ISSN 2618-1584 Vol. 1, Issue 3: 16-34, DOI: 10.25236/FMSR.2019.010303

Application of differential equation Model in Drug Propagation

Lin Jin[1], Ruige Chen[2]

1 School of Economics and Management, China University of Geosciences (Beijing), Haidian District, Beijing 10008, China
2 School of Mathematics and Mathematics, China University of Geosciences (Beijing), Haidian District,Beijing 100083, China

ABSTRACT. *This paper mainly considers the spread of opioids.By using the qualitative theory of differential equation and bifurcation theory as well as the related theories of reaction-diffusion equation, the drug diffusion model is established. The dynamic behavior of the model is analyzed, and finally the drug transmission model is established. Observing the process of heroin addiction in healthy subjects with non-linear incidence, this model is more reasonable than the existing model with richer and more complex dynamic behaviors. After linearization of the equation, through analysis of the stability model and branches, the toxicity balance of the system will become the possibility of multiple limit cycles under different conditions. These results suggest that many changes occur during drug transmission, so drug control is not as easy to spread as might be expected. Based on the findings, we find that reducing exposure to drug users is more effective in reducing drug transmission than treating drug users. In addition, reports of treated and untreated drug users can enable people to build up defense mechanisms that reduce the number of drug users. Reducing the speed of drug diffusion can also control the spread of drugs, playing the role of setting.*

KEYWORDS: *The multi-linear regression analysis of the related analysis of the drug-related differential equation.*

1. Introduction

In the late 1990s, the use of prescription and non-prescription opioid drugs rapidly increased in the United States and Canada, beginning and continuing throughout the next two decades. In 2017, it is the opioids for 49,000 of the 72,000 drug overdose deaths overall in the US. In 2016, a study estimated that the cost of prescription opioid abuse in the United States in 2013 was approximately $78.5 billion, largely due to health care and criminal justice spending, along with lost productivity. According to the data, from the aspect of health condition in society or of cost of medical cure, the spread of drug deserves to be paid attention.[1] In this

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question, we are supposed to make a continuous model and to identify the important parameters and strategies. Based on the Question C of 2019 American college students mathematical modeling contest, this article builds the differential equation model of drug spread, adding kinetic analysis and uses this model to discuss the drug spread. Combined with the DEA/NFLIS has published a data-rich annual report containing the results of drug identification and relevant information from drug cases analyzed by federal, state, and local forensic laboratories. Combined with the report, this problem is going to focus on specific counties located in five U.S. states including Ohio, Kentucky, West Virginia, Virginia, and Tennessee.

First, to build a model to describe the spread and characteristics of the synthetic opioid and heroin incidents in and between the five states and their counties over time and to identify the location where specific opioid might have started to be used in each of the five states. Second, with the provided U.S. Census socio-economic data, to find the way that the opioid use got to its current level, the group who are abusing it, the contributions to the growth in opioid use and addiction and the reason why opioid use persists despite its known dangers. Third, using a combination of our Part 1 and Part 2 results, to put forward a practical strategy for countering the opioid crisis. Use our model(s) to test the effectiveness of this strategy and to make summary of significant parameter bounds that success (or failure) is dependent upon. During the whole process, we can only use the data set supplied by specific file.

2. Basic Assumption

$T = S + U_1 + U_2$ & $E = \mu S + (\mu + \delta_1)U_1 + (\mu + \delta_2)U_2$; Wesupposed that the population is of constant size in the specific area during the modelling time period without huge moves in or out.All members of the population are assumed to be equally susceptible to drug addiction.Drug users not in treatment are infectious to susceptible and to users in treatment.Drug users in treatment are not infectious to susceptible.

3. Glossary & Symbols

3.1 Glossary

Basic copy number: R_0 is to investigate how many secondary infections are caused by an infected individual entering the susceptible population.

Nonlinear incidence: βS^{pIq} is always used in epidemic model when the bilinear incidence is not valid, which make it has a richer kinetic behavior.

Saddle node: In a differential equation, a singularity that is stable in one direction and unstable in the other is called a saddle pointCusp node: In a differential equation, a singularity that is stable in one direction and unstable in the other is called a saddle

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pointToxic equilibrium point: There are drug addicts in this equilibriumNon-toxic equilibrium point: There are no addicts in this equilibrium

3.2 Symbols

Symbols	Definition	Notes
T	Total quantity of population	
S	The number of susceptible people in general population	
U1	The number of drug users not in treatment	includes people who take drugs for the first time and drug users who are relapsed
U2	The number of drug users in treatment	
E	The number of people who go from normal to susceptible	
μ	The natural death rate of the general population	
$\square_1$	The removal rate of drug users in treatment	Includes rate of users in treatment stopping using drugs but are no longer susceptible and drug-related deaths rate of individuals not in treatment and a spontaneous recovery rate.
$\square_2$	The removal rate of drug users not in treatment	This includes the rate of death and successful recovery of drug-related users in treatment, as well as the recovery of drug-free life and immunity to drug addiction during modeling period time
$\square_1$	The possibility of becoming a drug user	
β	The infection rate of specific area	
α	Equals to The ratio of DrugReports / the TotalDrugReportsCounty in each county of each drug and the ratio of each DrugReports in a state totaling together / TotalDrugReportsCounty	Represents the drug use condition in specific area

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4. Model Construction and Optimization

4.1 Constant Differential Drug Spread Model

In 1981,heroin addiction was first defined as an infectious disease. Corresponding to the epidemic disease model, in the drug model, the total population is correspondingly divided into three categories: susceptible people, drug users not in treatment and drug users in treatment.[2]Through mathematical modeling, the biological system and ecological simulation of the three populations will be described. This paper builds the optimal model and researches its equilibrium point following four steps.

