ISSN 2616-5775 Vol. 5, Issue 2: 77-80, DOI: 10.25236/AJCIS.2022.050212

Research on Magic Square Construction Based on Genetic Algorithm

Xintian Cui, Xiangqi Cheng, Guoxuan Bu

Mechanical Engineering, Dalian Jiaotong University, Dalian, China

Abstract: *The construction of magic squares is a complex permutation and combination problem. It is very difficult to construct the magic square with the continuous minimum natural number, and its solution has the nature of diversity and increases exponentially with the increase of order. In this paper, the magic square is constructed by selecting specific real numbers in different situations. Firstly, the magic square model is realized by using genetic algorithm, and the population is selected, crossed and mutated, and the optimal solution is selected, so that the semi-magic square whose sum of rows and columns is magic sum can be generated. Then, the parent gene of diagonal magic population is generated again by row exchange and column exchange, and the data of diagonal magic population is improved. Then, the selection, crossover and diagonal local adjustment are carried out in turn, and finally, the N-order high-quality magic square in the interval of [1, N^2] natural number is obtained.*

Keywords: *Magic Square; Genetic Algorithm; Random Search*

1. Introduction

Magic squares, which originated in Nine palace map, an ancient Chinese Luo Chart, often appears in the form of Sudoku in modern life. With the development of computer science, it is widely used in combinatorial analysis, graph theory, programming and other fields. [1] At present, the magic square in a broad sense is a method of arranging numbers in a square grid so that the sum of numbers in each row, column and diagonal is equal. Magic square in narrow sense is a traditional Chinese game, which limits the range of elements to all natural numbers $1 \sim n^2$ on the basis of generalized magic square. Compared with the narrow magic square, the generalized magic square is more difficult to construct, and has deeper application value and discussion significance. [2]

2. Magic Square Model Based on Genetic Algorithm

For the convenience of description and understanding, all graphical data satisfying the following forms are called magic squares.

$$\begin{matrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{n4} \end{matrix}$$

When composing a magic square, the sum of numbers in each row, column and diagonal line is widely called "magic sum". Among them, the inferior magic squares are all magic squares with undetermined properties and magic squares that do not meet the condition that the sum of elements in each row and column is equal to magic sum. semi-magic square means that the algebraic sum of elements in each row and column is equal to magic sum, but the sum of elements on two diagonals has at least one magic square that is not equal to magic sum. A high-quality magic square is a magic square in which the sum of all elements in rows, columns and diagonal lines is equal to the magic sum. [3]

Compared with traditional optimization algorithms, evolutionary algorithm has high efficiency in solving complex problems. Considering that illegal individuals will be generated when magic squares are combined by hybridization and recombination in the construction process, mutation operator can be used as the main operator for evolution. In dealing with the construction of magic square, evolutionary algorithm is used to optimize the combination.

ISSN 2616-5775 Vol. 5, Issue 2: 77-80, DOI: 10.25236/AJCIS.2022.050212

In order to effectively limit the running time of the algorithm, limit the value range of magic square elements, generate a random real number array within the limited range, generate random numbers, and limit the change range of real numbers within the magic square. The number of array elements is, respectively:n^2

$$\begin{matrix} s_{11} & s_{12} & s_{13} & \cdots & s_{1n} \\ s_{21} & s_{22} & s_{23} & \cdots & s_{2n} \\ s_{31} & s_{32} & s_{33} & \cdots & s_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & s_{n3} & \cdots & s_{nn} \end{matrix}$$

Use random () command in MATLAB to generate a new random array for the original data. Fill in the magic square from left to right and from top to bottom, and its corresponding position number is:

$$\begin{matrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{n4} \end{matrix}$$

The number of parents of the first generation population is initially set at 1,000, and it is necessary to provide reasonable fitness to drive the evolution of the magic square of parents. The known fitness function is:

$$J_{n0} = \sum_{i=1}^{n}\left|S_{ij} - S\right| + \sum_{j=1}^{n}\left|S_{ij} - S\right| \quad (1)$$

And $J_0 \geq 1$, the reciprocal of the fitness should be taken. Therefore, using the roulette method, that is, the selection strategy based on the fitness ratio, the probability of the inferior magic square individual I_n being selected is $P(I_n) = \frac{\frac{1}{J_{n0}}}{\sum_{n=1}^{n}\frac{1}{J_{n0}}}$, obeying the distribution function $F(I_n) = ln(J_{n0})$. When inferior magic squares tend to be semi-magic squares, the probability of being eliminated is smaller, and the probability of being selected is greater. Therefore, the old individuals in the parent magic square are selected with a certain probability, and they can reproduce the next generation and form a new population through their own evolution.

