

Research on Strategies for Integrating Mathematics History into High School Mathematics Teaching

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Abstract: With the establishment of HPM (International Thematic Organization on the Relationship between the History of Mathematics and Mathematics Teaching), China's education has been continuously reformed and developed, but there are still some problems in the practice of integrating the history of mathematics into high school mathematics teaching. Based on the relevant literature at home and abroad in the past ten years retrieved by the Science Citation Database (WoS) and China Knowledge Network (CNKI), this paper systematically sorts out the current situation of integrating the history of mathematics into high school mathematics teaching, and then summarizes the resistance affecting the integration of the history of mathematics according to the current research and analysis, and puts forward corresponding strategies to provide help and reference for front-line teachers and related educators. Finally, the current research is reviewed, prospected, and summarized.

Keywords: History of mathematics, HPM, High school mathematics teaching, Status analysis, Strategy

1. Introduction

In today's implementation of quality education, the educational function of the history of mathematics has been widely recognized by all walks of life in education. Teachers can integrate the history of mathematics so that students can not only systematically learn mathematical knowledge, but also have a contextual understanding of the generation, formation, and development of mathematical knowledge, to form a correct mathematical thinking mode while cultivating students' mathematical history literacy. But now, for the principles and methods of integrating the history of mathematics, there is a lack of practical application in the first-line teaching due to the pressure of the college entrance examination, and there is no scientific and effective research plan for integrating the history of mathematics into mathematics teaching in the field of mathematics education research.

Therefore, how to formulate a set of scientific and effective programs, how to integrate the history of mathematics into the classroom, teachers' lack of basic literacy of mathematics history, students' wrong view of mathematics, and the compilation of textbooks are some of the problems that currently plague the integration of mathematics history into mathematics teaching. Therefore, this paper takes the literature searched by WoS and CNKI with 'history of mathematics', 'HPM', 'high school mathematics teaching', 'current situation analysis', and 'strategy' as the research object, summarizes the current situation at home and abroad, and analyzes the resistance and strategy of integrating the history of mathematics into high school mathematics teaching.

2. Foreign research overview of the history of mathematics in high school mathematics teaching

2.1. Why the history of mathematics in high school mathematics teaching

In recent years, some experts have discussed the concept of why the history of mathematics is integrated into mathematics education through classification analysis. Fauvel first summed up the fifteen main reasons for using the history of mathematics in mathematics education, and these are based on the principle of historical occurrence, which is also the main ideological basis for integrating the history of mathematics into mathematics education and teaching[1]. Tzanakis and Arcavi put forward seventeen reasons for the development of mathematics history and the enrichment and improvement of mathematics education from five perspectives: mathematics knowledge, research on mathematics principles and mathematics activity thoughts, teachers' academic background and basic preparation, mathematics

emotion, teaching, and art form appreciation.

The influence of the history of mathematics on students, such as Helfgott, Jahnake, and Kleiner, pointed out that the study of the history of mathematics can bring different perspectives and expression methods to classroom teaching, thus promoting students' understanding of mathematics[2]. Many researchers, such as Farmaki, Paschos, and Taimina, have emphasized that interesting facts in the history of mathematics can arouse and protect children's curiosity about mathematics, which has always been regarded as the best teacher [3].

2.2. How to integrate the history of mathematics into high school mathematics teaching

2.2.1. Methods of integrating mathematics history into mathematics teaching

There are many different ways to integrate the history of mathematics into the mathematics curriculum, which is mainly determined by the professor's teaching style, beliefs, and the choice of the history of mathematics. Fauvel provided nearly ten specific application methods, such as introducing the interesting deeds of past mathematicians, telling students the development history of new concepts, and setting up modern mathematics history courses[4]. Tzanakis and Arcavi summarized three main methods of using the history of mathematics in mathematics education: one is to give the most direct historical data; the second is to teach with the help of history, that is, the occurrence teaching method; third, by cultivating students' deeper understanding of the background of mathematics history, teachers can appropriately adopt these three teaching methods for teach. Jankvist identified three other approaches: heuristic, modular, and history-based[5]. Bid well puts forward three ways to use the history of mathematics: one is to introduce the deeds, photos, and stamps of some famous mathematicians in the classroom; second, add historical materials to teaching activities to increase the fun in teaching; the third is to regard the historical development stage as the content of teaching itself.

