

Research on Regional Economic Vitality Based on Mathematical Modeling

Zhang Zhe¹, Zhao Rui¹, Zhao Yueming¹, Zhang Zhaozhen², Yao Daohong¹

1 Qingdao Technological University Linyi Campus, Qingdao, China

2 Shanxian Vocational Secondary Vocational School, Shanxian, China

ABSTRACT. *The regional economic vitality of a city affects the development of a city. With the advent of economic globalization, inter-regional exchanges have gradually increased. This article analyzes how to improve the “regional economic vitality”. It uses a descriptive statistical method and multiple linear regression. Method, combined with software such as matlab for modeling research. This paper selects Beijing as the research city. First, it analyzes economic factors, population factors, corporate change factors, and foreign trade factors to obtain mathematical models that affect the vitality of Beijing's regional economy. The software predicts the regional economic vitality in Beijing and makes relevant suggestions on how to develop regional economic vitality in Beijing.*

KEYWORDS: *Regional economic vitality, Multiple regression models, Development suggestions, Mathematical modeling*

1. Introduction

1.1 Background

The regional (or urban or provincial) economic vitality is an important part of regional comprehensive competitiveness. In recent years, in order to improve the economic vitality, some regions have launched many preferential policies for stimulating the economy vitality, such as reducing the investment attraction approval steps, providing the capital support to start-ups and lowering the settlement threshold to attract the talented. However, due to different resource endowments, these policies have different effects in different regions. How to seize the key factors and effectively improve the regional economic vitality is a worth study topic.

1.2 Work

1. The regional (or urban or provincial) economic vitality is affected by variety of factors. Take a region (or city or province) as an example, please build the suitable relational model of influencing factors of economic vitality, and study the program of action to improve the regional economic vitality. Analyze the effects on the regional economic vitality change from the perspective of changing trend of population and enterprise vitality.

2. Select a region (or city or province), and analyze the short term and long term effects of economic policies transformation on the economic vitality of such region (or city or province) based on the suitable data surveyed by you.

2. Problem Analysis

2.1 Analysis of Question One

Question 1 requires us to take a region as an example to establish a reasonable model of influencing factors of economic vitality. First, we choose Beijing as the research city; second, we choose GDP to represent regional economic vitality, population change trends, enterprise change trends, and the average total production of residents. Value as an influencing factor; then use the total number of permanent residents to reflect the population change trend, use the number of enterprises and the output value of the enterprise to reflect the change trend of the enterprise, and use the per capita GDP; finally, establish a multivariate linear regression

model, and based on this model, give a plan to improve economic vitality.

2.2 Analysis of Question Two

Question two requires us to choose a region and analyze the short-term and long-term effects of the changes in economic policies on the region's economic vitality. First, we selected Anshan City as the research city. GDP of the three major industries in 2015; secondly, fitting the GDP and total GDP of the three major industries before and after adjustment, and seeing the short-term impact of economic policies on economic vitality from the fitted curve; Long-term impact on economic vitality.

3. Model

3.1 The Establishment and Solution of the Problem One Model

3.1.1 Collection and Inspection of Data

In the Beijing Statistical Yearbook, from 1999 to 2018, per capita GDP, total permanent population, enterprise output value, number of enterprises, and related data of these four indicators are shown in Appendix 1.

Make scatter plots of these four indicators to see if they have a linear relationship with y:

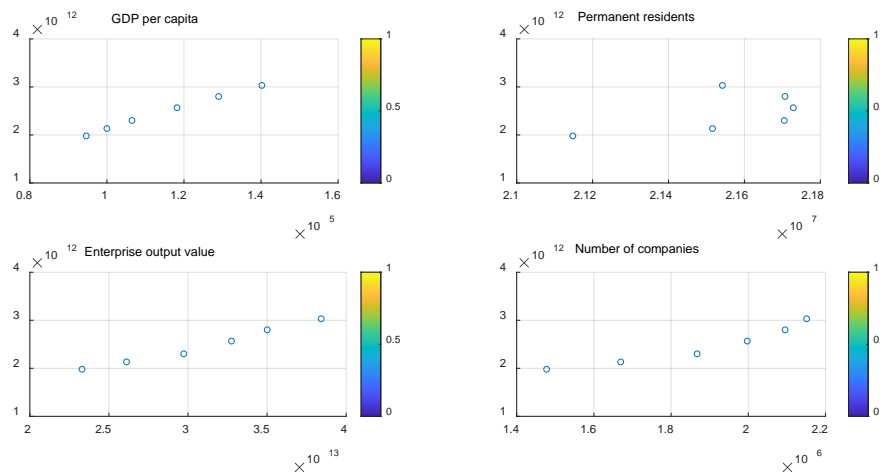


Fig.1 Four Indicators

As can be seen from Figure 1, all four indicators show a linear relationship, so a multiple linear regression model can be established to perform multiple linear regression analysis on each indicator variable.

