Optimization of the Number of Shared Parking Spaces based on Bottleneck Model

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Abstract: Based on the principle of bottleneck economics, this paper studies the impact of shared parking Spaces in work destinations on commuters' travel behavior, and solves the user equilibrium solution under two types of parking, and depicts the corresponding pattern diagram, and obtains analytical solutions of total social cost and total queue time of commuters. Through numerical example analysis, the following conclusions: setting up the shared parking can not only effectively reduce the social total cost, but also can reduce commuters general queuing time to alleviate the effect of road traffic congestion, while increasing the number of shared parking spaces is not always better. In addition, the optimal number of shared parking spaces is set differently based on different management goal.

Keywords: the number of shared parking spaces, bottleneck model, user equilibrium, morning commute

1. Introduction

At present, in the context of sharing economy, with the development of information technology and mobile Internet, a new parking way of shared parking arises at the historic moment. Shared parking refers to the integration of existing parking resources and the sharing of parking resources in a certain area by off-peak to realize the efficient use of parking resources. By the end of 2020, Shanghai had completed 660 shared parking projects, providing 32,000 shared berths. The vigorous development of shared parking will effectively alleviate the problem of parking difficulty in urban central area. However, the impact of the introduction of shared parking on road traffic should not be ignored while solving the problem of parking difficulty. Therefore, optimizing the number of shared parking Spaces with the goal of optimizing the road traffic environment is the focus of this paper.

Most of the researches on shared parking spaces are conducted from the perspectives of feasibility analysis¹, demand prediction² and parking allocation³, and most of them are based on solving the problem of parking difficulty. Few researchers study the impact on road traffic from the perspective of the number of open shared parking spaces. At the same time, the classical bottleneck model⁴ clearly describes the process of the emergence and disappearance of queue of commuters at rush hours and the decision-making departure time of commuters, which has strong practicability for solving traffic problems. Scholars have continuously added parking factors into the bottleneck model for research⁵⁻¹¹. Therefore, based on the principle of bottleneck economics, this paper will study the optimization of the number of shared parking spaces.

2. Problem description

The scenario studied in this section is that in a transportation network with single starting point and single ending point, the residential area and the work destination are connected by an urban road with limited capacity. There are two types of parking lots for commuters to choose after they arrive at the work destination by driving a car. It is assumed that the walking time between the central parking lot and the work destination is negligible; The second is shared parking Spaces, which are usually located near the workplace and require commuters to walk for a while to reach their work destination.

In the commuter transportation network in the morning rush hour, assuming that N commuters drive cars, the number of central parking Spaces in the work destination is M₁, and the number of shared parking Spaces is M₂, then N=M₁+M₂, the total number of parking Spaces can always meet the parking needs of all car commuters due to the opening of shared parking Spaces. This article provides a symbolic definition of the parameters involved, as shown in Table1:
In order to simulate the morning rush hour commuter scenario more effectively, this article makes the following assumptions about the model, as shown in Table 2:

**Table 2: The assumptions about the model**

<table>
<thead>
<tr>
<th>No.</th>
<th>Model assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Suppose the total number of morning peak travelers is (N); And the travelers are using cars to travel;</td>
</tr>
<tr>
<td>2</td>
<td>The pricing of central parking in the workplace is lower than that of shared parking, so users have priority to use the central parking lot and then the shared parking lot. In addition, users who choose central parking arrive early, and users of shared parking arrive early or late.</td>
</tr>
<tr>
<td>3</td>
<td>The parking lot is generally provided for the workplace, the parking space layout is dense, so ignore the walking distance between the parking spaces in the parking lot;</td>
</tr>
<tr>
<td>4</td>
<td>Shared parking is generally scattered near office buildings, where it is assumed that shared parking spaces are evenly distributed, and the number distributed from the workplace to the outside can be recorded as shared parking spaces (1,2,3,...n), and the time it takes for a car traveler to walk past a parking space after parking is recorded (\omega), then the (N) shared parking spaces to reach the workplace within walking time (\omega N);</td>
</tr>
<tr>
<td>5</td>
<td>Because the free flow time of commuters during the morning rush hour is equal, this paper assumes that the free flow time is zero, that is, the commuter arrives at the bottleneck as soon as he leaves the bottleneck, and arrives in the parking lot as soon as he leaves the bottleneck.</td>
</tr>
</tbody>
</table>

3. The parking model

The total number of car travelers during the morning rush hour commute is \(N\), all commuters want to reach the workplace at an identical preferred arrival time \(t^*\). Due to the shortage of parking spaces in the workplace, shared parking needs to be introduced as a supplement. Therefore, according to the way of parking chosen by the traveler, the traveler is divided into two categories: 1) the central parking users, i.e. \(M_1\); 2) the shared parking users, i.e., \(M_2\). Therefore \(N = M_1 + M_2\).

