ISSN 2616-5775 Vol. 7, Issue 5: 136-142, DOI: 10.25236/AJCIS.2024.070518

A Quantitative Assessment Model for Momentum in Tennis Based on Exponential Decay and EWM-TOPSIS Method

Zhuoyuan Li[1,*], Yitian Yin[2]

[1]School of Computer Science and Engineering, Beihang University, Beijing, 100083, China
[2]School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an, 710072, China
[*]Corresponding author

Abstract: *In sports, the concept of "momentum" describes how athletes or teams, inspired by positive factors during a match, perform better, leading to a "success breeds success" phenomenon. This study analyzes players' momentum in tennis, specifically using data from the 2023 Wimbledon Men's Singles Final. It focuses on quantifying momentum and determining its influence on player performance. The study developed the Player Performance Evaluation Model based on EWM-TOPSIS method, incorporating factors like winning status, movement distance, winning shots, and double faults. The model differentially weighs the winning incentives for servers and receivers and uses an exponential decay accumulation of evaluation indicators, akin to the Momentum algorithm in deep learning. The players' performance score at a certain score point, denoted as momentum score, were then derived using the EWM-TOPSIS evaluation algorithm. For the momentum score calculated by our model, we randomly selected a match to visualize the momentum score of both sides to show the consistency of momentum score and match conditions. Finally, this study examines the relationship between momentum score and the increase or decrease of winning probability to prove that momentum score has a significant correlation with the fluctuation of winning probability.*

Keywords: *Momentum in Tennis, Evaluation Model, Exponential Decay, EWM-TOPSIS Method*

1. Introduction

In physics terms, momentum is defined as the product of mass and velocity. Markman and Guenther (2007) introduced a theoretical framework to elucidate laypeople's perception of psychological momentum. They characterize velocity as either positive (advancing towards a goal) or negative (retreating from a goal), while mass is influenced by contextual factors that convey significance, immediacy, and importance. Recent studies further elucidate how individuals perceive and experience psychological momentum. These investigations highlight the dual role of perceptual velocity and mass in shaping the intensity and persistence of goal pursuit, suggesting that the trajectory towards achieving goals is not only propelled by successes and setbacks but also by the psychological weight of the challenges overcome[1].

Depken Craig A.'s research indicated that the influence of momentum in sports extends significantly to both strategic and psychological aspects[2]. It enhances team or athlete confidence, motivation, and concentration, thereby exerting pressure on opponents. Jordan Truman Paul Noel's research indicates that while momentum's predictive power may not be universally superior across all sports, instances in the National Hockey League (NHL) and European football leagues suggest its potential utility in enhancing pre-game prediction models[3]. This research also suggests a shift towards integrating momentum-based features with traditional statistical approaches to refine predictive analytics in sports. The dynamic nature of momentum, affecting both psychological and performance aspects of teams and players, underscores its significance as a multifaceted phenomenon within sports analytics.

Philippe Meier's research found that, in men's tennis, momentum exerts a substantial impact on matches, particularly due to its influence on individual performance[4]. In the 2023 Wimbledon Men's Singles Final, an intriguing encounter unfolded between the rising star Carlos Alcaraz and the veteran Novak Djokovic, characterized by its dramatic fluctuations and captivating moments. Based on the data of 2023 Wimbledon men's singles final 32 (from https://www.comapmath.com/MCMICM/), this study

ISSN 2616-5775 Vol. 7, Issue 5: 136-142, DOI: 10.25236/AJCIS.2024.070518

establishes a quantitative model of players' momentum in competition based on the entropy weight method (EWM) with TOPSIS (technique for order preference by similarity to an ideal solution), so as to effectively evaluate the players' performance status and comparative advantage.

2. Dataset preparation and preprocessing

2.1 Dataset preparation

The data set contains information about the settlement of every point in the game. For example, p1_score (player 1's score within current game), server (server of the point), and point_victor (winner of the point). In addressing the issue of missing or incomplete data within the match dataset, imputation techniques are employed to handle the missing data. Specifically, we impute missing values in 'speed_mph' (speed of serve, miles per hour) with the mean and fill missing values in 'serve_width' (direction of serve), 'serve_depth' (depth of serve), and 'return_depth' (depth of return) with the mode.

