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Research on the Application of Azimuth-only Passive Positioning Method in UAV Location

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Abstract: *This article uses the azimuth-only passive positioning method to locate the circular drone formation at the same height and the conical drone formation in space, to maintain the formation of the drone swarm during formation flight. After studying the direction information of the drone receiving the transmitted signal from the biased drone, the adjustment model of position deviation drone is established by using polar coordinate system and sine theorem, which we apply to the specific situation, and finally get the direction information adjustment scheme of the position deviation drone under the specific situation.*

Keywords: *Passive Positioning Method, Drone Formation, Direction Information, Adjustment Scheme*

1. Introduction

When the drone cluster flies in formation [1], it will emit electromagnetic wave signals as little as possible to avoid external interference. Therefore, azimuth-only passive positioning method [2] can be used to adjust the deviation of the drone position, so as to maintain the formation of the drone formation.

We study the position adjustment of circular formation drones at the same height. When the drone positions numbered FY00 and FY01 have no deviation and the number is known, it is found that only one drone that is evenly distributed on the circumference needs to transmit signals, and an effective positioning model can be established for the drone with a slight deviation in position. Based on this, the positioning model of the remaining drones with slight deviation can be established. Finally, by determining the combination of α_1 and α_2 of FY02 in the biased state, the positioning model is used to adjust it to the unbiased state, and the position adjustment schemes of the drone numbered FY03-FY09 are given respectively.

However, in actual flight situations, drone clusters have other formations, such as conical formations, and deviations may occur in space. At this time, the azimuth-only passive positioning method can still be used to design a plan for adjusting the drone position in the conical formation.

2. A drone localization model based on azimuth-only passive localization method

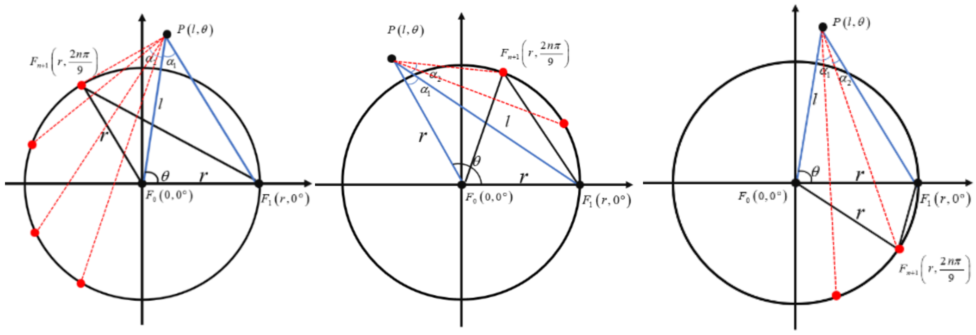
2.1 Passive localization in a circular formation of the same height

In order to clarify the information received by the drone, it is assumed that each drone in the formation has a fixed number FY01, FY02, FY03..., and the relative position between the drone remains unchanged. The direction information received by the drone is agreed: the angle $\alpha_m\left(m=1,2,\cdots\right)$ between the drone and any two transmission signal drone connections. For ease of understanding, we describe FY00, FY01 and FY02 drone points as $F_0\left(0,0°\right)$, $F_1\left(r,0°\right)$ and $F_2\left(r,40°\right)$.

2.1.1 Determine the number of drones that transmit signals without deviation

Because drone clusters should keep their secrecy when flying in formation, so they should choose a minimum number of drones to emit electromagnetic wave signals outward. Assuming that the current drones numbered FY00 and FY01 transmit signals and their positions are not deviated. Set the location of a drone with a slight deviation to be $P\left(l,\theta\right)$, the transmitting drone with unknown number to be

F_{n+1}, and the angle formed by F_0, F_1 and F_{n+1} emitted is α_1 and α_2. Through the analysis of various combinations of P and F_{n+1}, it can be found that when P and F_{n+1} move freely to different positions, the angle formed between P, F_1, F_0 and the angle formed by P, F_{n+1}, F_0 are not fixed. Therefore, it is necessary to establish a polar coordinate system to classify and discuss the different three situations, as shown in Figure 1.



(a) (b) (c)

Figure 1: Schematic diagram of three situations when P and F move freely to different positions

Scenario one: As shown in Figure 1(a), When F_{n+1} is the four unbiased points of the transmitted signal clockwise in the counterclockwise direction of P (as shown in the red point).