The above way of integrating the history of mathematics into mathematics teaching can be roughly summarized as directly using the history teaching method, integrating the history of mathematics into the whole process of teaching, and using heuristic teaching, so that students can experience the history of mathematics development, to cultivate students' strong interest. In the practical course, teachers should choose the corresponding way to integrate the history of mathematics for different courses, but they must also improve the level of integration, such as simply explaining the history of mathematics in the classroom to cultivate students' interests and hobbies, so students will feel that the history of mathematics is only an adjunct to the basic knowledge of mathematics he has learned, or that this is something that he has experienced a long time ago, and lacks the function and significance of the practice. On the contrary, in the mathematics curriculum, through a deeper integration method, the mathematical knowledge of various historical eras and fields can be linked together, so that learners can experience the generation and development of knowledge and regard mathematics as a product of human active creation.

2.2.2. The mode of integrating mathematics history into mathematics teaching

The integration of mathematics history into mathematics teaching should also consider other factors, such as the nature of teaching and the situation of students. Therefore, foreign researchers have formed several models of integrating mathematics history into mathematics education.

Ferlinghetti put forward the following teaching modes: mastering historical cultural resources→selecting appropriate topics→analyzing classroom teaching needs→designing classroom teaching practice activities→implementing plans→evaluating classroom teaching practice activities. The introduction of new concepts has created a major platform for the integration of modern mathematics history into mathematics classrooms[6]. For example, Grabiner and Davitt considered the four stages of derivative theory from the perspective of historical and cultural development, that is, the use-research-discussion/development-definition, named UDED (Use-Discover-Explore /Develop-Define) model, which is formed according to the order of historical culture of mathematics education thinking, especially suitable for the introduction of mathematics education theory[7]. Moreover, teachers can not only use the UDED model as a tool to obtain historical and cultural materials for the rapid development of mathematical experience but also use it as a teaching strategy in the classroom, so teachers should apply the integration model according to the nature of the course and the situation of students.

2.2.3 The strategy of integrating mathematics history into high school mathematics teaching for teachers

From the perspective of educational professional development, Ball, an American mathematics educator, pointed out the theory of Mathematical Knowledge for Teaching (MKT), which points out that

the basic knowledge required by the mathematics classroom consists of six parts, namely, general content basic knowledge, level content basic knowledge, professional content basic knowledge, content, and student knowledge, teaching content and classroom basic knowledge, and content and teaching basic knowledge. Based on the concept of MKT, Zopf and Kim carried out an in-depth study of the mathematical knowledge for Teaching Teachers (MKTT), which is necessary for educators[8]. They pointed out that MKTT covers the special knowledge points in MKT, as well as the knowledge of mathematics itself, covering the professional knowledge in the field of mathematical structure such as mathematical concepts, attributes, and theorems, and the understanding of the essence of subject professional knowledge. The origin, evolution, and creation of theoretical knowledge[9].

For teachers to use the history of mathematics in the classroom, Professor Ferlinghetti gives methods and strategies and points out the main ways to provide historical data for the classroom: First, consult the literature material about the history of mathematics; then, select historical fragments and important authors; again, carefully study the original data; finally, prepare classroom teaching materials. In this way, it means that in general, teachers' knowledge in reading mathematics history textbooks is insufficient, so teachers must deeply study the original literature of teaching content, because the original literature can help us to clarify and expand the connotation of the second-hand materials, and can explain the connotation and evaluate its function. The classroom materials obtained on this basis are also very effective tools, which help to successfully penetrate the history of mathematics into the mathematics classroom.

Therefore, as a first-line teacher, to have a perfect class, it is generally necessary to prepare students, prepare teaching materials and prepare teaching methods. Then, when applied to the classroom of integrating the history of mathematics, teachers should first improve their knowledge of the history of mathematics, and then think about how to enable students to truly understand this course from the perspective of historical development, and finally master some teaching methods of integrating the history of mathematics into the classroom.

