

Bearings only passive location for UAV in formation flight

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Abstract: *With the increasing development of UAV technology, UAV formation flight has also changed from single target task execution to multi-target cooperative task execution. When UAV is developing rapidly in the direction of clustering, there are too many external interference factors. Therefore, electromagnetic silence should be maintained for a long time in the whole formation flight, and the number of times when transmitting electromagnetic wave signals to the outside should be reduced as much as possible. Therefore, this paper uses the sine cosine theorem to adjust the position of UAV to the optimal position by using bearings only passive location method to maintain the formation of UAV multi-target flight. In this paper, the cosine theorem and sine theorem are used to locate and optimize the other UAVs that receive signals passively; By enumerating, the corresponding linear equations are listed by using the triangle formed by all UAVs; The position of each UAV shall be adjusted step by step according to the above conclusions, and the remaining UAVs receiving signals passively shall be adjusted to the optimal position to ensure the uniform distribution of nine UAVs on the circumference with a radius of 100m [1-3].*

Keywords: *Passive location; Sine theorem; Cosine theorem; Enumerative method*

1. Introduction

In recent years, with the continuous development of international and domestic science and technology, the UAV industry has also developed rapidly. With the continuous growth of people's demand for multi-point collaborative task solving, the single target of UAV navigation can not quickly solve the multi-directional complex tasks, so it is necessary to accomplish the tasks through multi-target cooperation. When UAVs are flying in a group formation, there are too many external interference factors, so electromagnetic silence should be maintained for a long time during the whole formation flight, and the number of times when sending electromagnetic signals to the outside should be reduced as much as possible [4]. Passive positioning does not need to transmit signals, but passively receives target radiation signals or reflected signals from other signal sources, so as to solve relevant parameters and obtain target location information [5]. The UAV location method based on passive location has smaller blind area, can work in bad weather, has the ability to detect stationary targets, carry out early warning and find UAV operators, and is more suitable for positioning UAVs [6].

When determining the position of the UAV at the center of the circle and the position of the other two UAVs on the circumference, and three UAVs transmitting signals without deviation, and determining the relative position relationship of ten UAVs with slight deviation on the position, when selecting the signal transmitted by the UAV with the number of FY00 and at most three UAVs on the circumference each time, it is necessary to establish a mathematical model to locate and adjust the other UAVs receiving signals passively.

2. Assumptions and symbols of the model

2.1 Assumptions of the model

(1) It is assumed that the position deviation of UAV can be continuously reduced until it reaches the expected position and reaches the precision of the predetermined included angle signal.

(2) Assuming that there are almost no external interference factors, the UAV will keep electromagnetic silence for a long time, and almost no electromagnetic wave signal will be sent out.

2.2 Symbol Description

The symbols in this document are shown in Table 1.

Table 1: Symbol Description Table

Symbol	Description
$\alpha, \beta, \alpha_1, \alpha_2, \varepsilon$	Angle between UAV and UAV connection
M, N, K	UAV number (any number from 1 to 9)
r, z, x, y, l	Distance between UAVs
ε, θ	step
n, k	Number of adjustments
A, B, C, D, H	Location identification of UAV
α'	Polar angle of UAV during adjustment
r'	Polar diameter of UAV during adjustment
p	Number of UAVs
t	Number of signal directions of signal receiving UAV

3. Establishment and solution of model

3.1 Application of Sine and Cosine Theorems

In addition to the signal transmitted by the circle center UAV FY00, it is necessary to select two UAVs to transmit signals. Suppose that the selected UAVs are FY0M and FY0N ($M > N$, M and N are any one of the numbers 1 to 9). For the UAV FY0K that needs to be located or has deviation, its signal receiving direction with the UAV FY00 and the UAV FY0N is α_1 , its signal receiving direction with the UAV FY0N and the UAV FY0M is α_2 , and its signal receiving direction with the UAV FY00 and the UAV FY0M is $(\alpha_1 + \alpha_2)$. The direction of UAV FY00 and UAV FY0N and UAV FY0M is $\beta = (M - N) \times 40^\circ$, and the direction of FY00 and FY0K and FY0N is ε . The distance between FY00 and FY0N and between FY00 and FY0M is r , the distance between UAV FY0M and UAV FY0N is z , the distance between UAV FY00 and UAV FY0K is x , the distance between FY0K and FY0M is y , and the distance between FY0K and FY0N is l .

(1) Situation 1

Make a circle with the distance z between FY0N and FY0M as the diameter, and locate the UAV outside the circle, as shown in Figure 1. At this time, the known conditions are $\alpha_1, \alpha_2, r, \beta = (M - N) \times 40^\circ$.