2.2 Feature extraction

For each score point in the dataset, the following features are extracted. It is postulated that these features contribute significantly to momentum incentives (two sets of feature data can be obtained for athletes on both sides of the competition for each score point):

(1) The number of sets or games or points won ahead. Leading in sets, games, and points impacts a player's momentum significantly due to the psychological boost of confidence and control, as well as the strategic advantage in dictating the pace and applying pressure on the opponent.

(2) Running distance. Running distance significantly impacts a player's momentum due to its strong association with physical fatigue. More running depletes energy, affecting shot precision, reaction time, and mental concentration. Distance run is a key indicator of both physical and mental exertion, directly influencing a player's momentum.

(3) The winner of a deciding point. The deciding point holds considerable sway over a player's momentum due to its heightened psychological importance. Success in this critical moment not only boosts immediate confidence but also carries the weight of potential victory or defeat, influencing the player's mindset and overall performance. The outcome of a deciding point serves as a potent catalyst for momentum shifts in a tennis match.

(4) Ace balls. Ace serves significantly impact a player's momentum by disrupting the opponent's rhythm and creating a strategic advantage. The unreturnable nature of aces leads to quick points, influencing both the server's morale and the opponent's psychological state, contributing to a shift in the match's momentum.

(5) Double fault. A double fault significantly impacts a player's momentum because it not only results in losing a point but also forfeits the server's advantage, disrupting the flow of the game and causing a notable psychological setback.

(6) Unforced errors. Unforced errors have a pronounced impact on a player's momentum due to their ability to disrupt the player's mental state, leading to frustration, loss of confidence, and a negative shift in psychological momentum during a match.

(7) Player 1 won the game player 2 is serving. When Player 1 wins the game while Player 2 is serving, it can significantly impact the player's momentum due to the psychological and strategic implications associated with breaking the opponent's serve. Psychologically, winning a game on the opponent's serve may boost Player 1's confidence, leading to increased focus and motivation. From a strategic standpoint, breaking the opponent's serve can alter the relative position in the match, creating a potential shift in momentum. This change in dynamics, both psychologically and strategically, contributes to the substantial impact on the player's momentum.

(8) Victory Incentive: For the serving player winning a match, this value is set to $\frac{1}{p}$; for the receiving player winning a match, it is set to $\frac{1}{1-p}$; and for non-winning scenarios, it is set to 0 (where p represents the prior probability of the serving player winning, empirically obtained from each data point, with a value of 0.6731).

The above calculated features are then labeled a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 in turn. The value

ISSN 2616-5775 Vol. 7, Issue 5: 136-142, DOI: 10.25236/AJCIS.2024.070518

of the athlete's feature i at point number t is denoted as $a_i(t)$.

3. Exponential decay model

Consider that an athlete's athletic status and scoring in the past can affect the present moment to varying degrees: individuals are more influenced by recent events and less motivated by past events. To quantify the diminishing influence of past events over time, we use an exponential decay model to describe how the impact of an event, or a factor decreases exponentially over time. The formula is as follows:

$$S_i(\mathrm{t}) = \lambda \cdot a_i(\mathrm{t}) + (1 - \lambda) \cdot S_i(\mathrm{t} - 1) \tag{1}$$

Where, $0 < \lambda < 1$, S_i is the result of the accumulation of a_i after exponential decay, and we use S_i instead of a_i as the eigenvalue used in the evaluation model. The decay constant λ determines accurately reflects how quickly the influence of past events diminishes in the specific setting of a tennis match. A higher value of λ indicates a faster decay of influence, meaning that the event's impact diminishes more quickly. We derived the value λ = 0.3 by analyzing historical match data, collecting information, and empirically deriving the value. This value is most consistent with the pattern of momentum fluctuations and successes we have observed in tennis.