Step 1

We chose a constant differential model of drug spread. According to the information of the study and the research of professionals in the relevant fields, the path of drug use career as shown in figure 1 was selected, matched with the model of susceptible-infectious diseases. Each square in the figure represents a social stage of the career, and the arrows represent the paths between the stages. First of all, by susceptible groups (E) into the system, S is the number of susceptible individuals in the crowd; μ is natural mortality rate; β_1 is the probability of drug addicts; β_3 says the probability of treated drug addicts restart taking drugs; U_1 is the number of drug addicts not in treatment; U_2 is the number of drug addicts in treatment;[3] T says the total population; δ_1 is removal rate for drug addicts in treatment;the δ_2 is for drug addicts not in treatment removal rate and p says the probability of drug addicts entering treatment. The constant differential equation was derived to characterize the whole system.

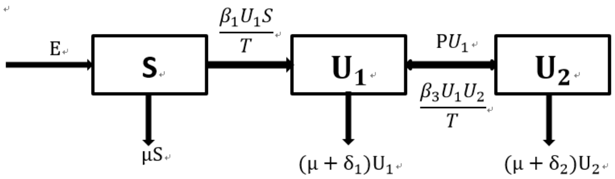

Figure 1 the path of drug use career

According to the path of drug use career, the differential equation is established as follows:

$$\begin{cases} \frac{dS}{dt} = E - \frac{\beta_1 U_1 S}{T} - \mu S \\ \frac{dU_1}{dt} = \frac{\beta_1 U_1 S}{T} - pU_1 + \frac{\beta_3 U_1 U_2}{T} - (\mu+\delta_1)U_1 \\ \frac{dU_2}{dt} = pU_1 - \frac{\beta_3 U_1 U_2}{T} - (\mu+\delta_2)U_2 \end{cases} \quad (1)$$

Basic copy number R_0 was proposed to investigate how many secondary infections are caused by an infected individual entering the susceptible population. If

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$R_0 < 1$, it means that the epidemic will not happen in the susceptible population because of an infected individual; Conversely, if $R_0 > 1$, it means that the susceptible population will be infected by an infected individual. If $R_0 = 1$, each infected person will infect a susceptible population, and the specific situation needs further investigation. However, in the opioid and heroin using lives, each addict leads at least one other person to take the drug, so in the following model usage, we set the condition $R_0 > 1$, which is very helpful to determine the existence of the equilibrium point.[4]

Step 2

In the case of the basic copy number $R_0 > 1$, we considered the difference between “light drug users” and “heavy drug users” because the duration of drug use has different effects on the difficulty of drug withdrawal. In the “light users” group, those who received treatment were more likely to escape treatment than those who did not. Therefore, it is easier and more natural to substitute $\frac{\beta_3 U_1 U_2}{T}$ in the formula (1) for σU_2. Meanwhile, it was also assumed that the “light drug users “ could spontaneously detox at a restitution rate of θ_1, while the “heavy drug users “ would detox at a treatment rate of θ_2.The formula after replacement is as follows:

$$\begin{cases} \frac{dS}{dt} = E - \frac{\beta_1 U_1 S}{T} - \mu S + \theta_1 U_1 + \theta_2 U_2 \\ \frac{dU_1}{dt} = \frac{\beta_1 U_1 S}{T} - pU_1 + \sigma U_2 - (\mu + \theta_1) U_1 \\ \frac{dU_2}{dt} = pU_1 - \sigma U_2 - (\mu + \delta_2) U_2 \end{cases} \quad (2)$$

Step 3

But this formula is linear. In general, a susceptible person will not become an addict because of a single drug use.[2] So in this paper, we assumed that an addict has used drug twice or more. As the result, it is necessary to consider the nonlinear incidence in the heroin model.[5] At the same time, according to the natural fact, drug addicts, once someone he will be hard to drug withdrawal through self-control, so we only considered the removal rate δ.[6]For the sake of simplicity, in this paper we considered the nonlinear incidence $\beta s u_1^2$, and eventually used model as follows:

$$\begin{cases} \frac{ds}{dt} = b - \beta u_1^2 s - \mu s + \delta u_2 \\ \frac{du_1}{dt} = \beta u_1^2 s - pu_1 + \sigma u_2 - \mu u_1 \\ \frac{du_2}{dt} = pu_1 - \sigma u_2 - (\mu + \delta) u_2 \end{cases} \quad (3)$$

The initial condition is $s(0) = s_0 \geq 0,\ u_1(0) = u_{10} \geq 0,\ u_2(0) = u_{20} \geq 0$.

The $s(t)$, $u_1(t)$, $u_2(t)$, respectively represents the number of susceptible people, untreated drug users and treated drug users at time t. Assume that all the parameter is positive, b refers to the added rate susceptible people, βu_1^2 is infection ability, β is the possibility of an infectious person be infected by a drug user, σ shows the proportion of drug users escaped from treatment process and δ refers to the proportion of drug addicts after treatment again into susceptible people.

