

Lithological Patterns Based on Arc Fitting

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Abstract: Map of geological cross section is the most intuitive expression of strata in the vertical direction. Realizing automatic filling of lithologic patterns and reflecting the changes of stratum curvature are important contents of map of geological cross section drawing. The traditional lithologic pattern filling method can not show the change of rock stratum dip angle. To solve this problem, an algorithm of lithologic pattern drawing based on arc fitting is proposed. By analyzing the mathematical relationship between the dip angle of the rock stratum and the boundary line of the rock stratum, the data needed for drawing can be increased, and then the curve and lithologic pattern can be drawn according to the dip angle of the rock stratum. The application results show that the lithologic pattern compiled by the algorithm changes with the change of rock dip, and the first derivative of a single curve in the lithologic pattern is continuous and the segments are circular arcs.

Keywords: Map of geological cross section, Lithologic pattern, Computer aided design, Stratigraphic dip, CAD secondary development

1. Introduction

1.1. Background

With the rapid development of modern information surveying and mapping technology, automatic drawing of geological profile [1-3] is gradually replacing the traditional manual drawing of geological profile as the main way to obtain geological profile. In recent years, the automatic drawing technology of geological profile has developed rapidly, and has made breakthroughs in many fields such as folds and faults. The realization of automatic filling of lithological patterns is an important content of automatic drawing of geological profile.

The lithological pattern is complex and changeable. How to draw the lithological pattern corresponding to the fold area is an important content of lithological pattern filling. There are two traditional lithologic pattern filling methods:

Method1 Fill in the lithological pattern by setting the "Area Parameters" in the GIS or CAD operating environment. For example, in the MapGIS operating environment, Perform topology processing in the filled area, Then set the "fill pattern", "fill color" and "stratigraphic dip" of the corresponding area to complete the filling of lithologic patterns. The lithological pattern of the fill is a single angle.

Method2 Lithologic patterns are drawn according to the stratigraphic boundary, Lithology dip not used in filling lithology patterns.

Therefore, the lithological patterns populated by the above two methods do not reflect the variation of stratigraphic dip.

1.2. Our Work

In order to solve the above problems, a lithologic pattern drawing algorithm based on arc fitting is proposed by deriving mathematical formulas and using CAD secondary development technology [4]. Use the dip angle data of the rock stratum and the curvature change of the rock stratum boundary to draw the curve, so that the curve changes with the dip angle change of the rock stratum, and then complete the lithologic pattern drawing according to the curve.

2. Traditional Algorithm Description

The upper and lower interfaces of rock layers are called stratification. The traditional lithology pattern filling method [5] treats rock layers as a combination of n rock layers and uses the rock layers as lithology pattern stratification for drawing lithology patterns. This method draws each level of the lithologic pattern in turn with known stratigraphic boundaries, and ensures that the spacing between adjacent lithologic pattern levels is a fixed value.

As shown in Figure 1, the line L₁ is known to be the stratigraphic boundary, and a sufficient number of points are collected on the line L₁, which are then used to produce a series of points equidistant from the line L₁, which are then fitted to a curve L₂ using a spline function.

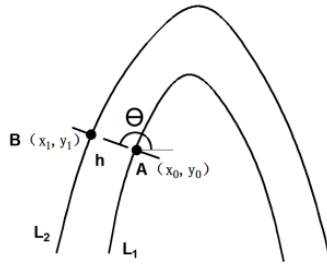


Figure 1: Schematic diagram of equal thickness layer compilation

When drawing the lithological pattern stratification, the (x₁, y₁) is obtained by making the normal AB of the curve past each point (x₀, y₀) on the line L₁ and going outward from (x₀, y₀) along AB with the length h. The calculation is as follows:

$$\begin{cases} x_1 = \cos\theta \times h + x_0 \\ y_1 = \sin\theta \times h + y_0 \end{cases} \quad (1)$$

3. Algorithm Description

3.1. Mathematical relationship between stratigraphic dip and stratigraphic boundaries

Analyze the mathematical relationship between the dip angle of the rock stratum and the boundary line of the rock stratum to increase the data needed for the drawing. This step can improve the drawing accuracy of the lithologic pattern.

As shown in Figure 2, the change in dip angle between borehole A and B is calculated based on the change in curvature of the rock boundary L. Dividing the adjacent boreholes A and B into n equal parts by horizontal coordinates on the rock boundary L. Its horizontal coordinates are (x₁, x₂, ..., x_n).