3.1.2 Establishment of Multiple Linear Regression Model

$$\begin{cases} y = \beta_0 + \beta_1 x_{i1} + \dots + \beta_m x_{im} + \varepsilon_i \\ \varepsilon_i \sim N(0, \sigma^2), i = 1, \dots, n \end{cases} \quad (1)$$

(1) In the formula, $\beta_0, \beta_1, \dots, \beta_m, \sigma^2$ is an unknown parameter that has nothing to do with x_1, x_2, \dots, x_m , and $\beta_0, \beta_1, \dots, \beta_m, \sigma^2$ is called a regression coefficient.

Now we have n to an independent observation data $(y_i, x_{i1}, \dots, x_{im}), i = 1, \dots, n, n > m$,

Obtained from (1)

$$\begin{cases} y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_m x_{im} + \varepsilon_i \\ \varepsilon_i \sim N(0, \sigma^2), i = 1, \dots, n \end{cases} \quad (2)$$

Referred to as

$$X = \begin{bmatrix} 1 & x_{11} & \dots & x_{1m} \\ \vdots & \vdots & \dots & \vdots \\ 1 & x_{n1} & \dots & x_{nm} \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$\varepsilon = [\varepsilon_1 \ \dots \ \varepsilon_n]^T, \beta = [\beta_0 \ \beta_1 \ \dots \ \beta_m]^T$$

Take-in (2)

$$\begin{cases} Y = X\beta + \varepsilon \\ \varepsilon \sim N(0, \sigma^2 E_n) \end{cases} \quad (3)$$

Where E_n is the identity matrix of order n .

(2) Parameter estimation

The parameter $\beta_0, \beta_1, \dots, \beta_m$ in the model (2) is still estimated by the least square method, That is, the estimated value $\hat{\beta}_j$ should be selected, When $\beta_j = \hat{\beta}_j$, When $j = 0, 1, 2, \dots, m$, Sum of squared errors,

$$Q = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_{i1} - \dots - \beta_m x_{im})^2 \quad (4)$$

To a minimum. To this end, let

$$\frac{\partial Q}{\partial \beta_j} = 0, \quad j = 0, 1, 2, \dots, m \quad (5)$$

Get

$$\begin{cases} \frac{\partial Q}{\partial \beta_0} = -2 \sum_{i=1}^n (y_i - \beta_0 - \beta_m x_{im}) = 0 \\ \frac{\partial Q}{\partial \beta_j} = -2 \sum_{i=1}^n (y_i - \beta_0 - \beta_m x_{im}) x_{ij} = 0, \quad j = 1, 2, \dots, m \end{cases} \quad (6)$$

Organized into the following normal equations

$$\begin{cases} \beta_0 n + \beta_1 \sum_{i=1}^n x_{i1} + \beta_2 \sum_{i=1}^n x_{i2} + \dots + \beta_m \sum_{i=1}^n x_{im} = \sum_{i=1}^n y_i \\ \beta_0 \sum_{i=1}^n x_{i1} + \beta_1 \sum_{i=1}^n x_{i1}^2 + \beta_2 \sum_{i=1}^n x_{i1} x_{i2} + \dots + \beta_m \sum_{i=1}^n x_{i1} x_{im} = \sum_{i=1}^n x_{i1} y_i \\ \beta_0 \sum_{i=1}^n x_{im} + \beta_1 \sum_{i=1}^n x_{im} x_{i1} + \beta_2 \sum_{i=1}^n x_{im} x_{i2} + \dots + \beta_m \sum_{i=1}^n x_{im}^2 = \sum_{i=1}^n x_{im} y_i \end{cases} \quad (7)$$

The matrix form of the normal equations is

$$X^T X \beta = X^T Y \quad (8)$$

When matrix X has full ranks, $X^T X$ is an invertible matrix, The solution of the above formula is,

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad (9)$$

Substitute $\hat{\beta}$ back to the original model to get the estimated value of y

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_m x_m \tag{10}$$

The fitted value of this set of data is $\hat{Y} = X\hat{\beta}$, and the fitting error $e = Y - \hat{Y}$ is called the residual, can be used as an estimate of the random error a , and

$$Q = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \tag{11}$$

Is the sum of residual squares (or the sum of residual squares), which is $Q(\hat{\beta})$.