The generalized travel cost of commuters departing at time \(t\) and choosing the central parking consists of three components: 1) the cost of queue time at the bottleneck \(\alpha T(t)\), \(\alpha\) is the unit cost of travel time, \(T(t)\) is the queuing delay time at the bottleneck at time \(t\); 2) the penalty cost of arriving early on time is \(\beta(t^* - t - T(t))\), \(\beta\) is the unit cost of arriving early, according to Small's\(^{[12]}\) findings are usually \(\beta < \alpha\); 3) the central parking fees, i.e., \(p_1\). As the formula (1) shows:

\[
C_{M_1}(t) = \alpha T(t) + \max \left\{ \beta(t^* - t - T(t)) \right\} + p_1 \tag{1}
\]

The generalized travel cost of commuters departing at time \(t\) and choosing the shared parking consists of four components: 1) the cost of queue time at the bottleneck \(\alpha T(t)\), \(\alpha\) is the unit cost of travel time, \(T(t)\) is the queuing delay time at the bottleneck at time \(t\); 2) the penalty cost of arriving early or later is \(\beta(t^* - t - T(t) - \omega n), \gamma((t + T(t) - t^*))\), \(\beta, \gamma\) is the unit cost of arriving early or later, according to Small's\(^{[12]}\) findings are usually \(\beta < \alpha < \gamma\); 3) the shared parking fees, i.e., \(p_2\); 4) the user of choosing the

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**Table 1: Definition of symbols**

<table>
<thead>
<tr>
<th>symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>(N)</td>
<td>The total number of cars traveling during the morning rush hour</td>
<td>(t^*)</td>
<td>Desired arrival time at work</td>
</tr>
<tr>
<td>(\delta)</td>
<td>capacity of bottleneck</td>
<td>(t)</td>
<td>Commuter departure time</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Unit cost of in-vehicle travel time</td>
<td>(n)</td>
<td>Share parking location</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Unit cost of arriving at work early</td>
<td>(\omega)</td>
<td>Time to walk past a parking space</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Unit cost of walking time</td>
<td>(p_1)</td>
<td>Central parking fees</td>
</tr>
<tr>
<td>(M_1)</td>
<td>The actual number of central parking spots</td>
<td>(p_2)</td>
<td>Shared parking fees</td>
</tr>
<tr>
<td>(M_2)</td>
<td>The actual number of shared parking spots</td>
<td></td>
<td></td>
</tr>
</tbody>
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No. \text{n} Shared parking space has the cost of walking time, i.e. $\lambda_{\text{on}}$, and the unit cost of walking time $\lambda$, and $\lambda > \beta$\textsuperscript{[8]}. As the formula (2) shows:

$$C_{M_1}(t, n) = \left\{ \begin{array}{ll}
\alpha T(t) + \max \{ \beta(t' - t - T(t)), \gamma(t + T(t) - t') \} + p_2 + \lambda_{\text{on}} - \beta_{\text{on}}, & \text{if early arrival} \\
\alpha T(t) + \max \{ \beta(t' - t - T(t)), \gamma(t + T(t) - t') \} + p_2 + \lambda_{\text{on}} + \gamma_{\text{on}}, & \text{if late arrival}
\end{array} \right. \quad (2)$$

According to the principle of user equilibrium, travelers choose the departure time and parking method to minimize the cost of travel, when the transportation network reaches equilibrium, that is $C_{M_1}(t) = C_{M_2}(t, n)$. No one can reduce their travel costs by changing departure times and parking methods.

4. Analysis of travel patterns under user equilibrium

4.1. User equilibrium with two types of parking users

When the work destination has both central parking lots and shared parking lots, users can be divided into two types during the morning rush hour commute, namely, users who choose to central parking lots in the workplace and users who choose shared parking lots near the workplace. This section only considering center pricing below Shared parking, car commuters choose center parking costs less than sharing, user preference center parking lot, early arriving, when after using center parking lot parking lots, and then the car travelers choose Shared parking, there are both early and late. The travel pattern diagram is shown in Figure 1.

