Obviously, this quantum coding method makes individuals have better diversity, and as $|\alpha|^2$ and $|\beta|^2$ tend to 0 or 1, chromosomes will converge to a single state.

2.2 Quantum genetic algorithm

Step1: Let $t = 0$, take the initial population $Q(t)$ with the population size of N;

Step2: Measure each individual of that initial population once to obtain a state $p(t)$;

Step3: Calculate fitness for each state;

Step4: Record the best individual and its fitness value;

Step5: While (termination condition not met)

1) $t = t + 1$;

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2) Measure each individual in population $Q(t)$ once to obtain a state $p(t)$;

3) calculate the fitness for each state;

4) update individual population by rotating quantum gate operation;

5) Record the best individuals and their fitness.

End

3. Hybrid quantum genetic algorithm based on spin

Heisenberg model is an important quantum spin system, which can be used to explain the quantum essence of many magnetic substances [6], so it has become one of the hot issues in people's research, and many scholars have studied it [7-8].

Hamiltonian of mixed Heiseberg model can be written as:

$$H = -J\sum_{\langle ij\rangle}\left[(1-\Delta)\left(S_i^x\sigma_j^x + S_i^y\sigma_j^y\right) + S_i^z\sigma_j^z\right] \tag{3}$$

Where J is the coupling coefficient of the nearest spin pair, and $J > 0$ corresponds to a ferromagnetic body; $J < 0$, corresponding to antiferromagnet; $S_i^\alpha(\alpha = x, y, z)$ represents the spin component of spin $S = 1$ on lattice point i; $\sigma_j^\alpha(\alpha = x, y, z)$ represents the spin component of spin $S = 1/2$ on lattice point j; $\sum_{\langle ij\rangle}$ represents the sum of all the nearest spins on the lattice, Δ is an anisotropic constant, Δ is a constant of $-\infty < \Delta \leq 1$, the model corresponds to isotropic Heisenberg model when $\Delta = 0$, Ising model when $\Delta = 1$, and isotropic XY model when $\Delta =$ -∞.

According to the average field theory, the average magnetic moments of two spins along the $\hat{z}$ direction are $m_1 = \langle S_1^z\rangle$ and $m_2 = \langle \sigma_2^z\rangle$ respectively, so the Hamiltonian of two spin groups can be written as:

$$H_{12}^{MFA} = -J\left[(1-\Delta)\left(S_i^x\sigma_j^x + S_i^y\sigma_j^y\right) + S_i^z\sigma_j^z\right] - J(q-1)\left(m_1\sigma_2^z + m_2 S_1^z\right) \tag{4}$$

Among

$$S_1^x = \frac{1}{\sqrt{2}}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}, S_1^y = \frac{1}{\sqrt{2}}\begin{bmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{bmatrix}, S_1^z = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}, \sigma_2^x = \frac{1}{2}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$
$$\sigma_2^y = \frac{1}{2}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}, \sigma_2^z = \frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \tag{5}$$

In the direct product representation of S_1^z and S_2^z, H_{12}^{MFA} can be written as a 6 $\times$6 matrix, and the partition function of two spin groups can be calculated as follows:

$$Z = 2\exp\left(\frac{K}{2T}\right)\cosh\left\{\left[x-(q-1)m_1\right]\frac{K}{2T}\right\} + 2\exp\left(\frac{K}{4T}(x-1)\right)\cosh\left(\frac{K}{2T}y\right) + 2\exp\left[-\frac{K}{4T}(X+1)\right]\cosh\left(-\frac{K}{2T}y\right) \tag{6}$$

Among

$K = \beta J(\beta = 1/(k_B T), k_B \text{ is the Boltzmann constant})$,

$x = 2(q-1)(m_1+m_2), y = \sqrt{[(q-1)m_2 - 1/2]^2 + 2(1-\Delta)^2}$ in a one-dimensional spin chain, the free energy of a two-spin group can be written as:

$$F(m_1, m_2, T) = J(q-1)m_1 m_2 - \beta^{-1}1nZ \tag{7}$$

The dimensionless free energy is:

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$$f\left(m_1,m_2,K\right)=\frac{F\left(m_1,m_2,T\right)}{J}=\left(q-1\right)m_1m_2-K^{-1}1nZ \tag{8}$$

According to the condition $\partial f/\partial m_1=0,\partial f/\partial m_2=0$ that the system energy has extreme value, two equations about m_1 and m_2 can be obtained. when the anisotropy parameter Δ takes different values, the relationship between magnetization m_1 and m_2 and reduced temperature k_BT/J can be obtained by numerical solution. By defining $m=\left(m_1+m_2\right)/2$ as the average magnetization of two spin groups, the relationship between m and reduction temperature k_BT/J can be obtained.