(3) Statistical analysis

The following results are given without proof:

(3.1) $\hat{\beta}$ is the linear unbiased minimum variance estimate of β . Means that $\hat{\beta}$ is a linear function of Y ; The expectation of $\hat{\beta}$ is equal to β ; In the linear unbiased estimation of β , the variance of $\hat{\beta}$ is the smallest.

(3.2) $\hat{\beta}$ follows the normal distribution

$$\hat{\beta} \sim N(\beta, \sigma^2(X^T X)^{-1})$$

Remember $(X^T X)^{-1} = (c_{ij})_{n \times n}$.

(3.3) for the sum of squared residuals Q , $EQ = (n - m - 1)\sigma^2$, and $\frac{Q}{\sigma^2} \sim \chi^2(n - m - 1)$

This results in an unbiased estimate of σ^2

$$s^2 = \frac{Q}{n - m - 1} = \hat{\sigma}^2 \tag{12}$$

s^2 is the residual variance (variance of the residuals), and s is called the residual standard deviation.

(3.4) Decomposing the total sum $SST = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$ of squares,

$$SST = Q + U, \quad U = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 \tag{13}$$

Where Q is the sum of squared residuals defined by (4), which reflects the effect of random error on y , and U is called the regression sum of squares, which reflects the effect of independent variables on y . The above decomposition makes use of the normal equations.

(4) Hypothesis Testing of Regression Models

It is necessary to test whether there is a linear relationship between the dependent variable y and the independent variable x_1, \dots, x_m as shown in model (1). Obviously, if all $|\hat{\beta}_j| (j = 1, \dots, m)$ is small, the linear relationship between y and x_1, \dots, x_m is not obvious, so the null hypothesis can be

$$H_0 : \beta_j = 0 (j = 1, \dots, m) \tag{14}$$

When H_0 holds, Q, U defined by decomposition (13) satisfies

$$F = \frac{U / m}{Q / (n - m - 1)} \sim F(m, n - m - 1) \tag{15}$$

There is an upper quantile $F_\alpha(m, n - m - 1)$ at the significance level α , if $F < F_\alpha(m, n - m - 1)$ accepts H_0 ; otherwise, it rejects.

(5) Hypothesis testing and interval estimation of regression coefficients

When H_0 is rejected above, β_j is not all zeros, but it does not exclude that several of them are equal to zero. Therefore, the following m tests should be further performed:

$$H_0^{(j)} : \beta_j = 0 \tag{16}$$

From $\hat{\beta}_j \sim N(\beta_j, \sigma^2 c_{jj})$, c_{jj} is the (j, j) -th element in $(X^T X)^{-1}$, and σ^2 is replaced by s^2 .

When $H_0^{(j)}$ holds.

$$t_j = \frac{\hat{\beta}_j / \sqrt{c_{jj}}}{\sqrt{Q / (n - m - 1)}} \sim t(n - m - 1) \tag{17}$$

For a given α , if $|t_j| < t_{\frac{\alpha}{2}}(n - m - 1)$, accept $H_0^{(j)}$; otherwise, reject.

can also be used to estimate the interval ($j = 0, 1, 2, \dots, m$) of β_j . At the confidence level $1 - \alpha$, the confidence interval of β_j is:

$$\left[\hat{\beta}_j - t_{\frac{\alpha}{2}}(n - m - 1) s \sqrt{c_{jj}}, \hat{\beta}_j + t_{\frac{\alpha}{2}}(n - m - 1) s \sqrt{c_{jj}} \right] \tag{18}$$

Among them $s = \sqrt{\frac{Q}{n - m - 1}}$.