2.2.4. Strategy research of interesting mathematics as classroom material

The research report of HPM-2016 Global Mathematics Education History and Contemporary Mathematics Education Conference, in-depth research on interesting mathematical ideological education works in the history of mathematics education, provides new direction materials for HPM in-depth research. The French mathematicians Lucas, Fourrey and C. A. Laisant's related works were selected by BARbin of Nantes College in Paris and GuitArt of Diderot College in Paris respectively. They studied their interesting mathematical problems from three aspects: history, education, and mathematics, and emphasized that they were inseparable at these three levels. For example, Professor Lucas used geometric methods to deal with the distribution of ancient Chinese heritage in his 'interesting arithmetic', which realized the unified visualization of the solution of arithmetic and algebraic problems[10]. It can be seen that this interesting mathematical problem is not only entertaining, but also inspiring, and has great teaching value.

In summary, from the above four aspects to explain the problem of 'how to integrate', mathematics historians and front-line teachers have jointly proposed some methods, principles, models, and strategies for integrating the history of mathematics into high school mathematics teaching. At this stage, front-line teachers are also trying to apply theory to practice, but the application is affected by various factors. It has not been able to meet expectations. Although there is no relatively unified step and complete set of solutions to say 'how to integrate', the core of integrating the history of mathematics into high school mathematics teaching is based on 'why to study'.

3. Summary of domestic research on the integration of mathematics history into high school mathematics teaching

3.1. Research on the current situation of integrating mathematics history into high school mathematics teaching

At the end of the 20th century, international HPM standards gradually entered the Chinese education market. However, since the 1930s, the research on the history of mathematics and mathematics education in China has gone through four historical stages of 'germination-development-reform-integration', and gradually evolved from 'history for history' to 'history for education'. At present, the work of HPM in China is vibrant and on the ascendant[11].

So, why infiltrate the history of mathematics into the teaching classroom of high school? Among them, Song said in the report of the Fifth China Contemporary Mathematics Education History and Mathematics Education Research Association: ' Mathematics history can encourage children to be interested in mathematics, understand mathematics, master mathematics, and finally learn mathematics well '. Zhang and Wang pointed out that the research on the history of mathematics oriented by modern mathematics teaching, the design of mathematics curriculum based on the history of mathematics, and the practice and exploration of integrating the research on the history of mathematics into the modern mathematics curriculum are the key aspects of HPM research[12]. Zhang also believes that the application of mathematical history research teaching can transform theoretical mathematical knowledge into knowledge of educational forms. It can not only clarify the basic laws of mathematical development, to establish a reasonable mathematical teaching concept, but also clarify the general process of mathematical development to make it more suitable for today 's teaching.

Domestic researchers have conducted a series of discussions on how to integrate the history of mathematics into the classroom in response to the problems raised by predecessors. It mainly uses the reconstruction history and the occurrence teaching method, that is, from the starting point of the learner's understanding, a set of teaching methods, teaching modes, and teaching principles have been developed. Among them, the most commonly used four methods of integrating the history of mathematics into the mathematics curriculum are additional, repetitive, adaptive, and reconstruction. The classroom teaching mode has two types explicit and implicit, and the teaching principles are scientific, learnable, effective, interesting, and humanistic. Under the theoretical support of the feasibility and necessity of the above series of mathematics history, the high school curriculum standards and textbooks have been adopted. The knowledge that is not easy to understand in high school can be integrated into the number.

3.2. Research on the resistance to integrating mathematics history into high school mathematics teaching

3.2.1. Research on teacher-level resistance

Some researchers have explored the problem of integrating the history of mathematics into high school mathematics teaching from the teacher level. Among them, Xu and Yang believe that the front-line teachers ' knowledge reserve of the history of mathematics is insufficient. Because the front-line teachers do not have enough training before they are in service, and the knowledge of the history of mathematics is not reasonably applied in practical teaching, this reason has a very large resistance to the process of integrating the history of mathematics into high school mathematics teaching. Zhou, Zhou and Chen, and others believe that teachers' ability to consciously use the history of mathematics is not high, and there is not much deep thinking about how to reasonably incorporate the history of mathematics into the mathematics curriculum, and most of the materials of the history of mathematics in the textbooks are ignored. They think that the addition of historical mathematics knowledge is worried about affecting the teaching progress. For a long time, the actual value of the history of mathematics cannot be stimulated in the classroom[13][14]. Some researchers also point out that teachers are too dependent on traditional teaching contents and methods, and teachers often adapt to the original traditional teaching methods. It is difficult to achieve the ideal teaching effect by incorporating the history of mathematics into mathematics teaching, so the integration of mathematics history into high school mathematics teaching is a mere formality.