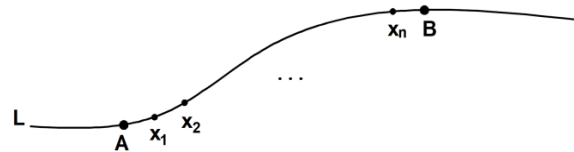


Figure 2: Schematic diagram of horizontal equipartition

The calculation of the inclination angle θ_m corresponding to the mth point is shown below:

$$\begin{cases} x_m = \frac{x_B - x_A}{n + 1} \times m + x_A \\ \theta_m = \frac{\arctan k_m - \arctan k_A}{\arctan k_B - \arctan k_A} \times (\theta_B - \theta_A) + \theta_A \end{cases} \quad (2)$$

k_A and k_B are the slopes of boreholes A and B on the formation boundary L; k_m is the slope of the formation boundary L at the mth point; θ_A and θ_B are the dip angles of the formation corresponding to boreholes A and B in that formation.

3.2. Curve plotting

According to the dip angle of the rock formation and the corresponding abscissa, construct a curve with continuous first derivative and segmented arcs. As shown in Figure 3, let the curves pass through $(x_1, y_1), (x_2, y_2), \dots, (x_p, y_p)$ in order, and L_1 and L_2 are the tangents to the circle.

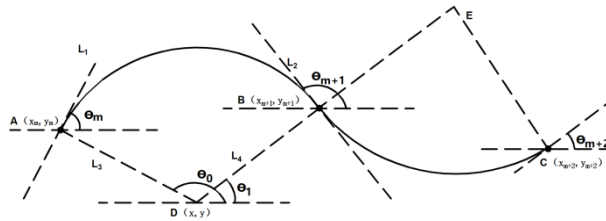


Figure 3: Schematic diagram of curve calculation

Knowing the values of x_1, \dots, x_p, y_1 , solve for y_2, \dots, y_p and the coordinates of the corresponding circle centers according to the mathematical relationship between the circular arc and the tangent line. Among them, the mathematical relationship between the circular arc and the tangents L_1 and L_2 is as follows:

$$\begin{cases} \tan \theta_0 = -1 \div \tan \theta_m \\ \tan \theta_1 = -1 \div \tan \theta_{m+1} \\ (x_m - x)^2 + (y_m - y)^2 = (x_{m+1} - x)^2 + (y_{m+1} - y)^2 \\ y_m - y = \tan \theta_0 \times (x_m - x) \\ y_{m+1} - y = \tan \theta_1 \times (x_{m+1} - x) \end{cases} \quad (3)$$

Equation (3) is transformed to obtain equation (4), and y_2, \dots, y_p and the coordinates of the circle center corresponding to each segment of the arc are calculated by equation (4).

$$\begin{cases} a \times y_{m+1}^2 + b \times y_{m+1} + c = 0 \\ a = 1 + 2 \times \tan \theta_{m+1} / (\tan \theta_m - \tan \theta_{m+1}) \\ b = 2 \times \frac{((x_{m+1} - x_m) \times \tan \theta_m + y_m) \times \tan \theta_{m+1} - 2 \times (y_m + \frac{x_m}{\tan \theta_m} + \frac{x_m - \tan \theta_m (x_{m+1} - y_m)}{(\tan \theta_m - \tan \theta_{m+1}) \times \tan \theta_m})}{\tan \theta_{m+1} - \tan \theta_m} \\ c = x_{m+1}^2 - x_m^2 - y_m^2 + 2 \times y_m \times (y_m + x_m / \tan \theta_m + \frac{x_m - \tan \theta_m (x_{m+1} - y_m)}{(\tan \theta_m - \tan \theta_{m+1}) \times \tan \theta_m}) + \\ 2 \times (x_m - x_{m+1}) \times (\tan \theta_m (x_{m+1} - y_m) - x_m) / (\tan \theta_m - \tan \theta_{m+1}) \\ y_{m+1} = (-b \pm \sqrt{b^2 - 4 \times a \times c}) / (2a) \\ x = ((y_{m+1} - y_m) \times \tan \theta_m \times \tan \theta_{m+1} + \tan \theta_m \times x_{m+1} - \tan \theta_{m+1} \times x_m) / (\tan \theta_m - \tan \theta_{m+1}) \\ y = y_m + (x_m - x) / \tan \theta_m \end{cases} \quad (4)$$

$A(x_m, y_m)$ is the starting point coordinate of arc of segment AB; x_{m+1} is the abscissa of the end point of the arc of segment AB; θ_m, θ_{m+1} is the dip angle corresponding to x_m and x_{m+1} . When there are two values of y_{m+1} , they are set as y_a and y_b respectively. Since a single arc on the curve cannot be located in the upper semicircle and the lower semicircle at the same time, which is closest to y_m among y_a and y_b is taken as the value of y_{m+1} , then solve the corresponding circle center coordinates according to y_{m+1} .