4. Results and discussion

At present, knapsack problem is widely used in industrial and financial investment fields, such as financial management, budget control, stock investment and cargo loading. For example, in an investment problem, the total number of currencies owned by an investor is C , and all these currencies are used to invest in n projects, which of course can be invested at will. In which the input of a certain project i is w_i , and the income obtained is p . The ultimate goal is to design an investment scheme to maximize the total profit. So the investment problem can be mapped into a simple knapsack problem.

This part mainly compares the three algorithms through three groups of experiments, and the first and second groups of experiments respectively compare the convergence speed and average calculation time of the three algorithms.

(1) Convergence speed of the algorithm

This group of experiments mainly compares the convergence speed of GA, QGA and the algorithm in this paper. This group of experiments mainly studies the influence of the number of items as the main parameter on the convergence speed of the algorithm. The convergence speed of the algorithm is shown in Figure 1.

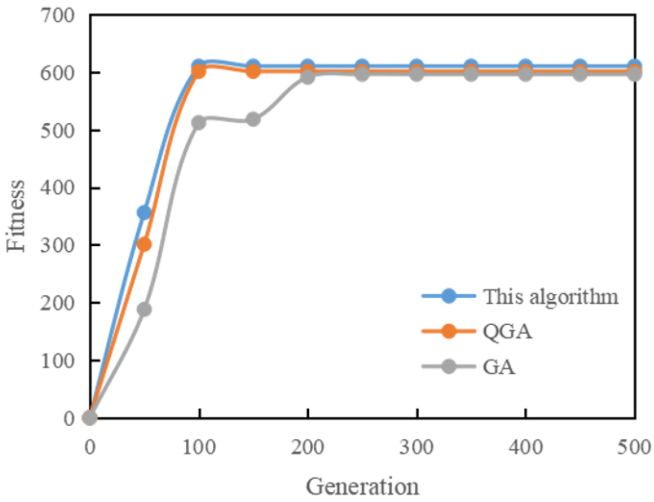

Figure 1: Contains 100 items

It can be seen from Figure 1 that the convergence speed of the proposed algorithm is obviously better than the other two algorithms. Through observation, the algorithm proposed in this paper generally converges in about 50 generations, and the convergence speed is relatively fast; The algorithm in this paper converges in about 150 generations, and the convergence speed is moderate; QGA generally converges in about 200 generations, which is not as fast as the previous two algorithms. On the whole, the algorithm in this paper fully complements the advantages of the two cooperative algorithms, and obtains a good convergence speed.

(2) Average computing time of algorithm

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Table 1: Average calculation time (s)

	m	n	This algorithm	QGA	GA
I_0	100	20	125.30	1.25	133.25
I_1	100	30	254.12	5.56	241.71
I_2	200	10	477.24	1.74	458.93
I_3	200	50	1405.63	8.83	1477.12
I_4	500	30	4426.85	2.01	4336.98
I_5	500	50	5670.19	14.29	5521.08

In this experiment, the average computing time was obtained after three algorithms evolved for 100 generations. It can be seen from Table 1 that the average calculation time of this algorithm is the shortest, followed by QGA, and the longest time is GA. This shows that the algorithm in this paper keeps the advantages of QGA and GA operators, and the calculation time is always short. QGA needs more time because of establishing probability model and sampling solution space. The algorithm in this paper has the longest time because of the co-evolution of two sub-populations and the need for information interaction between sub-populations. Although the algorithm in this paper takes a long time, it can be seen from the optimal results and average results that its optimization performance is the best. Therefore, the algorithm in this paper needs to co-evolve and increases the calculation time, but obtains better optimization ability.

5. Summary

In this paper, a hybrid framework of quantum genetic algorithm and traditional GA is proposed. A hybrid quantum genetic algorithm based on spin is proposed by using the hybrid of multi-modal space and multi-operation search (i.e., hybrid search of quantum search space and genetic code space and multi-directional hybrid search of quantum search and genetic search). The algorithm is applied to solve knapsack problem. In the experiment, different collections of objects and different populations are selected as parameters, and the convergence speed and calculation time are comprehensively analyzed. The results show that the spin-based hybrid quantum genetic algorithm has better performance.

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