Our approach, inspired by the Momentum-based Stochastic Gradient Descent (SGD) algorithm in deep learning[5], shares a conceptual parallelism with the dynamics of tennis matches. In deep learning, momentum SGD leverages the idea of building upon the direction of previous gradients to optimize learning. This analogy is akin to the concept of momentum in tennis, where a player's current performance is influenced by preceding events. Just as momentum in SGD smooths the learning path and accelerates convergence, in tennis, players build upon their recent successes or failures, influencing their subsequent performance. This correlation highlights a fascinating intersection between computational optimization strategies and sports psychology, demonstrating how principles from one field can metaphorically mirror phenomena in another.

4. EWM-TOPSIS-based Athlete Performance Score Evaluation Model

After constructing the evaluation indicators, the EWM-TOPSIS model[6] can be utilized to calculate the scores of each athlete at each moment. While TOPSIS method is proficient in ranking based on the proximity of a limited set of evaluation objects to idealized targets, it faces challenges in determining indicator weights when analyzing an athlete's "momentum". In this regard, the EWM method proves effective in mitigating this difficulty to a certain extent. TOPSIS excels in considering various factors occurring simultaneously, and EWM method provides a more comprehensive determination of the weights for each influencing factor[7].

4.1 Entropy weight method in tennis analytics

Information entropy serves as a metric to quantify the volume of information, offering an objective mirror to the inherent data. This metric shares an inverse relationship with the quantity of information, such that a rise in information entropy denotes a reduction in the significance of index data and, correspondingly, diminishes the effectiveness of a comprehensive assessment, and the converse is true as well. The pivotal role of information entropy in evaluations cannot be understated[8]. The Entropy Weight Method (EWM) capitalizes on this concept to gauge the discriminative power of various indices, thus establishing their respective weights. Unlike subjective weighting approaches like the Analytical Hierarchy Process (AHP) and the Decision-Making Trial and Evaluation Laboratory (DEMATEL), the EWM stands out for its objectivity, thereby yielding results with enhanced reliability and precision.

Employing the EWM, the weightage ties among various decision-making indices were quantified, laying a solid foundation for the nuanced comprehensive evaluation of a multitude of indices.

The process for establishing index weights through the Entropy Weight Method (EWM) can be delineated as follows:

(1) Assume that there are totally n sets of feature data and m distinct features (each feature data refers to the exponential decay features of one player at each score point, in our case, n=14568, m=8). Construct a decision-making index evaluation matrix X_{nm} comprising i rows and j columns. Here, the

ISSN 2616-5775 Vol. 7, Issue 5: 136-142, DOI: 10.25236/AJCIS.2024.070518

column j in X_{nm} represents the exponential decay feature S_j.

(2) Proceed to standardize every component within the evaluation matrix. Indexes that possess higher values, which favorably influence the decision-making objective, are recognized as positive indexes; conversely, those with lesser values are identified as negative indexes. In this example, we consider the features S_5 and S_6 as negative indexes, and the remaining features as positive indexes. Derive the normalized matrix x'_{nm} as follows: For positive indexes:

$$x'_{ij} = \frac{X_{ij} - min(X_{1j}, X_{2j}, \ldots, X_{nj})}{max(X_{1j}, X_{2j}, \ldots, X_{nj}) - min(X_{1j}, X_{2j}, \ldots, X_{nj})} \tag{2}$$

For negative index:

$$x'_{ij} = \frac{max(X_{1j}, X_{2j}, \ldots, X_{nj}) - X_{ij}}{max(X_{1j}, X_{2j}, \ldots, X_{nj}) - min(X_{1j}, X_{2j}, \ldots, X_{nj})} \tag{3}$$

(3) Determine the entropy weight e_j for the j^{th} index as:

$$e_j = -k \sum_{i=1}^{n} x'_{ij} ln(x'_{ij}) \ , j = 1,2,\ldots,n \tag{4}$$

where $k = -\frac{1}{ln(n)} > 0$. Ensure that $e_j \geq 0$.