Bring the data in Table 1 into (1)-(18), and use MATLAB software to get the multiple linear regression equation:

$$y = 2213.444x_1 - 1.425x_2 + 34.543x_3 - 0.010x_4 \tag{19}$$

Then test the regression model we established by SPSS software to get Tables 1 to 3

Table 1 Model Summary

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	1.000a	1.000	1.000	2686919607.0000000000
a. Predictors: (Constant), Number of companies, permanent residents, GDP per capita, Enterprise output value				

From the above table, r represents the goodness of fit. This value is used to measure the goodness of fit of the estimated model to the observed values. The closer it is to 1, the better the goodness of fit. Adjusted R Square is more accurate than R. Adjusted R Square = 1 in our model, indicating that the model has a good fit.

Table 2 Anovaa

ANOVAa					
Model	Sum of Squares	d	Mean Square	F	Sig.

			f			
1	Regression	8170105088000000000000.00	4	2042526272000000000000.00	28291.65	.004
	Residual	7219536976000000000.000	1	7219536976000000000.000		
	Total	8170177283000000000000.00	5			
a. Dependent Variable: GDP						
b. Predictors: (Constant), Number of companies, permanent residents, GDP per capita, Enterprise output value						

The table shows the results of analysis of variance. It is an overall test of the entire regression equation. It represents whether the entire regression equation has use value. The Sig value corresponding to F is less than 0.05. It proves that the regression equation is useful. From the above table, we It can be seen that Sig = 0.004, then this regression equation is suitable for measuring regional economic vitality.

Table 3 Coefficientsa

Coefficientsa						
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-1007562663000.000	883329060100.000		-1.141	.458
	GDP per capita	21023677.830	956508.413	.917	21.980	.029
	permanent residents	37560.146	44436.089	.020	.845	.553
	Enterprise output value	-.003	.004	-.041	-.722	.602
	Number of companies	183595.122	122892.174	.118	1.494	.376
a. Dependent Variable: GDP						

The table lists the significance test results of the independent variables. The Sig. Of the t-test in the last column is less than 1, indicating that the independent variables have a significant effect on the dependent variable. B represents the coefficient of each independent variable in the regression equation. The dimension and range of the independent variable are different. B cannot reflect the degree of influence of each independent variable on the dependent variable.

3.1.3 Proposed Solutions and Impact

From formula (19), we formulate the following scheme: first of all, through the spss test, we can easily know that per capita GDP is the most representative positive indicator that affects regional economic vitality. Therefore, we should increase per capita GDP. As the exemption and the progressive increase in tax rates will reduce the proportion of personal income tax to total tax revenue, The proportion of personal income tax in total tax revenue is closely related to regional economic growth. We can increase the average tax rate to increase the proportion of personal income tax in total tax revenue. We should adhere to the reform of the personal income tax system. At this stage we should gradually form a comprehensive Mixed income tax model with income tax as the main and classified income tax as the supplement; reduce tax rate levels, expand the tax base, increase the average tax rate, and gradually increase the proportion of individual taxes in total tax revenue; do not engage in a one-size-fits-all approach to expense deduction standards; vigorously develop And tertiary industry to promote technological innovation. The amount of foreign trade determines the degree of openness of a city. As a capital city, Beijing should vigorously develop trade, allow goods to be continuously exported, earn overseas funds, adapt to trade surpluses, and reduce trade deficits.

3.2 The Establishment and Solution of the Problem Two Model

In October 2003, China promulgated a fiscal policy to promote the adjustment of the industrial structure. Starting from the rationalization of the industry, with the intervention of the government, the industrial structure was adjusted to increase the overall economic intensification. In order to better understand the impact of government economic policy changes on regional economic vitality, we selected the GDP and total GDP values of the three major industries created by Anshan City in 1949-2000 before the 2003 policy adjustment (see Table

4).And GDP and total GDP created by the three major industries from 2005 to 2015 (see Table 5)