3.2.2. Research on the resistance to curriculum reform

Wang, and Li mentioned that the new curriculum standard requires the course 'Selected Lectures on the History of Mathematics '. However, due to the pressure of college entrance examinations, many colleges and universities have not realized the purpose of setting up elective courses in the history of mathematics even if such courses are arranged. In terms of improving the curriculum structure of mathematics departments in normal colleges and universities, some scholars believe that the current high school teachers' literacy of mathematics history is not high, which can be traced back to the pre-service teachers' normal education stage. Some mathematics departments do not offer mathematics history courses to students, or mathematics history appears in the training system in the form of elective courses. The curriculum design is not refined and the degree of implementation is not enough, resulting in poor teaching effect.

3.2.3. Research on resistance at the textbook level

Wang, Department of Mathematics, East China Normal University, proposed that the methods of applying the history of mathematics in mathematics courses mainly include additional, repetitive,

adaptive, and reconstructive[15]. However, through the analysis of the current editions of high school mathematics textbooks, some knowledge of mathematics history is still integrated with additional forms, and the other three forms of integration are relatively rare. Moreover, the frequency and form of the history of mathematics in different chapters are different, and there is an imbalance.

The above analysis of the resistance to the integration of mathematics history into high school mathematics teaching from three aspects can be summarized as subjective and objective aspects. On the subjective side, the first-line teachers' mathematics history literacy is low, and they do not attach importance to the arrangement of mathematics history courses. Even if the method of integrating mathematics history is adopted, it is only additional integration, which does not achieve the purpose of integration, resulting in students not mastering its methods. Objectively, due to the lack of a complete system and teaching materials for the integration of mathematics history into teaching, there are few cases that teachers can learn from, and because the college entrance examination puts more pressure on students and teachers, teachers are weak, and the quality of the integration of mathematics history in the college entrance examination questions is not high, so everyone does not pay enough attention to the history of mathematics, which has become some resistance to the integration of mathematics history into high school mathematics teaching.

3.3. The strategy of integrating mathematics history into high school mathematics teaching

According to the former's analysis of the current situation and resistance to the integration of mathematics history into high school mathematics, this paper summarizes the existing strategy research from three aspects: teachers, curriculum reform, and teaching materials, and finally puts forward its views.

3.3.1. Research on teacher-level strategies

Professor Zhu mentioned in the article at the teacher level that teachers should pay attention to when applying the history of mathematics: First, teachers should strengthen their knowledge of the history of mathematics, reasonably select, synthesize, modify and creatively process the data of the history of mathematics, and apply it in the classroom according to the principle of historical occurrence; second, advocate students to consult relevant materials of mathematics history on their own to stimulate students' interest; third, improve students' mathematical literacy and moral sentiment through the study of mathematical history[16]. Moreover, for the current situation of low literacy of front-line teachers in the history of mathematics, targeted post-service training and teaching and research activities are very effective measures. Chen suggested that mathematics teachers should strengthen their mathematical history literacy and actively participate in training and assessment[17]. Chao and Zhang mentioned that training should be targeted. In addition, it is necessary to clarify that learning theoretical knowledge is not the ultimate goal. Targeted training should be combined with first-line teaching, encouraging teaching and research activities among teachers in different schools, so that teachers can promote the improvement of mathematical history literacy in continuous reflection and discussion, and finally move from theory to educational practice[18].

Finally, some researchers put forward that the researchers of the history of mathematics need to cooperate with the front-line teachers of the school, set up a professional design team to integrate the study of the history of mathematics into high school education, develop resources and excellent cases for teachers to use, and finally summarize a systematic and effective scientific research method on the integration of the history of mathematics into mathematics education[19].

3.3.2. Research on the strategy of curriculum reform

Wang proposed that the introduction course guided by HPM theory is helpful to students. The teaching method is established by the introduction course, and the course is guided by the history of mathematics. It helps students to grasp in time what knowledge points will be introduced in a topic or module, and what is the connection between each knowledge point, which also lays a solid foundation for future teaching work[20].