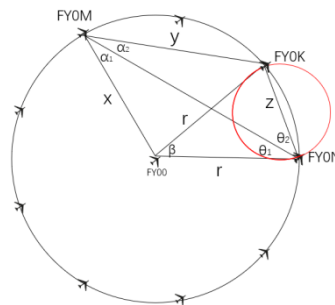


Figure 1: UAV Positioning Map (1)

In $\triangle OKN$

$$\cos \beta = \frac{2r^2 - z^2}{2r^2} \quad (1)$$

In $\triangle OMK$

$$\cos (\alpha_1 + \alpha_2) = \frac{x^2 + y^2 - r^2}{2xy} \quad (2)$$

In $\triangle OMN$

$$\frac{r}{\sin \alpha_1} = \frac{x}{\sin \theta_1} \quad (3)$$

In $\triangle NMK$

$$\frac{z}{\sin \alpha_2} = \frac{y}{\sin \theta_2} \quad (4)$$

$$\theta_1 + \theta_2 = \frac{\pi - \beta}{2} \quad (5)$$

The unknowns in the question are x, y, θ_1 and θ_2 the position and deviation of FY0K can be calculated according to formula (1) (2) (3) (4).

(2) Situation II

Make a circle with the distance z between FY0N and FY0M as the diameter to locate the UAV outside the circle, as shown in Figure 2. At this time, the known conditions are $\alpha_1, \alpha_2, r, \beta = (M - N) \times 40^\circ$.

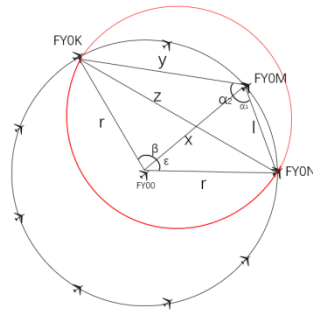


Figure 2: UAV Positioning Map (2)

In $\triangle NOK$

$$\cos \beta = \frac{2r^2 - z^2}{2r^2} \quad (6)$$

In $\triangle NOM$

$$\frac{r}{\sin \alpha_1} = \frac{l}{\sin \varepsilon} \quad (7)$$

In $\triangle MOK$

$$\frac{r}{\sin (\alpha_2 - \alpha_1)} = \frac{y}{\sin (\beta - \varepsilon)} \quad (8)$$

In $\triangle NMK$

$$\cos \alpha_2 = \frac{y^2 + l^2 - z^2}{2yl} \quad (9)$$

3.2 Application of Enumeration

Set the UAV numbered FY00 as point A, the UAV numbered FY01 as point B, and one UAV added on the circumference as point C, and so on as point D, point E, point F, etc. Set the position of unknown UAV receiving signal as H-point. Suppose one UAV is added first, that is, point C, then three triangles $\triangle AHC$, $\triangle BHC$ and $\triangle AHB$ can be formed. The signal angle of the UAV receiving the signal is $\angle AHB = \alpha_1$, $\angle BHC = \alpha_2$ and $\angle HBC = \beta$. In the three triangles, HA , HB , HC , BC and $\angle \beta$ are unknown quantities, and the direction angles $\angle \alpha_1$, $\angle \alpha_2$ and radius r of H point are known quantities. Then linear equations can be listed for solving by these quantities. Whether to continue to add UAVs can be judged according to the number of solutions. If necessary, repeat the above steps.

According to the description of the above problems and the establishment of the model, it is assumed to add an UAV first, as shown in Figure 3:

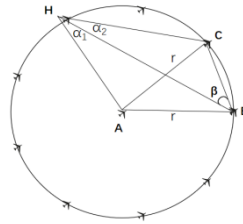


Figure 3: UAV Positioning Map (3)

In $\triangle AHB$

$$\cos \alpha_1 = \frac{HA^2 + BH^2 - r^2}{2HA \times BH} \quad (10)$$

In $\triangle BHC$

$$\cos \alpha_2 = \frac{HC^2 + BH^2 - BC^2}{2HC \times BH} \quad (11)$$

In $\triangle AHC$

$$\cos(\alpha_1 + \alpha_2) = \frac{HC^2 + HA^2 - AC^2}{2HC \times HA} \quad (12)$$

In $\triangle BHC$

$$\frac{BC}{\sin \alpha_2} = \frac{HC}{\sin \beta} \quad (13)$$

In $\triangle BHC$

$$\cos \beta = \frac{HB^2 + BC^2 - HC^2}{2HB \times BC} \quad (14)$$

Unknown quantities are BC , HB , HC , HA and β , the known quantities are α_1 , α_2 and radius r . According to Formula (10), (11), (12), (13) and (14), the linear equation group has a solution but is not unique, that is, the direction of the UAV receiving the signal cannot be determined, so another UAV is needed, as shown in Figure 4:

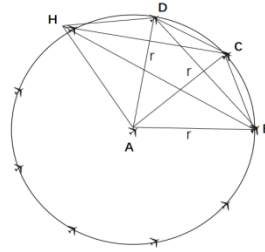


Figure 4: UAV Positioning Map (4)

According to the above figure, there are six triangles, namely $\triangle AHC$, $\triangle AHB$, $\triangle AHD$, $\triangle BHC$, $\triangle BHD$, and $\triangle CHD$. According to the known and unknown quantities, the corresponding linear equations can be listed. After analysis and calculation, the solution of the linear equations is the only solution, and the direction of the UAV receiving the signal can be determined, that is, the answer is the desired one.