The mutation of individual genes is beneficial to the algorithm to find the optimal solution and reduce the convergence speed. In the evolutionary process of the algorithm of local adjustment of the ranks of high-order magic squares, if the unified objective function is adopted, there will be a phenomenon that the crossover results will gradually converge with the increase of evolutionary algebra, and there is also a risk of falling into local optimum easily. The idea of mutation may break the deadlock and provide higher evolutionary efficiency. The method of individual gene variation is to complete the gene crossover when the magic squares do not meet the selection conditions, select any two magic squares as parents, and exchange columns or rows between them. Then, genetic mutation is carried out to make any number in the magic square change randomly, and this number is not equal to any real number of the mutant individual species. Each randomly generated array of inferior magic squares, semi-magic squares and high-quality magic squares is called an individual, and the set of all arrays of inferior magic squares, semi-magic squares and high-quality magic squares is called a population. With the evolution, different types of individuals will emerge in the population. Evolve the data set according to the variation rule to obtain the data set $I_0, I_1, I_2, I_3 \cdots I_n$, until exiting the evolution to obtain the data set. Exit evolution needs to achieve the objective function, so that all row magic sums are consistent with column magic sums.

After 10000 evolutions, if its objective function is $J_0 = 0$, the objective function is reached, that is, the semi-magic square is successfully constructed. However, in most cases of practice, random real numbers are difficult to form a perfect semi-magic square, so we allow some errors between the actual value and the theoretical value of the magic sum of the semi-magic square of real numbers, let $J_0 < 1$.

ISSN 2616-5775 Vol. 5, Issue 2: 77-80, DOI: 10.25236/AJCIS.2022.050212

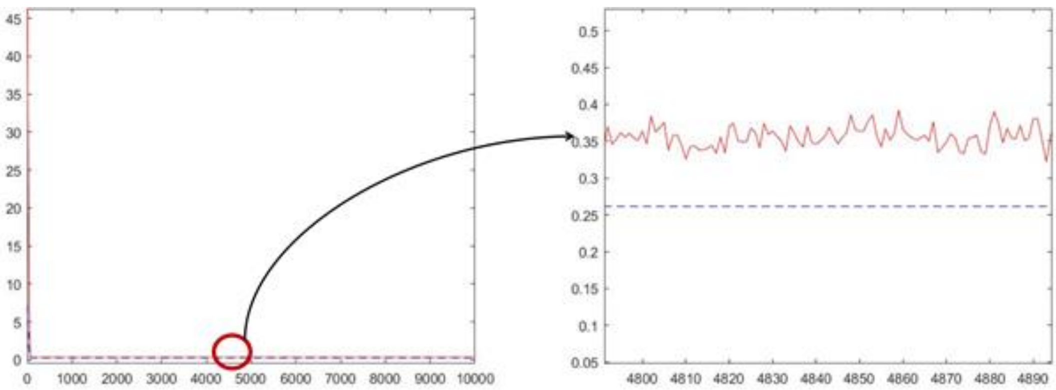

Figure 1: The generation process of inferior magic squares approaching to semi-magic squares

Decision variables are as follows:

$$J_{n1} = \sum_{i=1}^{n}\left|S_{ij} - S\right| + \sum_{j=1}^{n}\left|S_{ij} - S\right| \quad (2)$$

Drive 100 or more parents to construct magic squares.

3. Generation of High-quality Magic Squares Based on Local Variation

This magic square has a high probability of appearing in the form of inferior magic square, a low probability of appearing in the form of semi-magic square, and a very low probability of appearing in the form of high-quality magic square. Inspired by the equivalence of matrix after elementary row transformation and elementary column transformation, the construction of magic square can be divided into two steps. First of all, the semi-magic square with equal row magic sum and column magic sum is constructed to realize local adjustment of rows and columns, and then the semi-magic square is transformed by rows and columns to realize local adjustment of corners. Thereby realizing the construction of magic square. [4]

At the end of the construction process of semi-magic square, most of the rows and columns have already satisfied their element sum equal to magic sum, then conditional search is conducted in a few unsatisfied elements sum equal to magic sum, and at least one pair of elements are exchanged. Only one or two pairs of elements can be exchanged in this conditional search. Theoretically, three or more pairs of elements can be exchanged. However, this process costs a lot of time, so it is not proposed here. Among them, $1 \le k \le n,\ 1 \le j \le n,\ 1 \le s \le n,\ 1 \le t \le n,\ k \ne l,\ s \ne t.$