(4) Ascertain the entropy redundancy (coefficient of variation) d_j:

$$d_j = 1 - e_j, j = 1,2,\ldots,n \tag{5}$$

(5) Ascertain the weights ω_j by Ascertain utilizing the entropy redundancy d_j:

$$\omega_j = \frac{d_j}{\sum_{j=1}^{m} d_j}, j = 1,2,\ldots,n \tag{6}$$

4.2 Integrating Entropy Weights with TOPSIS

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [9] is widely utilized in the realm of multi-criteria decision-making. It operates on the principle of identifying rankings through proximity to an ideal solution, also known as the method of distance from both positive and negative ideal solutions. This technique adeptly leverages the entirety of data from the original dataset, ensuring that the distinctions among various assessment plans are precisely delineated. It is especially effective in the evaluation of intricate systems characterized by numerous indexes, offering a straightforward and manageable computation process, with well-defined concepts and high applicability[10].

In its application, TOPSIS establishes a base matrix, neutralizes dimensionality discrepancies through standardization, allocates weights to indices, and performs a holistic evaluation of green building design proposals, culminating in the identification of both ideal and anti-ideal solutions. The method then assesses the proximity to the ideal solution to ascertain the most favorable design approach. The methodology unfolds through the following steps:

Integrating TOPSIS with the entropy weight method for decision-making involves a seamless process where an initial judgment matrix x'_{nm} is first established, representing different alternatives and criteria. The initial judgment matrix is constructed the same as in step (1)(2) of the EWM. This matrix is normalized to:

$$r_{ij} = \frac{x'_{ij}}{\sqrt{\sum_{i=1}^{n} x'^2_{ij}}} \tag{7}$$

for comparability. Weights w_j, derived from the entropy method, are then applied, yielding $v_{ij} = r_{ij} \times w_j$. The method proceeds by identifying ideal (A^+) and negative-ideal (A^-) solutions, where $A_j^+ = \max(v_{ij})$ and $A_j^- = \min(v_{ij})$. The distances. Then the distances between each index value and the positive and negative ideal scheme values are calculated:

$$S_i^+ = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - A_j^+\right)^2}, S_i^- = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - A_j^-\right)^2} \tag{8}$$

Finally calculate the relative closeness C_i of each index value:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-} \tag{9}$$

ISSN 2616-5775 Vol. 7, Issue 5: 136-142, DOI: 10.25236/AJCIS.2024.070518

The final ranking is based on C_i, effectively combining the strengths of both methods. The momentum score of the subject player at the i^{th} score point corresponding to this set of data is defined as $M_i = C_i$.

5. Visualization of momentum score in match

Figure 1 shows the momentum score of the two players, Carlos Alcaraz and Nicolas Jarry, at each point during the first match. The dashed lines in the graph represent the boundaries of each set. From the graph, it can be observed that Carlos Alcaraz had higher momentum in sets 1, 3, and 4, while Nicolas Jarry had higher momentum in set 2. During the first set, Carlos Alcaraz had the upper hand, with his momentum score consistently higher than those of Nicolas Jarry. The outcome of the first set in this match also resulted in a victory for Carlos Alcaraz, and sets 2, 3, and 4 followed the same pattern. It can be seen that the trend of momentum score shown in the picture is consistent with the winning results of each game.

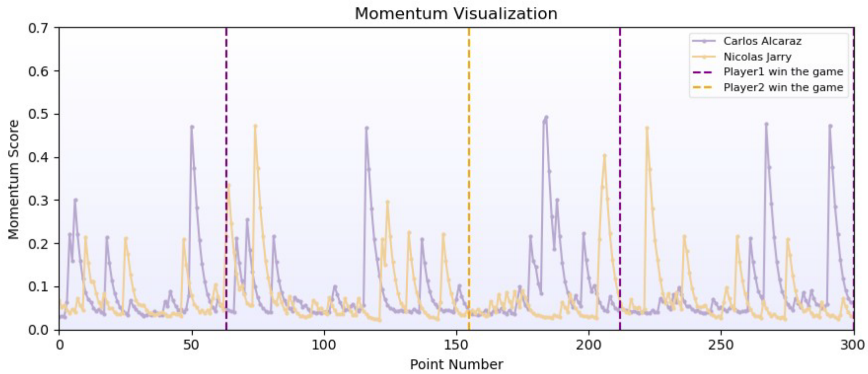

Figure 1: Line Graph of 2023-wimbledon-1301

Figure 2 displays the difference in momentum score between Player 1 and Player 2 in the first match. From the graph, it can be seen that, in general, they are closely matched. However, in specific sets, some differences are shown, which align with our hypothesis.