In $\Delta F_{n+1}F_1P$, ΔPF_0F_1 and ΔPF_0F_{n+1}, according to the sine theorem [3], and simplify the formula in the model, we can get the drone with a slight deviation in position to establish an effective positioning model $P(l,\theta)$, which are:

$$\begin{cases} l\sin\alpha_1 - r\sin(\alpha_1+\theta) = 0 \\ \sin\alpha_2\cdot\sin(\alpha_1+\theta) - \sin\alpha_1\cdot\sin\left(\alpha_2+\frac{2n\pi}{9}-\theta\right) = 0 \\ 2\sin\frac{n\pi}{9}\cdot\sin\alpha_1\cdot\cos\left(\alpha_2-\theta+\frac{n\pi}{9}\right) - \sin\theta\cdot\sin(\alpha_1+\alpha_2) = 0 \end{cases} \quad (1)$$

Respectively, scenario two 2 and scenario 3 are shown in (b) and (c) in Figure 1. Similarly, the effective positioning models are obtained.

To sum up, it can be seen that all three positioning models contain 3 equations and 3 unknowns. According to the substitution elimination method, this equation must have a solution. So it is only necessary to add another drone with a known number and no deviation to transmit signals, so as to effectively locate other drones with slight deviation.

2.1.2 Establishment of deviation point drone positioning model with known transmitting signal points

Based on the above, it can be seen that by selecting drones numbered FY00, FY01 and transmitting signals from up to one drone on the circumference, other slightly deviated drones can be effectively located. Therefore, we assume that FY00, FY01 and FY02 are the drones that transmit signals and their positions without deviation. When $\angle F_0PF_1=\alpha_1$, $\angle F_1PF_2=\alpha_2$ and $\alpha_3=\alpha_1+\alpha_2$. At this time, the direction information formed by α_1 and α_2 can determine the position of the deviation point P. However, when the point P is moved to a special position, and will be included with each other, this time only one angle direction information, the position of the deviation point P can not be determined, as shown in Figure 2.

ISSN 2616-5767 Vol.6, Issue 11: 82-89, DOI: 10.25236/AJETS.2023.061112

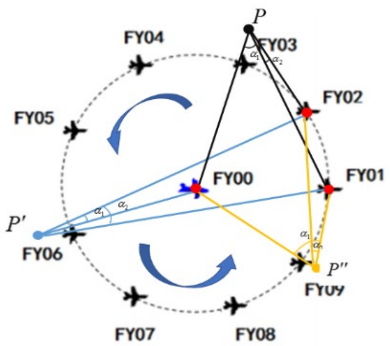


Figure 2: Motion diagram of deviation point P

As can be seen from the above figure, when point P moves to point P', there will be $\alpha_1 \subset \alpha_2 = \alpha_3$ situation and when point P moves to point P'', there will be $\alpha_2 \subset \alpha_1 = \alpha_3$ situation. For these two special cases, we should re-assume the α_1 and α_2 to carry out classification discussion.

Scenario one: When the deviation point of the drone is in the range of $(40°,180°)$, such as the deviation between the FY03, FY04, and FY05, we set $\angle F_oPF_1 = \alpha_1$, $\angle F_1PF_2 = \alpha_2$. Scenario two: When the deviation point of the drone is in the range of $(180°,220°)$, such as the deviation in the FY06, we set $\angle F_oPF_1 = \alpha_1$, $\angle F_2PF_0 = \alpha_2$. Scenario three: When the deviation point of the drone is in the range of $(220°,360°)$, such as the deviation between the FY07, FY08 and FY09, we set $\angle F_oPF_2 = \alpha_1$, $\angle F_1PF_2 = \alpha_2$, as shown in Figure 3.

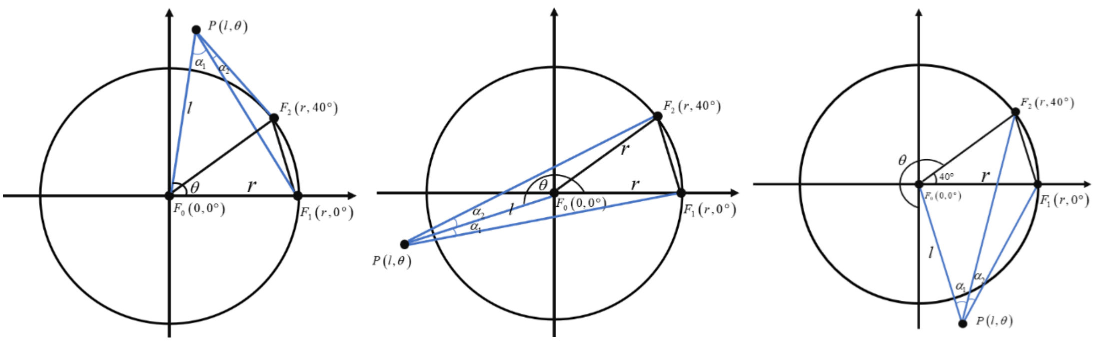


Figure 3: Diagram of deviation point location of known transmitting signal points

