Two Questions of Research on the Fairing for C-Bézier Curves

Zhongning Li Yanli Chen

Department of Mathematics, Yinchuan Energy Institute,Yinchuan750105,China

Abstract: This thesis talks about the preliminary study on the modification of the new C-Bézier curves, Asuccinct for mulafor calculating the derivative of the degree 3C-Bezier curve is given. It studies the four order C-Bézier curve fairing problem. Fairing C-Bézier curves are satisfied by adjusting the shape of the parameter α on the basis of applying energy.

Key Words: CAGD, C-Bézier curves, Fairing, Stitching

1. INTRODUCTION

A modification of the new C-Bézier curves in CAGD is producing in recent years, which design a broad and flexible approach shape[1], It can accurate representation of the quadratic curve, with many similar properties of Bézier curve. Several scholars have studied its shape changes, giving a segmentation algorithm and stitching, etc. [2-3], but the fairing problem was not involved. In this paper the problem of fourth-order fairing C-Bézier curve is studied.

2. THE CURVE FAIRING APPROXIMATION OF C-BÉZIER

In this section we study curve fairing approximation of C-Bézier.Literature [8] studies C-Bézier curve shape modification method ,and it is proposed to adjust the interpolation of a point of control C-Bézier curve method .It also shows the effect of moving control points on the curve.

Consider

Question 1: We know the data sequence of points $(i=0,1,\cdots,n)\in\Re^2 (i=0,1,\cdots,n)\in\Re^2$, seeking a C-Bézier curves: Γ :, $t\in[0,\alpha]t\in[0,\alpha]$,making the end of $p_i\in\Gamma$ radius vector.The solution is as follows:

First, we will parameter values of data points, which can be standardized cumulative chord length parameterization,

Denoted

as :, $L=\sum_{i=0}^{n} l_i L=\sum_{i=0}^{n} l_i$, $(i=0,1,\cdots,n)(i=0,1,\cdots,n)$,

thus we have:

$$\begin{cases} t_0 = 0 \\ t_1 = \alpha\left|\vec{p}_1-\vec{p}_0\right|/L \\ \vdots \\ t_n = \alpha\sum_{i=0}^{n} t_i / L \end{cases} \quad (1.1)$$

So, nonlinear equations is the solution for normalization of the above problems:

$(i=0,1,\cdots,n)(i=0,1,\cdots,n)$

$(k=0,1,\cdots,n)(k=0,1,\cdots,n)$

If we have written in matrix form

$AQ=P$ (1.2),

where

$[\vec{q}_0\vec{q}_0, \vec{q}_1\vec{q}_1, \vec{q}_2\vec{q}_2, \vec{q}_3\vec{q}_3]^{TT}$, $\vec{q}_0\vec{q}_0, \vec{q}_1\vec{q}_1, \vec{q}_2\vec{q}_2, \vec{q}_3\vec{q}_3$ $\in\Re^3\in\Re^3$ or $\Re^2\Re^2$ are four control points of the C-Bézier curve,.

$$A=\begin{bmatrix} Z_0(t_0) & Z_1(t_0) & Z_2(t_0) & Z_3(t_0) \\ Z_0(t_1) & Z_1(t_1) & Z_2(t_1) & Z_3(t_1) \\ \cdots & \cdots & \cdots & \cdots \\ Z_0(t_n) & Z_1(t_n) & Z_2(t_n) & Z_3(t_n) \end{bmatrix} A=\begin{bmatrix} Z_0(t_0) & Z_1(t_0) & Z_2(t_0) & Z_3(t_0) \\ Z_0(t_1) & Z_1(t_1) & Z_2(t_1) & Z_3(t_1) \\ \cdots & \cdots & \cdots & \cdots \\ Z_0(t_n) & Z_1(t_n) & Z_2(t_n) & Z_3(t_n) \end{bmatrix}$$

$Z_k(t)Z_k(t)$ $(k=0,1,\cdots,n)(k=0,1,\cdots,n)$ is basis functions of C-Bézier curve.

There is no exact solution on (1.2)equations under normal circumstances, there is no strict interpolated curve through these data points, but we can find these data points of the least squares approximation solution. That parameter polynomial curve Γ :,

$$\sum_{i=0}^{n}\left|\vec{B}_\alpha(t_i)-\vec{p}_i\right|^2=\min\sum_{i=0}^{n}\left|\vec{B}_\alpha(t_i)-\vec{p}_i\right|^2=\min \quad .$$

A is the full column rank, that $rank(A)=4$,so(1.1.2)orthogonal equations of corresponding Guass: $A^TAD=A^TP$ so:

$D=A^+P$, (1.3)

where : $A^+=(A^TA)^{-1}A^T$

Obtaining C-Bézier curve control points , $\vec{q}_1\vec{q}_1, \vec{q}_2\vec{q}_2, \vec{q}_3\vec{q}_3$ by Eq. (1.3) in order to get the solution of the problem 1.

Question 2: How to determine the parameters in order to correspond fairing of Γ : $\vec{r}=\vec{B}_\alpha(t)$ C-Bézier curve.