![Travel patterns of two types of parking lots](image)

\textbf{Figure 1:} Travel patterns of two types of parking lots

This travel mode describes the departure and arrival time of users choosing two types of parking and the cumulative number of arrivals at the corresponding time. The blue line represents the users who choose to build parking lots, and the red line represents the users who choose to share parking Spaces. AB/CD/DE is the arrival rate at the bottleneck; AC/CE represents the vehicle departure rate of the bottleneck; CE is the arrival rate of travelers arriving at the workplace. The blue area ABC represents the queuing delay time during the morning rush hour for users who choose to build optional parking lots, and the red area CDE represents the queuing delay time during the morning rush hour for users who choose to build shared parking lot.

According to the user equilibrium principle, the travel cost of users who choose shared parking is equal to that of users who choose central parking. $C_{M_1} = C_{M_2}$. And relevant parameters are solved. Let A represent the first commuter who chooses to central parking lot, then the departure and arrival time is $t_0$; B represents the last commuter who chooses to central parking lot, and the departure and arrival times are respectively expressed as $t_1, t_2$; C represents the first commuter who chooses the shared parking space, and the departure and arrival times are respectively $t_3, t_4$; D represents the commuters who arrive on time,
and the departure and arrival times are respectively represented as \( t_5 \) and \( t_6 \); \( E \) represents the last commuter to choose the shared parking space, and the departure and arrival times are respectively \( t_7 \) and \( t_8 \).

The travel mode, the first commuters work for choosing the destination user and arrive early at the center of the parking lot, the last one commuter Shared parking space is selected for the user and delay to arrive, first user when the user equilibrium and the last user are not line up, so all the car commuter leave the bottleneck in the \([t_0, t_7]\) time period and then arrived destination, available:

\[
N = s(t_7 - t_0)
\]

According to the user equilibrium principle, the travel cost of the first user \( C_A \) in the system is equal to the travel cost of the last user \( C_E \), namely:

\[
\begin{align*}
C_A &= \beta(t^* - t_0) + p_1 \\
C_E &= \gamma(t_1 - t^*) + (\lambda + \gamma)s(N - M_1) + p_2 \\
C_A &= C_E
\end{align*}
\]

In the simultaneous formula (3) and (4), the departure and arrival time of the first and last commuter in the travel mode is:

\[
\begin{align*}
t_0 &= t^* - \frac{\gamma N}{\beta + \gamma s} - \frac{\omega(N - M_1)(\gamma + \lambda)}{\beta + \gamma} - \frac{p_2 - p_1}{\beta + \gamma} \\
t_7 &= t^* + \frac{\beta N}{\beta + \gamma s} - \frac{\omega(N - M_1)(\lambda + \gamma)}{\beta + \gamma} - \frac{p_2 - p_1}{\beta + \gamma} \\
t_8 &= t_7 + \omega(N - M_1)
\end{align*}
\]

In \([t_0, t_2]\) period, the user first chooses to build a parking lot at the work destination and arrives early, that is, the last user \( B \) who chooses the central parking lot arrives early. After the parking Spaces of \( M_1 \) central parking lots are used up, the following \( N - M_1 \) users choose shared parking to arrive in advance and later, that is, the first user \( C \) who chooses shared parking arrives early; User \( D \) who arrives on time, chooses the shared parking space. Therefore, the expression of personal travel cost of user BCD is as follows:

\[
\begin{align*}
C_B &= \alpha(t_2 - t_1) + \beta(t^* - t_2) + p_1 \\
C_C &= \alpha(t_4 - t_3) + \beta(t^* - t_4) + p_2 \\
C_D &= \alpha(t^* - \omega n^* - t_3) + \lambda \omega n^* + p_2 \\
C_A &= C_B = C_C = C_D = C_E
\end{align*}
\]

Among them, the first \( M_1 \) users leave the bottleneck in sequence within \([t_0, t_2]\) periods, and the cumulative number of users arriving at the shared parking space at time \( t^* \) is denoted as \( n^* \), that is, \( n^* \) users leave the bottleneck in sequence within \([t_0, t_6]\) period, because the bottleneck flow is continuous in this scenario, that is, \( t_2 = t_4 \), so:

\[
\begin{align*}
M_1 &= s(t_2 - t_0) \\
n^* &= s(t_6 - t_0) \\
t_6 &= t^* - \omega n^*
\end{align*}
\]

By combining the above formulas, the departure and arrival time of the last user who selects the central parking lot and the first user who selects the shared parking lot, the departure and arrival time of
the quasi-arrival user, and the cumulative number of users who arrive at any time can be obtained 
\( t_1, t_2, t_3, t_4, t_5, t_6 \).