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Step 4

In order to simplify the model without affecting the establishment of the model, we did the following operations,

we made $s = \frac{b}{\mu}s,\ u_1 = \frac{b}{\mu}u_1, u_2 = \frac{b}{\mu}u_2, \beta = (\frac{\mu}{b})^2\beta$, (3) can be changed into,

$s = \frac{b}{\mu}s,\ u_1 = \frac{b}{\mu}u_1, u_2 = \frac{b}{\mu}u_2, \beta = (\frac{\mu}{b})^2\beta$, (3) can be changed into,

$$\begin{cases} \frac{ds}{dt} = \mu - \beta u_1^2 s - \mu s + \delta u_2 \\ \frac{du_1}{dt} = \beta u_1^2 s - p u_1 + \sigma u_2 - \mu u_1 \\ \frac{du_2}{dt} = p u_1 - \sigma u_2 - (\mu + \delta) u_2 \end{cases} \tag{4}$$

Add the equations together:$(s + u_1 + u_2)' = \mu - \mu(s + u_1 + u_2)$

So $\lim_{n\to\infty}(s + u_1 + u_2) = 1$

The population is assumed to be constant so that we can study the dynamic behavior on the plane $s + u_1 + u_2 = 1$

Therefore,by $s = 1 - (u_1 + u_2)$, (4) can be changed into:

$$\begin{cases} \frac{du_1}{dt} = \beta\big(1 - (u_1 + u_2)\big)u_1^2 + \sigma u_2 - (p + \mu) u_1 \\ \frac{du_2}{dt} = p u_1 - \sigma u_2 - (\delta + \mu) u_2 \end{cases} \tag{5}$$

Made length contract transformation:

$$u_1 = \frac{\delta + \sigma + \mu}{\beta} x$$

$$u_2 = \frac{p}{\beta} y$$

$$t = \frac{1}{\delta + \sigma + \mu} t$$

made:

$$A = \frac{\delta + \sigma + \mu}{\beta}$$

$$B = \frac{p}{\beta}$$

$$C = \frac{\sigma p}{(\delta + \sigma + \mu)^2}$$

$$D = \frac{p + \mu}{\delta + \sigma + \mu}$$

So (5) can be changed into:

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$$\begin{cases}\frac{dx}{dt}=(1-Ax-By)\mathrm{x}^2+Cy-Dx\\ \frac{dy}{dt}=x-y\end{cases} \quad (6)$$

In order to get to the equilibrium point, we made the right-hand side zero

$$0=(1-Ax-By)\mathrm{x}^2+Cy-Dx$$

$$0=x-y$$

As it illustrates,(0,0)is a Non-toxic equilibrium point,(a, a) is a Toxic equilibrium point and a is the root of function $0=(1-Ax-Bx)\mathrm{x}^2+Cx-Dx$. According to the discriminant $\Delta= 1+4(\mathrm{A}+\mathrm{B})(\mathrm{C}+\mathrm{D})$, to identify the number of equilibrium point.

We got three theorems:

For any parameter, the equilibrium point of people without drug addiction of the system is a stable node.

If $\mathrm{D}=\mathrm{C}+\frac{1}{4(A+B)}$, there is a unique positive equilibrium point $\mathrm{K}_1(\frac{1}{2(A+B)},\frac{1}{2(A+B)})$.

When $\mathrm{D}=\mathrm{C}+\frac{1}{4(\mathrm{A}+\mathrm{B})}$ and $\mathrm{B}\neq 4(\mathrm{A}+1)(\mathrm{A}+\mathrm{B})^2, \mathrm{K}_1$ is saddle node.

When $\mathrm{D}=\mathrm{C}+\frac{1}{4(\mathrm{A}+\mathrm{B})}$ and $\mathrm{B}=4(\mathrm{A}+1)(\mathrm{A}+\mathrm{B})^2, \mathrm{K}_1$ is cusp node.

If $\mathrm{C}<\mathrm{D}<\mathrm{C}+\frac{1}{4(A+B)}$, in the system, there is two positive equilibrium point $\mathrm{K}_2(\frac{1-\sqrt{\Delta}}{2(A+B)},\frac{1-\sqrt{\Delta}}{2(A+B)})$, $\mathrm{K}_3(\frac{1+\sqrt{\Delta}}{2(A+B)},\frac{1+\sqrt{\Delta}}{2(A+B)})$.

When $\mathrm{C}<\mathrm{D}<\mathrm{C}+\frac{1}{4(\mathrm{A}+\mathrm{B})}, \mathrm{K}_2$ is saddle node.

When $\mathrm{C}<\mathrm{D}<\mathrm{C}+\frac{1}{4(\mathrm{A}+\mathrm{B})}, \mathrm{K}_2$ has two cases:

If $2(3\mathrm{A}+2\mathrm{B})(\mathrm{A}+\mathrm{B})(\mathrm{D}-\mathrm{C})<\mathrm{A}+2(\mathrm{D}+1)(\mathrm{A}+\mathrm{B})^2+\mathrm{A}\sqrt{1-4(\mathrm{A}+\mathrm{B})(\mathrm{D}-\mathrm{C})}, \mathrm{K}_3$ is a stable unsaddle node.