① If $\sum_{i=1}^{n} a_{ki} - c = c - \sum_{j=1}^{n} a_{lj} = a_{ks} - a_{ls}$, then swap the two elements a_{ks} and a_{ls};

② If $\sum_{i=1}^{n} a_{ik} - c = c - \sum_{j=1}^{n} a_{jl} = a_{sk} - a_{sl}$, then swap the two elements a_{sk} and a_{sl};

③ If $\sum_{i=1}^{n} a_{ki} - c = c - \sum_{j=1}^{n} a_{lj} = a_{ks} + a_{kt} - a_{ls} - a_{lt}$, then swap the two pair elements a_{ks}, a_{ls} and a_{kt}, a_{lt};

④ If $\sum_{i=1}^{n} a_{ik} - c = c - \sum_{j=1}^{n} a_{jl} = a_{sk} + a_{tk} - a_{sl} - a_{tl}$, then swap the two pair elements a_{sk}, a_{sl} and a_{tk}, a_{tl};

⑤ If $\sum_{i=1}^{n} a_{ki} - c = c - \sum_{j=1}^{n} a_{lj} = a_{sk} + a_{ql} - a_{st} - a_{qt}$, then swap the two pair elements a_{sk}, a_{st} and a_{ql}, a_{qt};

⑥ If $\sum_{i=1}^{n} a_{ik} - c = c - \sum_{j=1}^{n} a_{jl} = a_{ks} + a_{lq} - a_{ts} - a_{tq}$, then swap the two pair elements a_{ts}, a_{ks} and a_{tq}, a_{lq}.

At the end of the stage of semi-magic square evolution, the matrix at this time has satisfied that the sum of rows and columns is equal to magic sum. Therefore, the exchange between rows and columns will not affect the magic square. We can adopt the method of local adjustment to simplify the final steps and improve the evolution speed.

ISSN 2616-5775 Vol. 5, Issue 2: 77-80, DOI: 10.25236/AJCIS.2022.050212

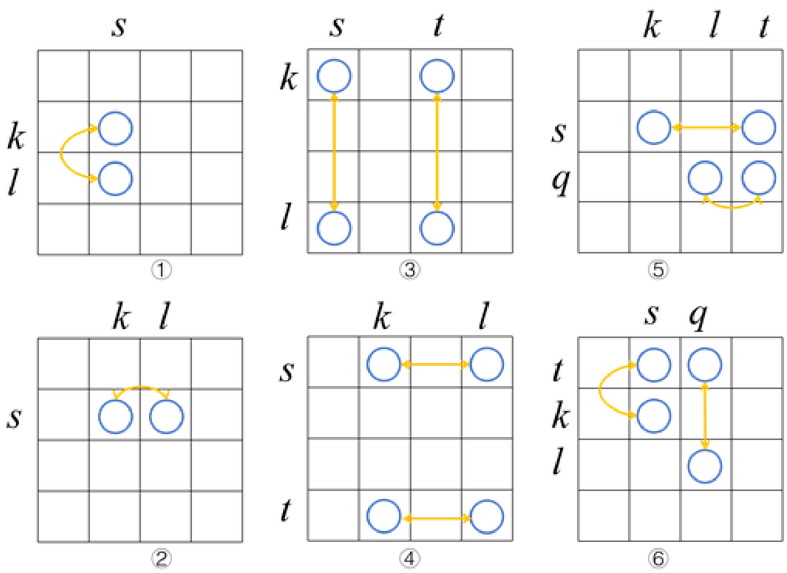

Figure 2: Schematic diagram of local adjustment of rows and columns

4. Conclusion

This model can realize the construction of fourth-order magic squares, but because of its strong randomness, only one high-quality magic square can be constructed in each run, and 880 all fourth-order magic squares can not be constructed in a short time, so the efficiency is low. The main analysis is that rapid contraction falls into the dilemma of approximate local optimum. Appropriate relaxation of genetic factors is helpful to slow down the population contraction speed. At the same time, adding tabu search algorithm is helpful to rebuild the population.

References

[1] Guo Ping, Liu Xingxiang. Definition and algebraic properties of sum magic matrix [J]. Journal of Yan'an University (Natural Science Edition), 2017, 36(01): 21-27.
[2] Zhang Shide, Li Cheng, Johnny. Several necessary and sufficient conditions of magic square [J]. Practice and cognition of mathematics, 2016, 46(04): 198-214.
[3] Wang Fei, Yao Huasheng. A method of magic square construction [J]. Journal of Huangshan University, 2009, 11(03): 28-32.
[4] Lu Zhenhong. Research on intelligent calculation method of magic square problem [D]. National University of Defense Technology, 2005.