The departure time \( t_1 \) and arrival time \( t_2 \) of the last user who selected the work destination with a parking lot are expressed as follows:

\[
\begin{align*}
  t_1 &= t_0 + \frac{(\alpha - \beta)M_1}{\alpha \cdot s} \\
  t_2 &= t_0 + \frac{M_1}{s}
\end{align*}
\] (9)

The departure time \( t_3 \) and arrival time \( t_4 \) of the first user who selects a shared parking space are expressed as follows:

\[
t_3 = t_4 = t_0 + \frac{M_1}{s}
\] (10)

The accumulative number of users \( n^* \) arriving at time \( t^* \) is equal to the \( M_1 \) users who choose central parking lots plus the accumulative number \( n \) of users who choose shared parking lots to arrive on time. The expression is as follows:

\[
n^* = M_1 + n = M_1 + \frac{s(t^* - t_0) - M_1}{1 + s\omega}
\] (11)

The departure time \( t_5 \) and arrival time \( t_6 \) of on-time users \( n^* \) are expressed as follows:

\[
\begin{align*}
  t_5 &= t^* - \frac{(\alpha + \beta - \lambda)\cos + \beta}{\alpha(1 + \cos)}(t^* - t_0) + \frac{(\alpha - \lambda)\cos}{\alpha(1 + \cos)} + \frac{p_2 - p_1}{\alpha} \\
  t_6 &= t^* - \omega n^*
\end{align*}
\] (12)

Meanwhile, in the travel mode discussed in this section, user C who first chooses shared parking doesn’t need to queue up, that is, he will arrive at his work destination immediately after starting out. Moreover, the last user who chooses to build a parking lot and the first user who chooses shared parking just meet at the bottleneck, \( t_2 = t_3 = t_4 \), it can be concluded:

\[
p_2 - p_1 = \frac{\beta(N - M_2)}{s}
\] (13)

According to the user equilibrium principle, the travel cost of each commuter is equal, and the expression of the individual travel cost of the commuter can be obtained:

\[
PC = C^* = \beta(t^* - t_0) + p_1
\]
\[
= \frac{\beta \gamma}{\beta + \gamma} \frac{N}{s} + \frac{\beta \omega(N - M_2)(\gamma + \lambda)}{\beta + \gamma} + \frac{\beta(p_2 - p_1)}{\beta + \gamma} + p_1
\] (14)

The total social cost is equal to the travel cost of all commuters in the system, that is, the total number of people multiplied by the personal travel cost under the commuter equilibrium, minus the total income, that is, minus the total income of the parking lot. The expression is as follows:

\[
TSC = PC \cdot N - M_2 \cdot p_1 - M_2 \cdot p_2
\]
\[
= \frac{\beta \gamma}{\beta + \gamma} \frac{N^2}{s} + \frac{\beta \omega(N - M_2)(\gamma + \lambda)}{\beta + \gamma} + \frac{\beta \omega(N - M_2)(\gamma + \lambda)}{\beta + \gamma} + \frac{\beta(N - M_2)^* M_2}{s}
\] (15)
The total queuing time of commuters is equal to the queuing time of commuters who choose central parking lot plus the queuing time of commuters who choose shared parking lot. The expression is as follows:

\[ TQT = TQT_{M_1} + TQT_{M_2} = S_{ABC} + S_{CDE} \]
\[ = \frac{1}{2} (t_3 - t_6) * M_1 + \frac{1}{2} (t_6 - t_3) * M_2 \]  

(16)

4.2. Two special cases

This section introduces two special situations, namely, when the work destination has only central parking lots, as shown in Figure 2-a; when the work destination has all shared parking lots, as shown in Figure 2-b.

![Figure 2: Travel patterns in two special cases](image)

a) travel patterns of central parking lot users  
b) travel patterns of shared parking lot users

In this section, two special cases are given as the comparative reference for the adjustment of the number of shared parking Spaces. The solution of key parameters follows the user equilibrium principle. Similarly, the solution process in the previous section is not repeated here.

In the travel pattern diagram with only central parking lots,  \( N = M_1 \), the final expression to the total social cost is as follows:

\[ TSC_{M_1=N} = \frac{\beta \gamma}{\beta + \gamma} \frac{N^2}{s} \]  

(17)

In the travel pattern diagram with only central parking lots,  \( N = M_1 \), the expression of queuing time of commuters is finally solved as follows:

\[ TQT_{M_1=N} = \frac{1}{2} A_B \frac{\beta \gamma}{\beta + \gamma} \frac{N^2}{s} \]  

(18)

In the travel pattern diagram with only shared parking Spaces,  \( N = M_2 \), the final expression to the total social cost is as follows:

\[ TSC_{M_2=N} = \frac{N \gamma}{\beta + \gamma} \left( \frac{N}{s} + \omega N \right) - \frac{\lambda \omega N^2}{\beta + \gamma} \]  

