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Modal Parameter Identification of an Arch Bridge Based under Ambient Excitation

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ABSTRACT. *The theory of modal parameter identification based on self-cross spectrum density method was introduced. Taking an arch bridge as the research background, the modal test of the bridge was carried out by using the locomotive measure point method under ambient excitation. The modal frequencies, shapes and damping ratios of the first six orders of the bridge were identified by self-cross spectrum density method. At the same time, the software of ANSYS was used to establish the finite element model of the bridge, and the corresponding frequencies and mode shapes were obtained by analysis. The comparison of measured and analyzed modal parameters shows that the results of the two methods are very close.*

KEYWORDS: *arch bridge, locomotive measure point method, ambient excitation, model analysis*

1. Introduction

The modal parameters of bridges mainly include the natural frequencies, modes and damping. They are important dynamic response information of bridge structures. The measured modal parameters can be used for bridge damage identification and finite element model updating. The modal parameters analysis of the structure is also the premise of other dynamic analysis. Therefore, it is very important to obtain accurate modal parameters of bridge structure [1].

Modal parameters can be obtained by modal test. Traditional modal test is to identify modal parameters by measuring the excitation and structural response under artificial excitations. But it is difficult and expensive to impose artificial incentives in practical application, especially in some large structures. In addition, due to the influence of environment on the structure, the artificial excitation is not the same as the actual excitation on the structure, which affects the accuracy of modal parameter identification. Another is that the traditional methods need to interrupt traffic, causing great social influences and economic losses. The modal test based on ambient excitation can identify modal parameters by using the response of

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environment and driving to bridge excitation. It needs neither artificial excitations nor traffic interruptions, so it is more economical. And the modal parameters obtained by this method are in the actual operational state of the bridge, which can better reflect the actual situation of the structure and solve the difficulties that traditional methods can not overcome. Modal tests under ambient excitation have been widely used in engineering [2-3], which has important practical significance.

In this paper, the modal test of an arch bridge was carried out by using the locomotive measure point method under ambient excitation, and the modal parameters are identified by using the self-cross spectrum density method. The measured results are compared with the results of the finite element model analysis.

2. Principle of Modal Test and Analysis

2.1 The locomotive measure point method

Move the measuring ordering can measure only two points at a time. Firstly, a reference point is determined at the position where the bridge amplitude is larger, and a sensor is arranged at the reference point to pick up the response. This reference point is fixed, and then another sensor is used to measure the response at the test point in turn [4]. This measurement method needs less equipment, simple layout and low cost, and it can continue to increase the number of measuring points for supplementary testing when the test results are not ideal.

Modal parameter identification based on ambient excitation usually assumes that ambient excitation is white noise. On this premise, the principle of moving point method is introduced.

Resonance method is used in the traditional modal test method based on ambient excitation to identify parameters. The mathematical model is as follows [5]:

$$H_{lp}(\omega) = \omega \sum_{i=1}^{N} \frac{D_{lp}}{1-\left(\frac{\omega}{\omega_i}\right)^2 + 2j\xi_i\frac{\omega}{\omega_i}} \tag{1}$$

In formula (1), ω is the excitation circle frequency; $H_{lp}(\omega)$ is the frequency response function of the l-point output to the p-point input; i is the modal order, $i = 1,2,3,\cdots,N$; ξ_i is the damping ratio of the i-th order mode; When $\omega = \omega_r$, Formula (1) can be written as:

$$\left|H_{lp}(\omega_r)\right| = \frac{(D_{lp})_r}{2\xi_r} = \frac{\Phi_{lr}\Phi_{pr}}{2\xi_r k_r} \tag{2}$$

In formula (2), Φ_{lr} is the l-point mode component of the r-th mode; k_r is the first order modal stiffness; $D_{lp} = \frac{\Phi_{lr}\Phi_{pr}}{k_r}$; Vector form of Formula (1) is as follows:

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$$\begin{Bmatrix} |H_{l1}(\omega_r)| \\ |H_{l2}(\omega_r)| \\ \cdots \\ |H_{lp}(\omega_r)| \end{Bmatrix} = \begin{Bmatrix} \frac{\Phi_{lr}\Phi_{1r}}{2\xi_r k_r} \\ \frac{\Phi_{lr}\Phi_{2r}}{2\xi_r k_r} \\ \cdots \\ \frac{\Phi_{lr}\Phi_{pr}}{2\xi_r k_r} \end{Bmatrix} \tag{3}$$

Assuming that the ambient excitation is white noise, formula (3) can be expressed as:

$$\begin{Bmatrix} |G_{11}(\omega_r)| \\ |G_{22}(\omega_r)| \\ \cdots \\ |G_{Pp}(\omega_r)| \end{Bmatrix} = \begin{Bmatrix} \frac{\Phi_{lr}\Phi_{1r}}{2\xi_r k_r} \\ \frac{\Phi_{lr}\Phi_{2r}}{2\xi_r k_r} \\ \cdots \\ \frac{\Phi_{lr}\Phi_{pr}}{2\xi_r k_r} \end{Bmatrix} \tag{4}$$