Table 4 GDP and Total GDP of the Three Major Industries from 1949 to 2000

years	Regional GDP (100 million yuan)				Proportion (%)		
	Total	primary industry	Secondary industry	Tertiary Industry	primary industry	Secondary industry	Tertiary Industry
1949	0.95	0.37	0.38	0.2	38.50%	40.10%	21.40%
1952	6.83	0.82	5.28	0.73	12.10%	77.20%	10.70%
1957	14.64	0.88	12.36	1.4	6.10%	84.40%	9.50%
1962	13.18	1.02	10.57	1.59	7.80%	80.20%	12.00%
1965	20.65	1.28	17.58	1.79	6.20%	85.10%	8.70%
1970	23.03	1.46	19.65	1.93	6.30%	85.30%	8.40%
1975	25.05	2.49	19.92	2.64	10.00%	79.50%	10.50%
1978	32.79	3.50	25.97	33.23	10.70%	79.20%	10.10%
1980	38.42	3.44	30.96	4.02	8.90%	80.60%	10.50%
1985	60.74	4.75	45.99	10.00	7.80%	75.70%	16.50%
1990	106.83	9.49	60.86	36.48	8.80%	57.00%	34.20%
1995	278.94	23.44	147.30	108.20	8.40%	52.80%	38.80%
2000	461.36	38.72	223.24	199.40	8.40%	48.40%	43.20%

From the analysis in Table 4, we can find that: after the founding of the Republic (1949 ~ 1957), after the founding of the Republic, Anshan established the “steel capital” of New China. In 1952, Anshan Iron and Steel produced 790,000 tons of steel. In 1957, the output of iron, steel and steel was 3 times, 2.7 times and 3.3 times that of 1952, respectively. In 1957, the GDP of Anshan reached 1.46 billion yuan, 15.4 times that of 1949, with an average annual growth rate of 35.7%. The three major industries have achieved great development, of which the secondary industry has an average annual growth of 46%. From 38.5: 40.1: 21.4 in 1949 to 6.1: 84.4: 9.5, the secondary industry has become an absolute pillar industry.

During the Great Leap Forward and the Cultural Revolution (1958-1977), the “Great Leap Forward” in 1958, and the subsequent three years of natural disasters, Anshan's economy declined. From 1961, Anshan City adjusted its national economy. In 1965, the GDP of Anshan reached 5.86 billion yuan, and the composition of the three industries became 6.2: 85.1: 8.7. Compared with 1952, the secondary industry increased by 7.9%. The “cultural revolution” slowed down the pace of economic development. In 1978, the GDP of Anshan area was 3.28 billion yuan, an annual increase of only 1.1% compared with 1965.

Table 5 GDP and Total GDP of the Three Major Industries from 2005 to 2015

years	Regional GDP (100 million yuan)				Proportion (%)		
	Total	primary industry	Secondary industry	Tertiary Industry	primary industry	Secondary industry	Tertiary Industry
2009	1018.01	56	560.08	401.93	5.50%	55.00%	39.50%
2010	1730.5	82.6	913.5	734.4	4.40%	54.30%	41.30%
2011	2420.1	110.5	1302.4	1007.2	4.60%	53.80%	41.60%
2012	2429.3	124.4	1293.2	1011.7	5.10%	53.20%	41.70%
2013	2296.5	130.3	1189.9	976.3	5.70%	51.80%	42.50%
2014	2385.9	131.1	1206.4	1048.4	5.50%	50.60%	43.90%
2015	2349.0	136.5	1116.9	1095.6	5.80%	47.50%	46.70%

Table 5 shows that after the reform and opening up (1978-2010), after the Third Plenary Session of the Eleventh Central Committee of the Party, the Party and the state transferred their work centers to economic construction. The reform and opening up led to great economic development in Anshan. In 2010, Anshan's GDP reached 173.05 billion yuan, which was 51.8 times that of 1978. The development of productive forces and various national support policies have expanded the space for the development of the tertiary industry in Anshan. The structure of the three industries changed from 10.7: 79.2: 10.1 in 1978 to 4.4: 54.3: 41.3 in 2010, and the proportion of the tertiary industry increased significantly. During this period, although the proportion of the tertiary industry has risen, the proportion has been much lower than that of the secondary industry. The secondary industry, especially the steel industry, has been the main source of economic growth in Anshan. Fit the above data (Figure 2, Figure 3)