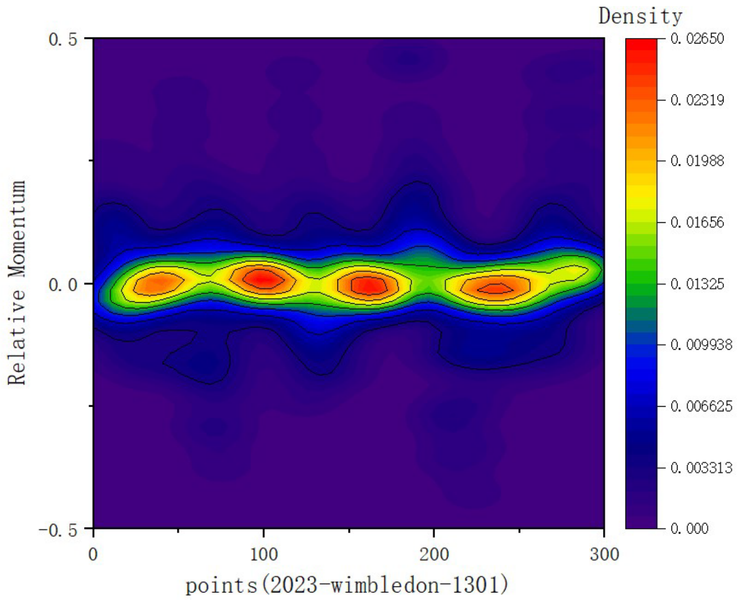

Figure 2: Heatmap of 2023-wimbledon-1301

6. Results and analysis

To examine the efficacy of the momentum score derived from our model, it is essential to validate whether the side with the advantage at a certain moment in a match is more likely to achieve a higher win probability in subsequent plays than before. This validation involves a nuanced analysis to mitigate the impact of discrepancies in player abilities on the outcomes.

In an effort to control for potential biases due to differences in player strengths, we selected data from matches involving pairs of players with similar pre-match odds and anticipated win probabilities. These pairs included: (Alexander Zverev, Matteo Berrettini), (Frances Tiafoe, Grigor Dimitrov), (Christopher Eubanks, Christopher O'Connell), (Alexander Bublik, Maximilian Marterer), (Andrey Rublev, Alexander Bublik), and (Roman Safiullin, Denis Shapovalov), corresponding to matches 2023-wimbledon-1302, 2023-wimbledon-1303, 2023-wimbledon-1307, 2023-wimbledon-1314, 2023-wimbledon-1407, 2023-wimbledon-1406, respectively.

For each match, we designated one player as the subject P and their opponent as Q. At every score point t in the match, we computed the momentum score for both players. We defined the relative momentum score for player P at score point t as:

$$M(t) = M_P(t) - M_Q(t) \tag{10}$$

where $M_P(t)$ is the momentum score of player P at score point t, and $M_Q(t)$ is the momentum score of player Q at score point t. Subsequently, we calculated the win probability for player P in the next n plays, denoted as $P_{next}(t)$, to measure the performance of the player under the influence of the current momentum score. The value of n was set to 12 to balance the influence of randomness (which is more pronounced for smaller n) and the immediacy of momentum impact (which diminishes for larger n). Furthermore, we computed the win probability for player P up to the current score point t, denoted as $P_{prev}(t)$, considering only the moments after the first 30 points of the match to reduce the influence of randomness. Both win probabilities were calculated as the average of the win probabilities as the server and the receiver.

A score point t was considered to demonstrate the effectiveness of momentum if $M_R(t) > 0$ and $P_{next}(t) > P_{prev}(t)$ or if $M_R(t) < 0$ and $P_{next}(t) < P_{prev}(t)$. These points were termed as effective momentum points. The proportion of effective momentum points, denoted by η, was then calculated by dividing the number of effective momentum points by the total number of score points in the dataset.