According to the sine theorem and simplifying the equation, we can finally get the drone positioning model of the known transmitting signal points and the establishment of deviation points in three different situations at this time, which are:

$$\begin{cases} r\cdot\sin(\theta-40°+\alpha_1+\alpha_2)-l\sin(\alpha_1+\alpha_2)=0 \\ r\cdot\sin(\alpha_1+\theta)-l\sin\alpha_1=0 \end{cases} \begin{cases} r\cdot\sin(\alpha_1-\theta)-l\sin\alpha_1=0 \\ r\cdot\sin(\alpha_2+\theta-40°)-l\sin\alpha_2=0 \end{cases} \begin{cases} -r\cdot\sin(\theta-\alpha_1-\alpha_2)-l\sin(\alpha_1+\alpha_2)=0 \\ r\cdot\sin(40°-\theta+\alpha_1)-l\sin\alpha_1=0 \end{cases} \quad (2)$$

2.2 Passive localization of conical formation at spatial level

Considering that in practice, drone groups may not continue to maintain the same height and the formation is diversified. Therefore, based on the previous research on the positioning model of circular formations at the same height, we extend this scheme to the passive positioning of conical formations at the spatial level [5].

2.2.1 Adjustment model of drone position deviation in space

It is known that three drones that transmit signals and have no deviation in position can effectively locate other drones with slight deviation. Firstly, it is determined that the numbers FY01, FY02 and FY03 are the drones that transmit signals without deviation, according to the principle of determining the plane

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at three points, so they fly at the same altitude. These three fixed drones are combined with the spatial rectangular coordinate system [4] to adjust the deviation of the remaining drones from the spatial deviation to the same horizontal height, as shown in step one in Figure 4.

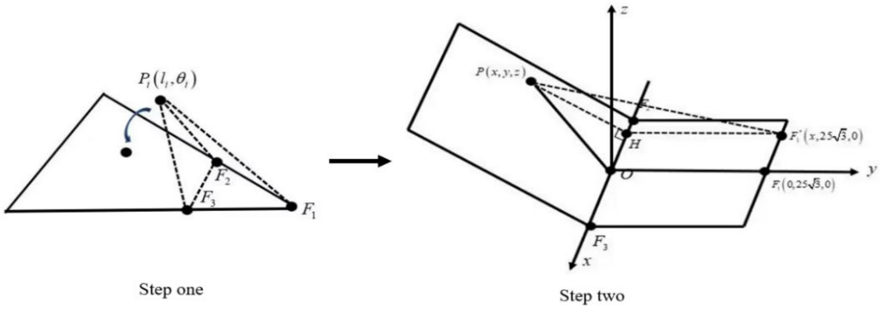


Figure 4: Schematic diagram of drone spatial position deviation steps

Based on the diamond positioning model, the fourth point of the diamond is adjusted by the known FY01, FY02 and FY03, and the other points are adjusted to the same level in turn in combination with the adjusted points. Take the midpoint of F_2 and F_3 as the origin, and the line connecting it with F_1 is the Y-axis, F_2 and F_3 is connected to the X-axis, to establish a spatial rectangular coordinate system, as shown in step two in Figure 4.

In the space rectangular coordinate system, the drone coordinate with slight deviation is $P(x,y,z)$, and the drone coordinate without deviation is $F_1\left(0,25\sqrt{3},0\right)$. The crossing point P is a vertical line perpendicular to the X axis, and the vertical point is $H(x,0,0)$. Translating the point F_1 to $F_1'\left(x,25\sqrt{3},0\right)$, using the cosine formula finally to get.

$$\cos\angle PHF_1' = \frac{PH^2 + HF_1'^2 - PF_1'^2}{2PH\cdot HF_1'} = \frac{y^2+z^2+\left(25\sqrt{3}\right)^2-\left[\left(y-25\sqrt{3}\right)^2+z^2\right]}{2\sqrt{y^2+z^2}\cdot 25\sqrt{3}} \tag{3}$$

In the above, $PH=\sqrt{y^2+z^2}$, $HF_1'=25\sqrt{3}$, $PF'=\sqrt{\left(y-25\sqrt{3}\right)^2+z^2}$.