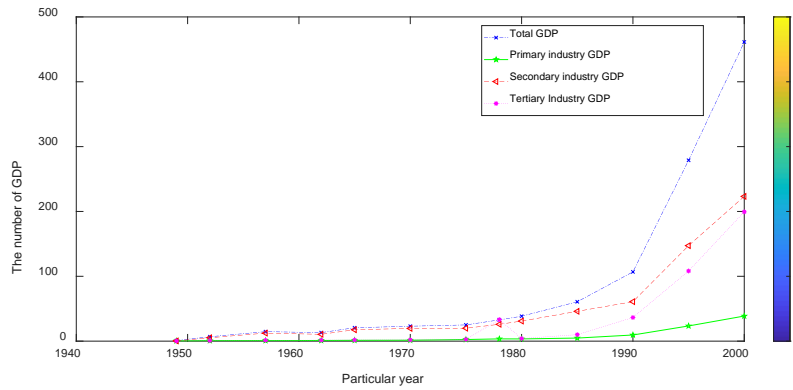


Fig.2 1949-2000 Gdp and Total Gdp Created by the Three Major Industries from 1949-2000

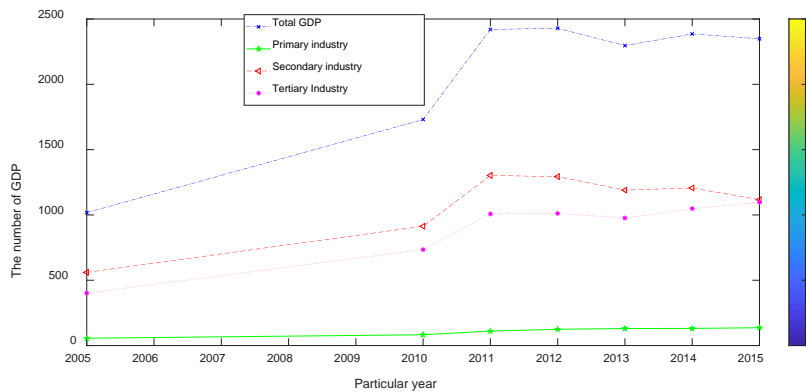


Fig.3 Gdp and Total Gdp Created by the Three Major Industries from 2005 to 2015

It can be seen from Figures 3 and 4 that each GDP value in Figure 4 is higher than that in Figure 3 each year. It can be seen that due to changes in government economic policies, GDP has grown rapidly, thereby increasing the economic vitality of the city of Anshan; Selecting the fastest growth rate from 1990 to 2000 in Figure 3, the slope of the total GDP is about 35, and the slope of the relatively slow growth rate in the beginning of Figure 4 is 150. For the first industry, the change is relatively gentle. However, the overall trend is increasing; for the secondary industry, the slope is 15 before the reform and the slope is 60 after the reform; for the tertiary industry, the slope is 15 before the reform and after the reform, the slope is 60, so It can be seen that changes in government economic policies will not only significantly increase the total GDP, but also increase its growth rate significantly within a certain interval. Therefore, government economic policies are conducive to enhancing regional economic vitality.

4. Suggestions

The first is to ask for impetus for reform and increase the vitality of development. We must continue to promote reform and opening up, adhere to the transformation and upgrading of scientific and technological innovation, and adjust the industrial structure. Fully implement the “dual-core drive” development strategy. Guided by cracking development problems, taking reforms in key areas and key links as breakthrough points, efforts are made to rationalize institutional mechanisms, stimulate market vitality, expand development cooperation, enhance radiation capabilities, optimize business environment, promote regional integrated development, and accelerate Anshan's accelerated upgrade development of.

The second is to take infrastructure construction as a guide and build a comprehensive transportation channel. At present, backward transportation facilities, inadequate collection and distribution systems, and low levels of interoperability within and outside the city, coupled with operational management and service efficiency and

quality issues, have become major bottlenecks that restrict the “dual-core” driving capability. In accordance with the requirements of modern ports, we must accelerate the construction of hub ports, improve the collection and distribution system, and realize the connection of port operation areas with major industrial parks, logistics parks, railways, and highways.

To enhance the economic vitality of Anshan City, it is necessary to continuously adjust and optimize the industrial structure. While vigorously developing the tertiary industry, it is also necessary to strengthen the primary and secondary industries; improve the level of social security, and actively encourage insurance companies to launch preferential and practical products to meet more The needs of residents; increase investment in public infrastructure, and rationally allocate government investment while encouraging the development of private investment and foreign investment; improve the economic efficiency of industrial enterprises; strengthen the opening of cities to the outside world; expand employment, increase employment Income, enhancing residents' well-being, etc. So that the city is full of vitality, vitality and sustainable economic development.

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