In the case of known $x_1, y_1, x_2, \theta_1, \theta_2$ find out y_2 and the corresponding circle center coordinates according to equation (4), then substitute y_2 and $x_2, x_3, \theta_2, \theta_3$ into equation (4) and continue to solve y_3 and the corresponding circle center coordinates, and so on until y_p and the corresponding circle center coordinates are found out. Then, starting from (x_1, y_1) , all circular arcs are drawn in order according to the coordinates of two adjacent points and the corresponding circle center coordinates, and finally all arcs are connected into one curve.

3.3. Stratification plotting

Imitating the traditional lithological pattern filling method. Draw each stratification according to the above curves to complete the filling of the lithological pattern.

Known the coordinates of the points on the circle and the coordinates of the corresponding circle center, make the radius of the first arc increase h ($h > 0$). Solve for the coordinates of the end point of the arc after the radius change. If the coordinates lie between the start point of the next arc and the center of the arc, then the radius decreases by h , otherwise the radius increases by h , and so on, finally connect the segments of the arc into a curve. Repeat the above steps until the filling of the lithological pattern is completed. The solution of arc endpoint coordinates is as follows:

$$\begin{cases} \theta = \arctan\left(\frac{y_1 - y_0}{x_1 - x_0}\right) \\ x_2 = \cos\theta \times h + x_1 \\ y_2 = \sin\theta \times h + y_1 \end{cases} \quad (5)$$

Where (x_0, y_0) are the coordinates of the center of the circle, (x_1, y_1) are the coordinates of the initial end point of the arc, and (x_2, y_2) are the coordinates of the end point after increasing the radius of the arc by h .

4. Conclusions

Based on the above algorithm, the authors carried out a secondary development for CAD to achieve automatic filling of lithological patterns in the defined area of the geological profile. Considering that the current lithology pattern is filled mainly by setting the "zone parameter", and that the lithology pattern filled according to the stratigraphic boundary does not use the dip data, the lithology pattern filled according to the stratigraphic boundary is not compared.

As shown in Figure 4, Where (1) is to fill the lithology pattern by setting the "zone parameter". Where (2) is the lithological pattern filling by the method in this paper.

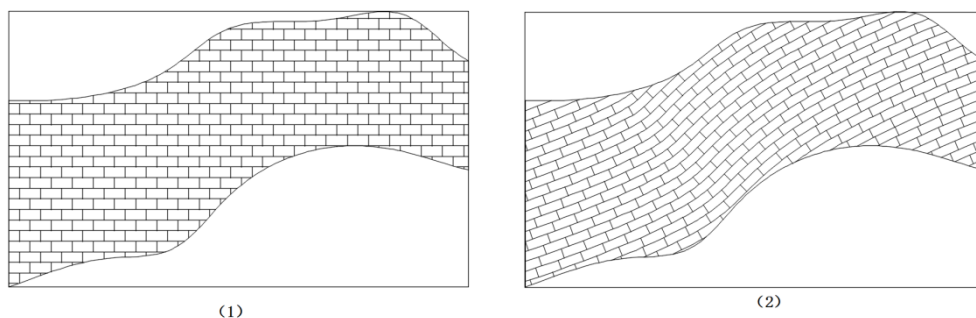


Figure 4: Limestone filling schematic

Compared with the traditional lithology pattern filling method, the lithology pattern drawing algorithm based on circular arc fitting in this paper has the following advantages:

(1) The lithological pattern drawn varies with the dip angle of the rock formation, achieving the effect of simulating manual drawing.

(2) By analyzing the mathematical relationship between the dip angle of the formation corresponding to adjacent boreholes and the formation boundary, multiple dip angle data are obtained by calculation based on two stratigraphic dips, making the lithologic pattern curve between adjacent boreholes a combination of multiple arcs, which is closer to the real situation.

However, the results obtained in this paper still need to be improved, and the relationship between stratigraphic dip and rock boundaries needs to be further studied.

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