The shape of C-Bézier curve is related with α parameters. For fixed controling vertices, curve is moving away from the control polygon and tending to flatten with parameters of α gradual increases.

According to fairing criteria, applied energy method may establish fairing constraint equation

$$E = \int_0^{\alpha} \left\| \overrightarrow{B''}_{\alpha}(t) \right\|^2 dt = \min \qquad (1.4)$$

Here we are seeking specific expression of E fairing amount of fourth-order C-Bézier curve ,By (1.1.1) it is

$0 < \alpha \le \pi 0 < \alpha \le \pi$, $0 < t \le \alpha 0 < t \le \alpha$ (1.5)

Where, $\overrightarrow{T''} = [-\sin t, -\cos t, 0, 0]$,D and B are same with (1.4) formula,which has the

$$E = (BD)^T \begin{bmatrix} -\frac{1}{4}\sin^2\alpha + \frac{1}{2}\alpha & \frac{1}{2}\sin^2\alpha & 0 & 0 \\ \frac{1}{2}\sin^2\alpha & \frac{1}{4}\sin^2\alpha + \frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} (BD)$$

We will consider deviation curve and minimum of fairing amount during the fairing, so integrating issues (1) and issues (2) we can let the energy of the whole system as $E \to \min$

So α parameters and control points can be obtained after fairing, but we hope we control curve points through the whole designing.Let , $\vec{p}_i = \vec{q}_3 \vec{p}_i = \vec{q}_3$, we can get the E parameters and the , $\vec{q}_2 \vec{q}_2$ partial derivatives of control vertices,so as to achieve the purpose of the fairing.

From the variational principle it has

$$, \frac{\partial E}{\partial \vec{q}_1} = 0 \frac{\partial E}{\partial \vec{q}_1} = 0 , \frac{\partial E}{\partial \vec{q}_2} = 0 \frac{\partial E}{\partial \vec{q}_2} = 0 \quad . \qquad (1.6)$$

Integrating issues (1) and issues (2) we give the C-Bézier curve fairing and approximation algorithm .It is as follows:

Step1:Method using cumulative chord length parameterization of data points

Step2: , $\vec{q}_1 \vec{q}_1$, $\vec{q}_2 \vec{q}_2$ determined by the (1.6) equation

Step3:Let the values of , $\vec{q}_1 \vec{q}_1$, $\vec{q}_2 \vec{q}_2$ into (1.2),thereby we can obtain the corresponding curve of fairing and approximation Γ : $\in \Re^2 \in \Re^2$, $t \in [0, \alpha] t \in [0, \alpha]$

Note: The fairing and approximation for higher-order C-Bézier curves and surfaces can be similarly treated. It is omitted here.

3. CALCULATION EXAMPLE

A set of data points $(i = 0,1,\cdots,9)(i = 0,1,\cdots,9)$ are given as the following table:

Table 2.1 A set of data points

ii	0	1	2	3	4	5	6	7	8	9
x_i	0.5	1	0.5	1.5	1.2	2.7	1.7	2	4.5	6
y_i	0.5	1	1	0.8	2.1	2.2	2.3	2	2	1

Given a set values of α , we will get a different energy, as shown in Table 2.2:

Table 2.2 The energy changes of the value of α are shown:

α	0.54	0.8	1	1.137	2.0	2.6
E	37.55	33.27	26.26	23.28	24.52	31.219

The fairing minimum energy is 23.28 after fairing from the table when $\alpha = 1.137$.

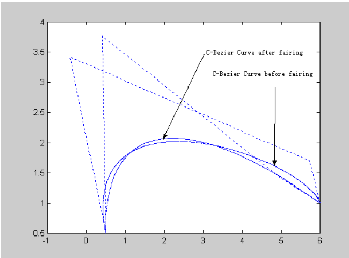

Fig.4.5 Fairing around the C-Bézier curve

4. CONCLUSIONS

The nature and definition of C-Bézier curves are given firstly and carefully studied when $\alpha \to 0^+$,the Curve of $B(t;\alpha_1)$ and $B(t;\alpha_2)$ are intersecting($\alpha_1 \ne \alpha_2$). On the basis of the energy law, making the curve is given by adjusting the minimum energy so as to achieve the purpose of fairing.

REFERENCES

[1] Buqin Su.Some Notes about three parameters spline curve [J]. Applied Mathematics Journal, 1976, 1, 49-58

[2] Buqin Su.A theorem on three parameters spline curves [J]. Applied Mathematics Journal. 1977, 1 ,49-54

[3] Buqin Su. On affine invariant Bézier curves [J]. Computational Mathematics, 1980, 2, 289-298.

[4] Ye Zheng lin, Wei Sheng min, Feng Guo sheng. On properties of planar parametrictension [J]. Journal of North western Polytechnical University, 1995, 13(3): 458-462

[5]Buchin Su. Hesheng Hu, Yuanlong Xin. Differential geometry [M]. Beijing: Higher Education Press, 1990

[6] Jianming Zheng, Thomas W. Gaussian and mean curvatures fractional Bézierpatches [J]. CAGD20, 2003: 297-301