Solve to:

$$\angle PHF_1' = \arccos\frac{y}{\sqrt{y^2+z^2}} \tag{4}$$

The obtained $\angle PHF_1'$ is the degree of the dihedral angle formed by the drone coordinate with a slight deviation and the surface $F_1F_2F_3$. When it is adjusted to 180°, point P falls on the same plane as FY01, FY02 and FY03, and combines with the adjusted points to adjust the next deviation point in turn, then the drone position deviation adjustment model in the plane is established.

2.2.2 Horizontal drone position deviation adjustment model

After all the spatial deviations are adjusted to the same horizontal height, the drone numbered FY01 is used as the pole coordinates to establish the pole coordinate system and locate the slightly deviated drone coordinates. It was found that 6 drones could not be accurately located in this model, so adjust the pole coordinates to FY05. Since the deviation coordinates P_i of the rest of the drones are different from the angle α formed by the connection of FY02, FY03 and FY05, they are divided into two situations for discussion. The horizontal drone position deviation adjustment model is established to adjust its horizontal angles α_1 and α_2 .

ISSN 2616-5767 Vol.6, Issue 11: 82-89, DOI: 10.25236/AJETS.2023.061112

Scenario one: Polar coordinate system with point F_1 as the pole

Point $P_i\left(l_i,\theta_i\right)$ is a position where a slight deviation of the drone point, where $i=5,8,9,12,13,14$. The specific schematic diagram is shown in Figure 5.

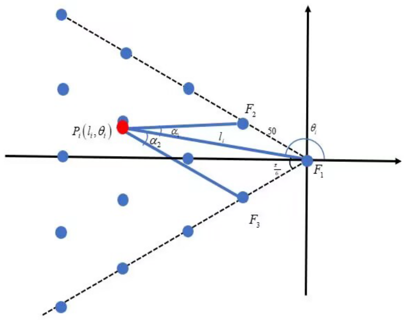


Figure 5: Polar coordinate system with point F_1 as the pole

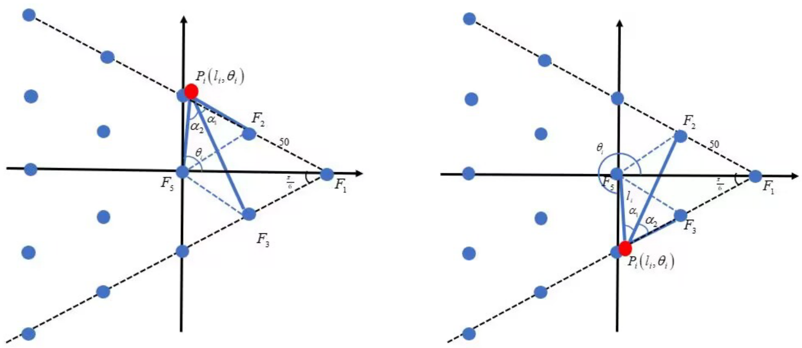
As shown in Figure 5, set $\angle F_2PF_1=\alpha_1$, $\angle F_1PF_3=\alpha_2$ and establish polar coordinate system with F_1 as pole. After analyzing the remaining points, it can be seen that when the positions of FY01, FY02 and FY03 are known, only the positioning models of FY05, FY08, FY09, FY12, FY13 and FY14 can be established.

As above, using the sine theorem, the positioning models of point $P_i\left(l_i,\theta_i\right)$ on points FY05, FY08, FY09, FY12, FY13 and FY14 can be obtained as follows:

$$\begin{cases} 50\sin\left(\alpha_1+\theta_i-\dfrac{5\pi}{6}\right)-l_i\sin\alpha_1=0 \\ 50\cos\left(\alpha_1+\alpha_2+\theta_i\right)-l_i\sin\alpha_2=0 \end{cases} \tag{5}$$

Scenario two: Polar coordinate system with point F_5 as the pole

When FY02, FY03 and FY05 are fixed, the six drones FY04, FY06, FY07, FY10, FY11 and FY15 can be positioned. At this time, the polar coordinate system is established with the point FY05 as the pole. As the positioned points are different, α_1 and α_2 will change. The sums of FY04, FY07, FY11 and FY06, FY010, FY15 are calculated successively, and the above two situations are classified and discussed. The schematic diagram is shown in Figure 6.