If $2(3\mathrm{A}+2\mathrm{B})(\mathrm{A}+\mathrm{B})(\mathrm{D}-\mathrm{C})>\mathrm{A}+2(\mathrm{D}+1)(\mathrm{A}+\mathrm{B})^2+\mathrm{A}\sqrt{1-4(\mathrm{A}+\mathrm{B})(\mathrm{D}-\mathrm{C})}, \mathrm{K}_3$ is an unstable unsaddle node.

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5. Model Extension and Simulation Analysis

5.1 Analysis and Solving of Problem part one

5.1.1 Application of equilibrium

Then, we could use the equilibrium point that we had got to describe the spread and characteristics of the synthetic opioid and heroin incidents in and between the five states and their counties during given time. The ratio of DrugReports to the TotalDrugReportsCounty in each county of each drug and the ratio of each DrugReports in a state totaling together to TotalDrugReportsCounty in each state of each drug was calculated to show the drug use. Then the calculated ratio is used to establish a time relationship according to model (4) to observe the trend of the drug in this region (state or county). We took heroin as an example and got the following figures,

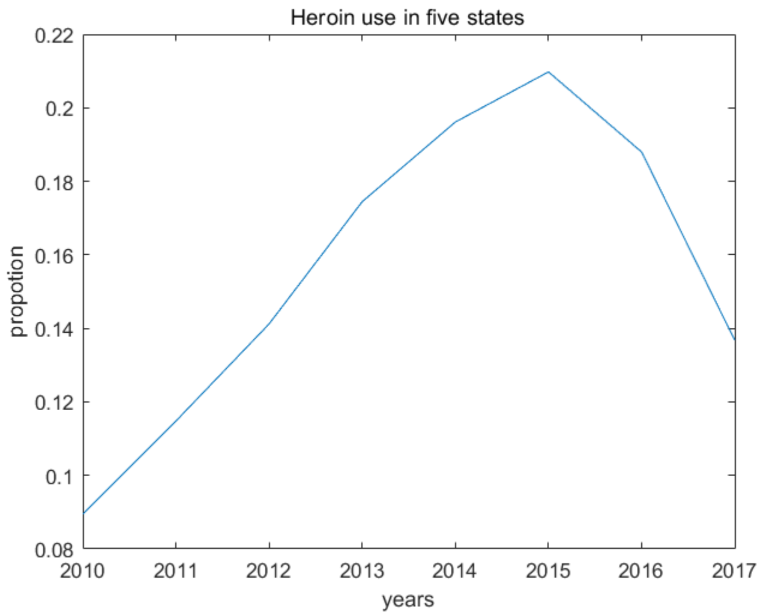

Figure.2 Variation of total quantity of Heroin use over time in five states

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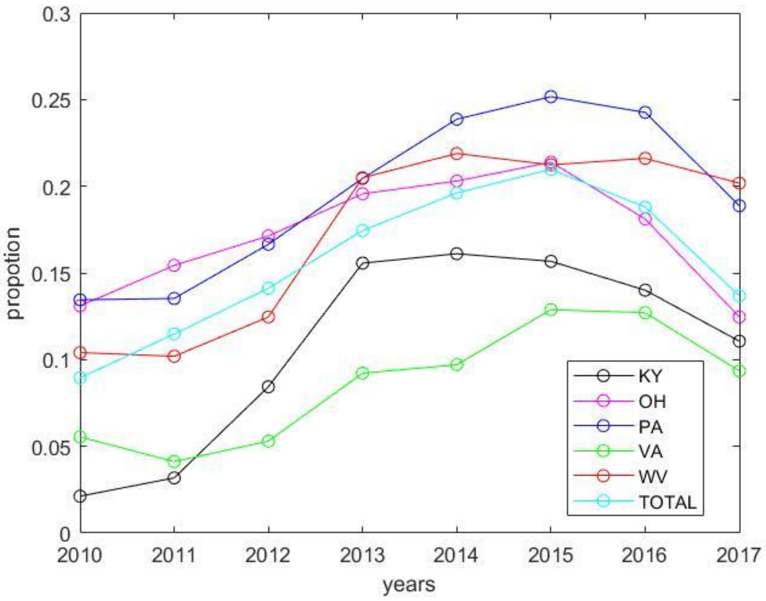