Out of 1026 points analyzed, 592 were identified as effective momentum points, yielding an η of 57.70%. This indicates that the momentum score at a given moment indeed influences the subsequent performance of the player. To further substantiate this finding, we hypothesized that the momentum score at score point t has no impact on the subsequent win probability, positing an expected effective momentum proportion η_0 of 0.5. A binomial test against this null hypothesis revealed that the actual proportion of $\eta = 592/1026$ significantly deviates from 0.5, with a p-value of 9.07×10^{-7}, well below the threshold of 0.05, leading to the rejection of the null hypothesis.

These results affirm the validity of the defined momentum score, suggesting that a superior momentum at a given moment increases the likelihood of achieving a higher win rate in subsequent plays, and conversely, an inferior momentum decreases this likelihood.

7. Conclusions

This paper conducts an analytical study on the momentum of tennis players based on the match data from the 2023 Wimbledon Men's Singles top 32 final. It quantifies the momentum score of a player at a certain scoring point. A Player Performance Evaluation Model was developed, taking into account various factors such as the player's win conditions, degree of lead in the match, distance covered, winning shots made, and double faults, as evaluative indicators. Notably, considering the significant difference in prior probabilities of winning between the server and the receiver, different weights were assigned to the winning incentives of serving and receiving. Additionally, acknowledging that humans are more influenced by recent events and the impact of more distant factors gradually diminishes, we applied an exponential decay accumulation to the evaluative indicators, inspired by the Momentum optimization algorithm used in deep learning. Subsequently, using the entropy weight method and the TOPSIS evaluation algorithm, we calculated the performance scores of the players. These scores were defined as the players' momentum scores, which were then utilized as a standard for visualizing the match conditions. The paper concludes by examining whether the advantage of momentum scores is linked to an increase in players' win rates. If a player's momentum score exceeds that of the opponent and is followed by an increased win rate in subsequent points, momentum is considered to be effective. Multiple

ISSN 2616-5775 Vol. 7, Issue 5: 136-142, DOI: 10.25236/AJCIS.2024.070518

samplings were conducted to count the occurrences of momentum effectiveness, and through a binomial test, it was established that there is a relationship between the momentum score and the fluctuation in players' win rates.

References

[1] Briki W, Markman D K .Psychological momentum: The phenomenology of goal pursuit [J].Social and Personality Psychology Compass, 2018, 12(9):n/a-n/a.
[2] A. C D, M. J G, A. D S .Set-level Strategic and Psychological Momentum in Best-of-three-set Professional Tennis Matches [J].Journal of Sports Economics, 2022, 23(5):598-623.
[3] Zhang, Yun B .A Quantitative Network Situation Assessment Method Based on Stochastic Model[J]. Applied Mechanics and Materials, 2014, 513-517:768-771. DOI:10.4028/www.scientific.net/AMM.513-517.768.
[4] Meier P, Flepp R, Ruedisser M, et al. Separating psychological momentum from strategic momentum: Evidence from men's professional tennis [J].Journal of Economic Psychology, 2020, 78 (prepublish).
[5] Xie, Xingyu, et al. "Adan: Adaptive Nesterov Momentum Algorithm for Faster Optimizing Deep Models." arXiv preprint arXiv: 2208.06677 (2022).
[6] Liu W, Fu S, Yu L .Evaluation of light pollution risk level based on EWM-TOPSIS method[J]. Population, Resources Environmental Economics, 2023, 4(1).
[7] Ting H X, Min H L .Assessment of vulnerability to waterlogging in subway stations using integrated EWM-TOPSIS [J].Smart Construction and Sustainable Cities, 2023, 1(1).
[8] Xu X , Xie J , Wang H ,et al.Online education satisfaction assessment based on cloud model and fuzzy TOPSIS[J].Applied Intelligence, 2022, 52(12):13659-13674. DOI:10.1007/s10489-022-03289-7.
[9] Lin L, Wei S, Shuyu C, et al.A Dynamic Adaptive and Resource-Allocated Selection Method Based on TOPSIS and VIKOR in Federated Learning[J].Neural Processing Letters, 2024, 56(2)
[10] Muhammet Gül.A Quantitative Occupational Risk Assessment Methodology Based on TOPSIS-Sort with its Application in Aluminum Extrusion Industry[J].International Journal of Pure and Applied Sciences, 2021(1).DOI:10.29132/IJPAS.943612.