(a) $P_i\left(l_i,\theta_i\right)$ is above the F_5 *(b) $P_i\left(l_i,\theta_i\right)$ is below the F_5*

Figure 6: Polar coordinate system with point F_5 as the pole

According to the sine theorem, the positioning models of point $P_i\left(l_i,\theta_i\right)$ on points FY04, FY07, FY11 and points FY06, FY10, FY15 can be obtained respectively:

ISSN 2616-5767 Vol.6, Issue 11: 82-89, DOI: 10.25236/AJETS.2023.061112

$$\begin{cases} 50\sin\left(\alpha_2+\theta_i+\dfrac{\pi}{6}\right)-l_i\sin a_2=0 \\ 50\sin\left(\alpha_1+\alpha_2+\theta_i+\dfrac{\pi}{6}\right)-l_i\sin\left(\alpha_1+\alpha_2\right)=0 \end{cases} \quad \begin{cases} 50\sin\left(\alpha_1-\theta_i+\dfrac{\pi}{6}\right)-l_i\sin a_1=0 \\ 50\sin\left(\alpha_1+\alpha_2-\theta_i-\dfrac{\pi}{6}\right)-l_i\sin\left(\alpha_1+\alpha_2\right)=0 \end{cases} \tag{6}$$

Substituting the point $P_i\left(l_i,\theta_i\right)$ into the above positioning model, the direction information model with deviation can be obtained.

3. Application of drone positioning model

3.1 Positioning adjustment of circular formation drones

Assuming that the radius of the circumference is one hundred meters, the points with known drone numbers FY00 and FY01 are already on the circumference with uniform distribution and no deviation. This is referred to as the standard positioning state of the drone, while the positions of the remaining 8 drones are deviated. The initial positions of the currently known drone are shown in Table 1.

Table 1: The initial position of the drone

Drone number	Polar coordinates $(m,°)$	Drone number	Polar coordinates $(m,°)$
0	(0,0)	5	(98,159.86)
1	(100,0)	6	(112,199.96)
2	(98,40.10)	7	(105,240.07)
3	(112,80.21)	8	(98,280.17)
4	(105,119.75)	9	(112,320.28)

First, according to the standard position of the drone FY00, FY01 and the drone position slightly deviated FY03, we can determine the combination of α_1 and α_2 of FY02 when there is deviation, as shown in Figure 7.

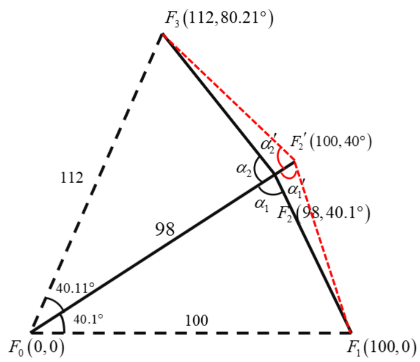


Figure 7: Detailed illustration of F_2 by F_3 amendment

According to the above figure and combined with the sine theorem, the combination of α_1 and α_2 in the deviation state is obtained by simultaneous solution.

Similarly, we can find the combination of α_1 and α_2 of FY02 in the standard positioning state, and adjust the combination to this state when there is deviation, to get the combination of α_1' and α_2' in this standard state.

$$\begin{cases} \alpha_1=\arctan\dfrac{50\sin 40.1°}{49-50\cos 40.1°} \\ \alpha_2=\arctan\dfrac{8\sin 40.11°}{7-8\cos 40.11°} \end{cases} \quad \begin{cases} \alpha_1'=70° \\ \alpha_2'=\arctan\dfrac{28\sin 40°}{25-28\cos 40°} \end{cases} \tag{7}$$

Combined with the positioning model established above and the positions of FY00, FY01, FY02 in

ISSN 2616-5767 Vol.6, Issue 11: 82-89, DOI: 10.25236/AJETS.2023.061112

the standard state under the polar coordinate system, they are substituted into the positioning model in turn according to three different situations. Finally, we can get the adjustment scheme of FY03-FY09 for each deviation point. The direction information adjustment scheme of the circular formation drone is shown in Table 2.