(19)

In the travel pattern diagram with only shared parking Spaces,  \( N = M_2 \), the final expression of queuing time for commuters is as follows:
\[ TQT_{M_1,\ldots,N} = S_{ABC} = \frac{1}{2} \left( \frac{\beta \gamma}{\alpha (\beta + \gamma)} \right) \frac{N^2}{s} - \frac{N^2 \gamma}{(\beta + \gamma)(1 + \omega s)} \] (20)

5. A number example

Based on the user equilibrium principle of economics, bottleneck section on depicting the travel of the coexistence of two types of parking lot model, and two special cases of patterning parking is only one kind of travel, and solving the social total cost and commuter line up the analytical solution of the total time, this section will be combined with concrete example analysis number of Shared parking Spaces for social total cost and the influence of commuters queuing time.

The basic model parameters are as follows, A total demand of \( N = 240 \) vehicles commute in morning peak hour and go through a freeway bottleneck with capacity \( s = 120 \) veh/min, so that the morning peak commuting time lasts around 2 h. \( \alpha = 1 \$/h \), \( \beta = 0.6 \$/h \), \( \lambda = 1.2 \$/h \), \( \omega = 0.0015 \) h are chosen according to the literature.

5.1. Analysis of the impact of the number of shared parking Spaces on the total social cost

Figure 3 shows the trend of the total social cost as the number of shared parking Spaces increases. The blue line indicates that there are two types of parking lots, the red line indicates that there are only central parking lots (central parking lots) in the workplace, and the yellow line indicates that there are only shared parking lots in the workplace. As can be seen from the figure, when all the parking Spaces for the purpose of work are provided by the central parking lot, the total social cost is as high as $310.5. When the central parking space is reduced by adding shared parking Spaces, the total social cost decreases first and then increases. When all parking Spaces are provided by shared parking, at this time, the total social cost is $269.2, which is lower than the situation where all parking Spaces are central parking Spaces, but not optimal. The optimal number of shared parking Spaces is 136. At this time, the lowest total social cost is $211.11, which is 32% optimized compared with the highest total social cost.

It can be concluded that setting shared parking Spaces at work destinations will help reduce the total social cost, but the more shared parking Spaces are not the better. Therefore, managers should set the number of shared parking Spaces reasonably according to the total number of travelers to achieve the goal of reducing the total social cost.
5.2. Analysis of the impact of the number of shared parking Spaces on the total queuing time of commuter

![Graph showing the influence of the number of shared parking Spaces on the total queuing time of commuters in rush hours]

Figure 4 shows the variation trend of the total queuing time of commuters in the system with the number of shared parking Spaces. Similarly, the blue line indicates that there are two types of parking lots, the red line indicates that there are only central parking lots (central parking lots) in the workplace, and the yellow line indicates that there are only shared parking lots in the workplace. By the figure, when work destination all parking Spaces are supplied with center parking lot, the commuter total longest queue times for 240 minutes, by adding Shared parking space to reduce the amount of center parking lots, appeared the social total cost down and the rising trend, when adding to all parking is provided by the Shared parking, At this time, the total queuing time in the system is 103 minutes, which is lower than the central parking space with all parking Spaces, but not optimal. The optimal number of shared parking Spaces is 168. At this time, the total queuing time of commuters is 72 minutes, which is 70% optimized compared with the longest queuing time.

Thus, work in the destination set Shared parking space is beneficial to ease road traffic congestion, but it is not the more the better, Shared parking management according to the total number of travel to optimize share same number of parking space, it is worth noting that the social total cost minimum and the general line of commuter time yes Shared parking lot number is different, therefore, Managers should also set the number of shared parking Spaces differently according to the actual road conditions and specific management objectives.

6. Conclusions

Based on the principle of bottleneck economics, this paper analyzes the impact of the setting of shared parking Spaces at work destinations on users' travel behaviors, and depicts the corresponding travel patterns. According to the user equilibrium principle, the key parameters are solved, as well as the analytical solutions of the total social cost and the total queuing time of commuters. Finally, a concrete numerical example, it is concluded that, in the destination set up Shared parking can not only reduce the social total cost, but also can alleviate road congestion out of reducing commuters queuing time. It is worth noting that the number of shared parking Spaces should not be set too much and should be differentiated according to different management objectives.

Finally, this study only considers the limitations of car commuters during the morning rush hour, and future research can consider the addition of public transportation to study the optimization of the number of shared parking under multi-mode travel.
References