Figure.3 Variation of Heroin use over time in each of states

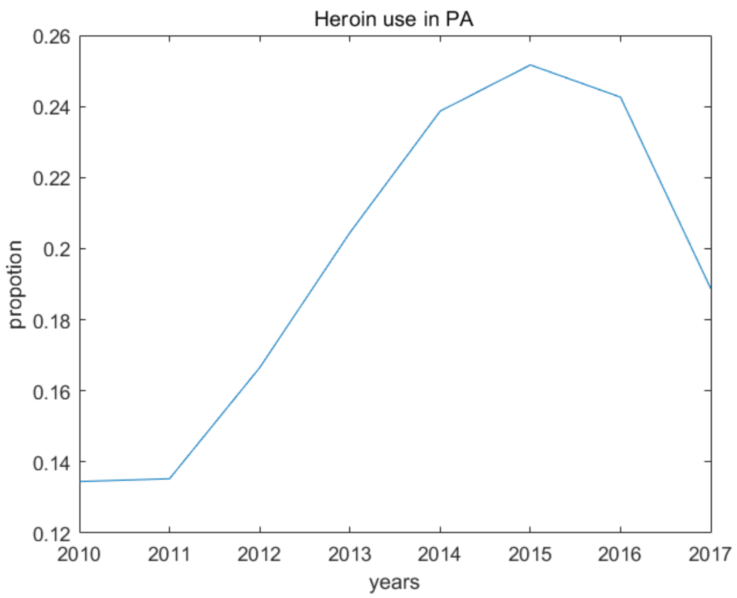

Figure.4 Variation of Heroin use over time in Pennsylvania

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In figure 2, it illustrates that the change trend of the total heroin use over time is basically "inverted u-shaped", indicating that the total heroin use in the five states increases with the change of time, but the increase rate decreases. [6]The total heroin use reached the saturation point around 2015, and then decreased, that is, the total heroin use rate became negative. In figure 3, we can see that heroin use in each state also shows an inverted "U" trend over time. For example (figure 4), in Pennsylvania, heroin use in the region was below the equilibrium point from 2010 to 2015 and from 2016 to 2017, heroin use crossed the equilibrium point and showed a downward trend.

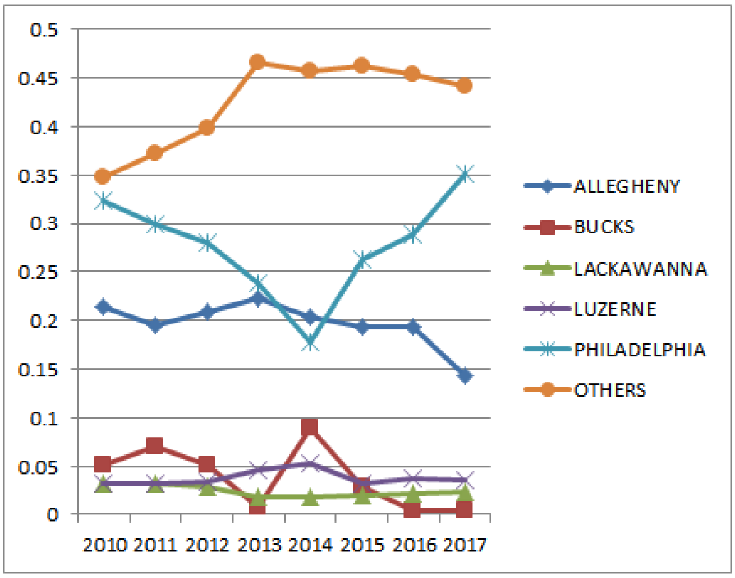

Figure. 5 Variation of Heroin use over time in each of counties in Pennsylvania

The same principle can be applied to counties in each state to find the equilibrium point and carry out communication analysis. Figure 5 shows the change in heroin use in specific counties in Pennsylvania. Among them, the equilibrium point of Allegheny during this period is around 2013, the equilibrium point of bucks is around 2014, the equilibrium point of Lackawanna should be before 2010, the equilibrium point of Luzerne is around 2014, the equilibrium point of Philadelphia is before 2010 or after 2017, and other counties are generally around 2013.

5.1.2 Parameter calculation

According to simulations and figures we made and economy and social objective law, the equilibrium point is cusp node. So in order to get parameters in the model, this paper used proper conditions of cusp node to make equations as following,

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$$\begin{cases} D = C + \frac{1}{4(A+B)} \\ B = 4(A+1)(A+B)^2 \\ x = y = \frac{1}{2(A+B)} \end{cases} \quad (7)$$

Then with the data of the drug use α (DrugReports/TotalDrugReportsCounty and each DrugReports in a state totaling together/TotalDrugReportsCounty) and time of the equilibrium points in five states, we carried out the value of A,B,C,D using least square method.

Because,

$$\begin{cases} A = \dfrac{\delta + \sigma + \mu}{\beta} \\ B = \dfrac{p}{\beta} \\ C = \dfrac{\sigma p}{(\delta + \sigma + \mu)^2} \\ D = \dfrac{p + \mu}{\delta + \sigma + \mu} \end{cases}$$

So we can determine b, δ , σ , μ , β , p with A,B,C and function (3),

$$b = 0.0041$$

$\delta = 0.0234$

$\sigma = 0.0357$

$\mu = 0.0459$

$\beta = 0.0501$

$$p = 0.962$$

5.1.3 Possible locations where specific opioid use might have started

Continuing to observe figure 5, combined with the previous calculation, it can be seen that the curve of Philadelphia and Allegheny is basically above the curve of the other three counties, indicating that these two counties can be considered as the source of heroin in Pennsylvania. The same can be said of other states. After making the chart and observing it, combined with specific states, we found that these 15 counties can be considered as the origin place: Hamilton in Ohio, Butler in Ohio, Boone in Kentucky, Campbell in Kentucky, Kenton in Kentucky, Kanawha in West Virginia, Jefferson in Kentucky, Allegheny in Pennsylvania, York in Virginia, Cuyahoga in Ohio, Prince William in Ohio, Fairfax in Virginia, Luzerne in Pennsylvania, Lackawanna in Pennsylvania, Mineral in West Virginia.