Some people think that modifying the writing form of mathematics history in college entrance examination questions is also helpful to integrate mathematics history into high school mathematics. With the further development of the history of mathematics in China, the proportion of the history of mathematics has also been increased in the college entrance examination, but there are some problems in the preparation of the test questions. Wang and Li gave suggestions: the selection of material background should be simplified, the history of mathematics should be 'hidden', and the core literacy of students should be examined based on the essence of the history of mathematics[21].

In terms of improving the curriculum structure of the mathematics department in normal universities, according to the above resistance analysis, Chen put forward some suggestions on the application of mathematics history to middle school mathematics education: the curriculum structure of the mathematics department in normal universities needs to be adjusted. Based on retaining the original curriculum, the relevant courses of mathematics history should be opened in the form of compulsory or elective courses for mathematics majors, the course credits should be stipulated and the academic assessment should be carried out[17]. Some scholars believe that teachers' examinations should be included in the examination of the history of mathematics, which not only increases the knowledge of teachers' history of mathematics, but also provides help for better teaching in front-line positions in the future.

3.3.3. Strategy research from the textbook level

Wang and Wang pointed out that the design of mathematics history in high school mathematics textbooks should have: first, the content of the textbook is clear and clear, so that students can form a systematic understanding before and after reading the knowledge of the textbook, with a certain continuity; secondly, if there is relevant knowledge containing important mathematical ideas, it should be reflected in the textbook; thirdly, for famous mathematicians, their images should be drawn in the teaching materials, so that students have respect and solemnity for mathematicians[22]. For the choice of the content of the history of mathematics, Wang and Wang also mentioned, first, should reflect the development process of all ethnic groups, and how to integrate and promote; second, reflect the different solutions to the same problem, so that students understand the idea of multiple solutions to a problem and different ways of thinking under different cultural backgrounds; thirdly, it reflects the setbacks and experiences encountered by mathematicians in the process of exploring truth, so that students can deeply understand the perseverance of mathematicians; fourth, increase readability; fifth, change the form of historical knowledge introduction, teachers can selectively let students complete the historical title, too deepen students' understanding of the history of mathematics, arouse students' enthusiasm for mathematics.

In summary, through the strategy research on teachers, curriculum reform, and teaching materials, the above three strategies can not only improve students' interest in learning the history of mathematics, lay a solid foundation for future mathematics learning, but also open up a new development path for teachers' teaching methods and achieve a win-win situation.

4. Discussions

4.1. Review of the integration of domestic and foreign mathematics history into high school mathematics teaching

Although the research on the integration of mathematics history into high school mathematics teaching at home and abroad has made great achievements, there are still some shortcomings. At present, the application of mathematics history still shows the current situation of 'high evaluation, low application', mainly because there is no suitable teaching method, and the attention of schools and teachers is not high. At the same time, it can also be found that there are many repetitive studies on the application of mathematics history to high school mathematics teaching, and there are many theories about the integration of mathematics history, but the specific implementation strategies of mathematics history and the integration of mathematics history into high school mathematics teaching are not deep enough. Then, based on summarizing the results of previous studies, this paper puts forward the corresponding specific strategies from the existing resistance. Next, I will continue to study the evaluation of the integration of mathematics history into classroom teaching. I hope that through the strategic research of this paper, we can improve students' enthusiasm for learning mathematics and help front-line teachers in teaching practice.

4.2. Prospect of integrating mathematics history into high school mathematics teaching

Based on the analysis of historical experience and scientific research status, we can summarize the future development trend of the history of mathematics: (1) pay attention to the objective evaluation of integrating the history of mathematics into teaching; (2) Deepen the discussion of HPM on teacher growth; (3) Strengthen HPM research and application research on students' core literacy; (4) Exploring HPM research from an interdisciplinary perspective. The field of HPM research spans the two major disciplines of mathematics history and mathematics education. It is an international academic field with

outstanding characteristics and great development prospects. Since entering the 21st century, the field of HPM research in China has been flourishing.

5. Conclusion

In summary, in foreign research, the main discussion is 'why' and 'how'. The former has been more mature and widely recognized. The latter is reviewed from the existing integration methods, models, principles, and other aspects. The domestic research, mainly discusses the current situation of domestic scholars' research and the resistance encountered. According to the resistance, it analyzes and summarizes the strategic research on teachers, curriculum reform, and teaching materials.

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