Therefore, at least 2 UAVs are required to transmit signals to achieve effective positioning of UAVs.

3.3 Positioning and adjustment of optimal position

Let the radius of the circle be r , the position of the UAV numbered FY00 is point A, the position of the UAV numbered FY01 is point B, the final position of the UAV numbered FY02 is point C, and the actual position of the UAV numbered FY02 is point D, $\angle DAB$ is α , $\angle ADB$ is β . According to the analysis and model establishment of the first question, if 9 UAVs are evenly distributed on a circle, it can be concluded that $\triangle ABC$ is an isosceles triangle with $\angle CAB$ as 40° , so $\angle ACB = \angle ABC = 70^\circ$. Assume that the position of an unknown UAV is evenly distributed on the circumference, so we can adjust the specific position of an unknown UAV step by step with the UAVs numbered FY00 and FY01 according to the angle inferred above, and then use the same analogy to adjust the position of an unknown UAV with two determined UAVs each time, Adjust the remaining UAVs receiving signals passively to the optimal position to ensure that the nine UAVs are evenly distributed on the circumference with a radius of 100 m.

First, determine the position of the UAV numbered FY02.

From the above reasoning, it can be concluded that:

Assume that the UAV numbered FY02 is at point C, as shown in Figure 5:

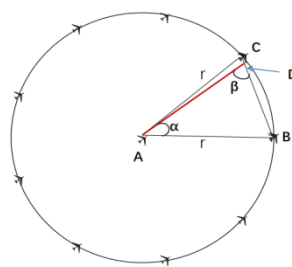


Figure 5: UAV Positioning Map (5)

In isosceles $\triangle ABC$, $\angle CAB = 40^\circ$, $\angle ACB = 70^\circ$.

But in fact, the position of the UAV is not so accurate, so we need to adjust it according to the known angle information.

(1) If $\angle \alpha < \angle CAB = 40^\circ$, the UAV shall move counterclockwise according to the circular arc of the circle with the radius of AD;

If $\angle \alpha = \angle CAB = 40^\circ$, the UAV is just on the extreme diameter of the circle;

If $\angle\alpha > \angle CAB = 40^\circ$, the UAV shall move clockwise according to the circular arc of the circle with AD as the radius.

After adjustment in this step, the UAV numbered FY02 can be adjusted to the extreme diameter of the circle, and then the UAV will be adjusted to the circle.

(2) If $\angle\beta < \angle ACB = 70^\circ$, the UAV shall move inwards along the polar radial direction;

If $\angle\beta = \angle ACB = 70^\circ$, the UAV is just on the circumference;

If $\angle\beta > \angle ACB = 70^\circ$, the UAV shall move outward along the extreme diameter;

After these two steps of adjustment, one UAV can be adjusted to the correct position.

In (1), set the step size of $\theta = 0.01$, and then $\alpha' = \alpha \pm n \times 0.01$ (n is the number of adjustments) after each adjustment

Then judge whether the adjusted $\angle\alpha$ is equal to $\angle CAB (\angle CAB = 40^\circ)$. If it is exactly equal to $\angle CAB (\angle CAB = 40^\circ)$, the adjustment will be ended and the current position information will be updated.

In (2), set the step size of $\varepsilon = 0.01$, then $r' = r \pm k \times 0.01$ after each adjustment (k is the number of adjustments);

Then judge whether the adjusted r is equal to 18. If it is exactly equal, the adjustment will be ended and the final position information will be updated.

After the above steps, the position of the UAV numbered FY02 has been determined. After each UAV position is adjusted, use the UAV and the UAV numbered FY00 to adjust each UAV with slight deviation one by one according to the above steps, and adjust the remaining UAVs receiving signals passively to the optimal position to ensure that the nine UAVs are evenly distributed on the circumference with a radius of 100 m.

4. Conclusion

This paper uses sine and cosine theorems and bearings only passive location method to locate UAVs. The UAV location method based on passive location has smaller blind area, can work in bad weather, has the ability to detect static targets, carry out early warning and find UAV operators, and is more suitable for positioning UAVs; Through the combination of numbers and shapes, the reader can easily understand and understand the process of model establishment and solution, which is more readable; As well as references to relevant knowledge, it enhances the reliability of the article and is more persuasive.

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