In order to verify the accuracy of data fitting, we find the particular given five states in the United states from the map. The drug use in some years (that is, the proportion calculated above) is presented in different colors. The higher the

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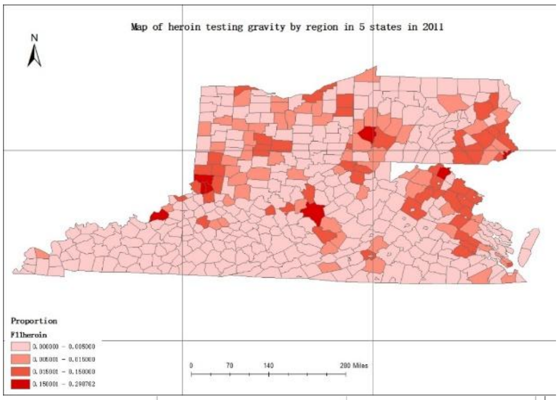
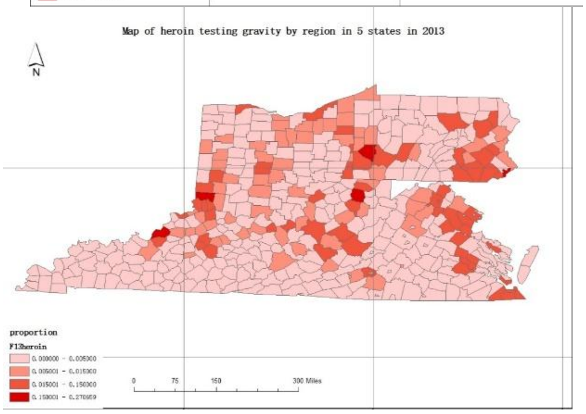
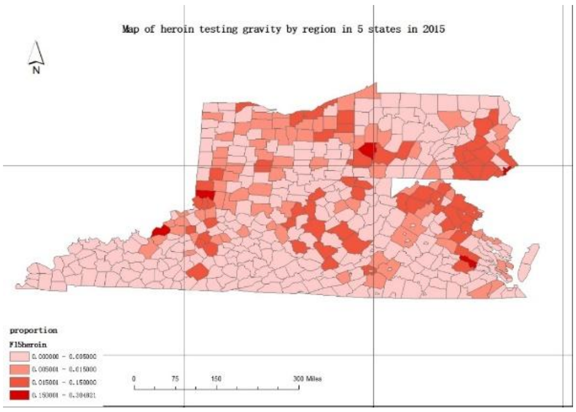
proportion, the darker the color. As it shown in the figure below.







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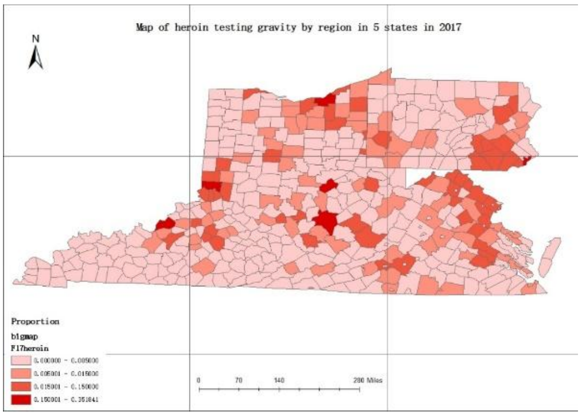

Figure. 6 the variation of the drug use in some years in given five states

To sum up, we can see from the picture that the region with the deepest color duration within the given period of time is where specific opioid use might have started, which is highly consistent with the counties calculated by our model.

So far, our model is practical and sustainable. We realized that one of the concerns of the U.S. government should have is the source of the drug source place. After consulting information, the federal and state and local governments in the United States are mostly concerned with drug addiction treatment and controlling individual abuse. However, due to market supply and demand and huge profit factors, drug trafficking and spread are also very serious, so the source of infection should also be considered by the U.S.[8] government. Its threshold value is the equilibrium point from simulation between data of time and usage (proportion) and model fitting.

So when and where will the future shift from equilibrium occur? Obviously, if a region's usage has almost but not yet reached the predicted equilibrium point by observing the current trend, then the region and the point when it will reach the equilibrium point in the future are the answers to the problems to be solved.

In the same way, with the chart and observation, we define three potential counties that may be the opioid resource as following,

Wood in West Virginia, Kanawha in West Virginia, Halifax in Kentucky, Mecklenburg in Kentucky, Cuyahoga in Ohio and Summit in Ohio.