Table 2: Orientation information adjustment scheme for circular formation drones

Drone number	$[\alpha_1,\alpha_2]$ in the deviation state	$[\alpha_1',\alpha_2']$ in the standard state
2	$[71.5338°,80.2943°]$	$[70°,78.8390°]$
3	$[45.8366°,14.8969°]$	$[50°,20°]$
4	$[29.2208°,18.9076°]$	$[30°,20°]$
5	$[10.3132°,20.0535°]$	$[10°,20°]$
6	$[9.4080°,9.4481°]$	$[10°,10°]$
7	$[9.7861°,19.4405°]$	$[10°,20°]$
8	$[30.4183°,20.3572°]$	$[30°,20°]$
9	$[46.2606°,14.9714°]$	$[50°,20°]$

3.2 Positioning adjustment of cone formation drones

Table 3: Direction Information Adjustment Scheme with F_1 *as Pole*

<table>
<tr><th>Drone number</th><th>$[\alpha_1,\alpha_2]$ combination in deviation state</th><th>$[\alpha_1',\alpha_2']$ combination in standard state</th></tr>
<tr><td>05</td><td rowspan="6">$\begin{cases}\alpha_1=\arctan\dfrac{50\sin\left(\theta_i-\dfrac{\pi}{6}\right)}{l_i-50\cos\left(\theta_i-\dfrac{\pi}{6}\right)}\\ \alpha_2=\arctan\dfrac{50\cos(\alpha_1+\theta_i)}{l_i-50\sin(\alpha_1+\theta_i)}\end{cases}$</td><td>$[30°,30°]$</td></tr>
<tr><td>08</td><td>$[10.9781°,19.1081°]$</td></tr>
<tr><td>09</td><td>$[19.1081°,10.9187°]$</td></tr>
<tr><td>12</td><td>$[21.2828°,42.2686°]$</td></tr>
<tr><td>13</td><td>$[10.9781°,19.9781°]$</td></tr>
<tr><td>14</td><td>$[42.2686°,21.2828°]$</td></tr>
</table>

Table 4: Direction Information Adjustment Scheme with F_5 *as Pole*

<table>
<tr><th colspan="3">Scenario one: $P_i(l_i,\theta_i)$ is above the F_5</th><th colspan="3">Scenario two: $P_i(l_i,\theta_i)$ is below the F_5</th></tr>
<tr><th>Drone number</th><th>$[\alpha_1,\alpha_2]$ combination in deviation state</th><th>$[\alpha_1',\alpha_2']$ combination in standard state</th><th>Drone number</th><th>$[\alpha_1,\alpha_2]$ combination in deviation state</th><th>$[\alpha_1',\alpha_2']$ combination in standard state</th></tr>
<tr><td>04</td><td rowspan="3">$\begin{cases}\alpha_1=\arctan\dfrac{50\sin\left(\theta_i-\dfrac{\pi}{6}\right)}{l_i-50\cos\left(\theta_i-\dfrac{\pi}{6}\right)}-\alpha_2\\ \alpha_2=\arctan\dfrac{50\sin\left(\theta_i+\dfrac{\pi}{6}\right)}{l_i-50\cos\left(\theta_i+\dfrac{\pi}{6}\right)}\end{cases}$</td><td>$[30°,30°]$</td><td>06</td><td rowspan="3">$\begin{cases}\alpha_1=\arctan\dfrac{50\sin\left(\dfrac{\pi}{6}-\theta_i\right)}{l_i-50\cos\left(\dfrac{\pi}{6}-\theta_i\right)}\\ \alpha_2=\arctan\dfrac{50\sin\left(\theta_i+\dfrac{\pi}{6}\right)}{50\cos\left(\theta_i+\dfrac{\pi}{6}\right)-l_i}-\alpha_1\end{cases}$</td><td>$[30°,30°]$</td></tr>
<tr><td>07</td><td>$[10.9781°,19.1081°]$</td><td>10</td><td>$[19.1081°,10.9187°]$</td></tr>
<tr><td>11</td><td>$[42.2686°,21.2828°]$</td><td>15</td><td>$[42.2686°,21.2828°]$</td></tr>
</table>

For the three cases of conical formation, the point P is substituted into its respective positioning model to obtain the direction information model with deviation, and then the $[\alpha_1',\alpha_2']$ combination in the non-deviation state can be obtained according to the conical formation in the ideal state, as shown in Table 3 and Table 4.

4. Conclusion

To sum up, the model reduces the number of drones that need to be increased as much as possible through trial calculation, thus ensuring the premise of electromagnetic silence. In order to make the angle

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variable participate in the equation better, the polar coordinate system is used to establish the model and discuss it in various situations, which fully takes into account the situation that the angle variable in the model changes with the change of the anchor point. However, in the adjustment plan at the spatial level, the solution is only generalized and roughly given, and more complicated factors need to be combined when applied to practice.

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