5.2 Analysis and Solving of Problem part two

5.2.1 Identification of important economy and society elements

On the basis of the large number of data identified in the problem, we identified and analyzed the correlations between each parameter and the “county-by-state” ratio of drug use situation (the proportion of identified drugs in each county

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accounting for identified by state) and identified coefficients were identified. Take the top 20 socio-economic factors with the highest correlation coefficient as the correlation factors that really affect the use or use trend, and then through the same kind of screening, the regression equation of 7 economic factors and 7 factors and the county-state ratio of drug use status can be obtained finally,

y=-0.00047*x1-0.00182*x2-0.00045*x3+0.00061*x4+0.00159*x5+0.00065*x6 +0.01596*x7

The following table shows the meaning of the equation and the correlation between positive and negative

Table 1 the meaning of the equation and the correlation

Economic indicator	The corresponding mathematical variable	Coefficient of regression equation	Correlation
Married-couple family	$\square_1$	-0.00047	negative correlation
Spouse	$\square_2$	-0.00182	negative correlation
Never married	$\square_3$	-0.00045	positive correlation
Percent bachelor's degree or higher	$\square_4$	0.00061	negative correlation
Different house in the U.S. - Same county	$\square_5$	0.00159	positive correlation
Born in United States	$\square_6$	0.00065	negative correlation
Language other than English - Asian and Pacific Islander languages	$\square_7$	0.01596	positive correlation
Total Drug Reports County/Total Drug Reports State	y		

5.2.2 Further discussion of correlation

Because the existence of socio-economic factors has a great impact on the parameters, we modified the parameters by considering the factors. In the previous step, the meaning of each parameter was determined and the value of each parameter without considering social and economic factors was obtained. By finding the seven values that have a large impact on factors, the parameters were closely related to social and economic factors through regression equation [7]. The proportion of each factor is calculated, so that the parameters and social and economic factors had a strong coupling. Finally, the drug transmission model is modified. We took β for example.

In order to better couple the selected socio-economic factors with the infection rate, we establish a linear regression equation $\beta = \beta_0 + k_1 + k_2 y$ for the data set of infection rate and socio-economic factors. In the equation, β is the infection rate of specific area(county or state) and β_0 is estimated β in part 1 and y is the data set

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of important socio-economic factors. We used eviews 8 to calculate and get the result:

β = 0.028 + 2.387*y

so,

$$\begin{cases} \beta = \beta_0 + 2.387 * y - 0.0216 \\ \beta_0 = 0.0492 \end{cases} \quad (8)$$

So we have the infection rates associated with important socio-economic factors, and we have the extent to which drug use and transmission are correlated with the socio-economic data provided by the U.S. census.

5.2.3 Model modification

Using the modified β (8) in 2 to modify the model (4) used in part 1 and we can get,

$$\begin{cases} \frac{ds}{dt} = b - (\beta_0 + 2.387 * y - 0.0216)u_1^2 s - \mu s + \delta u_2 \\ \frac{du_1}{dt} = (\beta_0 + 2.387 * y - 0.0216)u_1^2 s - pu_1 + \sigma u_2 - \mu u_1 \\ \frac{du_2}{dt} = pu_1 - \sigma u_2 - (\mu + \delta)u_2 \end{cases} \quad (9)$$

5.3 Analysis and Solving of Question Three

Through the analysis of problem two, we identified seven socio-economic factors related to the spread of drugs. We can change the value of these factors to reduce the infection rateβ, or to increase the recovery rate δ to reach the control point and then to solve the opioid crisis.

We believed that improving the level of basic education of citizens can well solve the opioid crisis. Through the second asked regression equation for solving the beta, based on $\alpha_4$coefficient become bigger, can reduce the balance, can effectively prevent drug explosive growth, or make drug use level swiftly.

By discussing the model equilibrium point, we can solve the problem well.

From the above conditions, we can transform K_3 from an saddle node to a cusp node by β changes, which enables it to change from unstable state to stable state. Again, we need to transformK_3 from an unstable unsaddle node to a stable unsaddle node simply by changing. Then we solved the opioid crisis.

This paper simulates several counties in the five states after taking measures to improve citizens' basic education level.[7]

We adjusted the coefficient from 0.00061 to 0.001 and simulated the data. The simulation results are as follows:

ISSN 2618-1584 Vol. 1, Issue 3: 16-34, DOI: 10.25236/FMSR.2019.010303

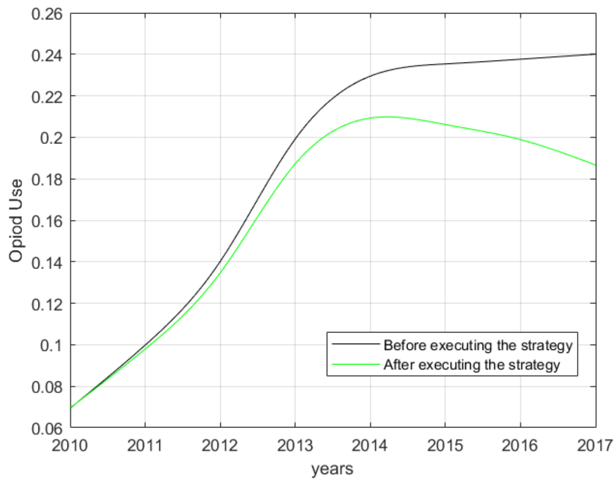

Figure.7 Simulations of several counties in the five states after taking measures

The result shows that these measures can bring down drug use levels quickly.

6. Sensitivity Analysis

6.1 Sensitivity Analysis

This paper also analyzed the sensitivity of the infection rate β. In Ohio, for example, firstly by changing differential infection rate β of the ratio of heroin addicts, to numerically resimulate the annual number of people becoming drug users. The figure 8 shows that when β decreases 10%, the state of the positive equilibrium point has fallen by 12% while when the β is increased by 10%, the results of numerical simulation found addicted people ratio after the original balance did not tend to be saturated, it still keep growth trend. This means that if the infection rate in the region rises above a certain threshold, if the government's measures to prevent drug transmission are not changed, the spread of drugs will be rampant. Therefore, we should take measures to reduce the transmission rate of heroin and prevent the uncontrolled growth of drug infections

Furthermore, we established a elasticity coefficient of infection rate and drug use to make a more accurate quantitative assessment. The formula is $\frac{\Delta\beta/\beta}{\Delta\alpha/\alpha}$. According to the calculation, we found that the elasticity coefficient is approximately equal to 0.442, indicating that the change rate of infection rate is about 10 units for every 4 units of drug use, which describe the sensitivity.[8]

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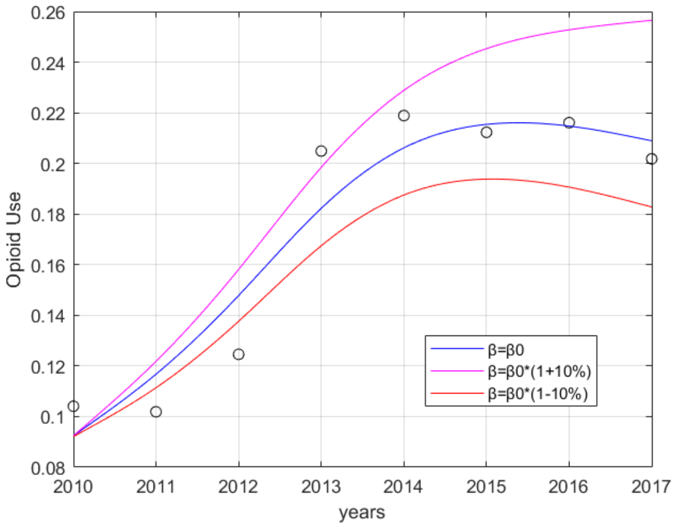

Figure. 8 The sensitivity of the infection rate β *in Ohio*

7. Evaluation and Promotion of Model

7.1 Strength and Weakness

In this paper, a more accurate model of drug transmission was established by studying the law of drug transmission, discussing its dynamic behavior and state, and modifying the parameters through the influence of social and economic factors on the parameters. Based on the above analysis, the advantages and disadvantages of this model are as follows:

7.1.1 Strength

1) Extensibility. The model divided the population into three types, and established the differential equations. Therefore, when discussing the impact of other factors on the model, the model can be simply modified. The analysis method is the same but has strong expansibility.

2) Compatibility. In addition to the study of drug transmission, when discussing similar models, such as the establishment of economic transmission model, the equation and model of desire can be obtained by changing the meaning of parameters.

3) Stable model. There is more than one mode of transmission of drug addiction, influenced by politics and economy situation. The discussion on the balance point is particularly important. Through the discussion on the balance point, we can clearly get the change of state in different situations.

ISSN 2618-1584 Vol. 1, Issue 3: 16-34, DOI: 10.25236/FMSR.2019.010303

7.1.2 Weakness:

1) The subject only provided data from 2010 to 2017. Due to the insufficient data, it has certain influence on the parameter determination of the model, resulting in errors.

2) Considering the limitation of drug rehabilitation resources, we should replace the commonly used linear recovery rate in the infectious disease model with a non-linear recovery rate, which could not be solved due to limited time.

8. Conclusions and Future Expectations

8.1 Conclusions of the problem

In this article, we mainly considered the spread of new drugs. Drug Spread model was constructed, using the theory of ordinary differential equation qualitative theory and branch as well as the reaction diffusion equations related theory to analyze the dynamic behavior of the model. The heroin model was set up, using the nonlinear incidence to describe the process of healthy subjects with heroin addiction, which was more reasonable compared with the existing model with richer and more sophisticated dynamic behavior. After linearizing the equation, through analysis of the model of stability and analysis of branch, under the different condition, system of Toxic Balance will be the possibility of multiple limit cycles. These results suggested that drugs many changes would appear in the process of transmission, so the spread of drug control is not so easy as imagine.

According to the results, we found that minimize contact with drug addicts than in treatment for drug addicts to reduce the spread of drugs more effective. In addition, the reports of treated and untreated drug users can make people come up defensive mechanisms, so that the number of drug users will be fewer. Reducing the drug diffusion speed can also control the spread of the drug have the effect of setting.

8.2 Future Expectations

Opioid crisis is not only limited in many regions of the United States, but also a problem of people’s daily life all over the world. It is closely associated with health of people’s life and economy and society conditions of a country, hence it demands much attention from publics as well as governments.

Our team found a lot of mechanisms worth improving and subjects worth studying: First, the current drugs (especially new drugs) transmission is differ and domestic production is more and more frequent, so people can consider that when modeling, the effect of the changes in input mode for the spread of drug. Second, drug addicts are often infected other pathological conditions, such as HIV/AIDS, hepatitis b because of blood transmission of the virus diseases. A common

ISSN 2618-1584 Vol. 1, Issue 3: 16-34, DOI: 10.25236/FMSR.2019.010303

understanding is that the drugs often leads to the spread of disease. How big is the effect it? Can we describe it quantitatively with a mathematical model? These are also meaningful subjects. Third, during the establishment of the model, the time delay of the model had not been not taken into account, which tests the accuracy of the model. The time delay model worth being thinking.

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