Formula (4) shows that the mode shapes of the system are determined by the amplitude of the output response spectrum. If a measuring point is taken as a reference point, the following formula can be obtained.

$$\begin{Bmatrix} \frac{|G_{11}(\omega_r)|}{|G_{kk}(\omega_r)|} \\ \frac{|G_{22}(\omega_r)|}{|G_{kk}(\omega_r)|} \\ \cdots \\ 1 \\ \cdots \\ \frac{|G_{pp}(\omega_r)|}{|G_{kk}(\omega_r)|} \end{Bmatrix} = \begin{Bmatrix} \frac{\Phi_{1r}}{\Phi_{kr}} \\ \frac{\Phi_{2r}}{\Phi_{kr}} \\ \cdots \\ 1 \\ \cdots \\ \frac{\Phi_{pr}}{\Phi_{kr}} \end{Bmatrix} \tag{5}$$

According to Formula (5), it is feasible to carry out modal tests by using locomotive measure point method.

2.2 Modal parameter identification method based on self-cross spectrum density method

Self-cross spectrum density method is a simple and fast method to identify modal parameters in frequency domain based on environmental vibration. It is based on the fact that the natural frequencies of structures will peak in their frequency response functions, and the peak is a good estimation of the characteristic frequencies. As the excitation force under ambient excitation is unknown, the frequency response function loses its significance. Instead, in this case, the self-cross power spectrum between the ambient excitation response and the reference point response could be used [6]. So, the natural frequency would be determined only by the peak value on the average regularized power spectral density curve. When the input signal and the measured structure satisfy the idealized assumption (Input signals should be stationary random), the modal parameters of the system could be identified by using the self-power spectrum of the structure response point and the amplitude, phase, coherence function and transfer rate of cross-power spectrum of the reference point [7].

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Assuming that the structure is a real modal system(small damped), the frequency response function can be obtained according to the relationship between excitation and response:

$$h_{ik}(\omega) = \frac{x_i}{f_k} = \sum_{r=1}^{N} \frac{\phi_{ir}\phi_{kr}}{k_r - \omega^2 m_r + j\omega c_r} = \sum_{r=1}^{N} \frac{\phi_{ir}\phi_{kr}}{(j\omega - \lambda_r)(j\omega - \overline{\lambda_r})} \tag{6}$$

In Formula (6), x_i is the steady-state response of the system at i point; f_k is the excitation amplitude of k point; N is the modal order; ϕ_{ir} is the mode vector at point i of the mode of the r-th order; ϕ_{kr} is the mode vector of the first order mode at k point; λ_r and $\overline{\lambda_r}$ are a pair of conjugate eigenvalues of structures.

Formula (6) shows that the frequency response function contains all modal information of the structure [8]. Under environmental excitation, the response is known and the excitation force is unknown. Assuming that the response of a reference point in the structure is an excitation, there is a linear correlation between the response of other measuring points and the response. The transfer function between the assumed reference point and other measurement points is established to identify the modal parameters. If the fixed reference point on the structure is p, the transfer rate is:

$$\alpha_i(\omega) = \frac{x_i(\omega)}{x_p(\omega)} \tag{7}$$

In formula (7), $x_i(\omega)$ is the displacement response of point i on the structure. According to formula (6), it can be expressed as:

$$x_i(\omega) = \sum_{r=1}^{m} h_{ik}(\omega) f_k(\omega) \tag{8}$$

Assuming that the input signal under ambient excitation is white noise, the excitation signal is streightness spectrum. Therefore, its power spectral density function is approximately uniform distribution in all frequency arrange, and the excitation force at each point in the structure satisfies:

$$f_k(\omega) = f(\omega) = C_1 \tag{9}$$

C_1 is a constant in formula 9. Bring Formula (9) into Formula (8):

$$x_i(\omega) = f_k(\omega) \sum_{r=1}^{m} h_{ik}(\omega) = C_1 \sum_{r=1}^{m} h_{ik}(\omega) = C_1 h_i(\omega) \tag{10}$$

In formula (10), $h_i(\omega)$is a lumped frequency response function. From formula (10), the response $x_i(\omega)$ of the structure is equivalent to the lumped frequency response function $h_i(\omega)$, so the natural frequency of the structure can be obtained directly from the response $x_i(\omega)$.

Finally, it is assumed that the real modes of the structure could be effectively separated from each other, and there were no coupling or very small coupling between them. Therefore, the system response at the natural frequencies of order r is dominated by the vibration of the mode of order r, and the contribution of other modes is neglected. Bringing Formula (10) into Formula (7):

$$\alpha_i(\omega) = \frac{x_i(\omega)}{x_p(\omega)} = \frac{\sum_{r=1}^{m} h_{ik}(\omega)}{\sum_{r=1}^{m} h_{pk}(\omega)} = \frac{\frac{\phi_{ir}}{(j\omega-\lambda_r)(j\omega-\overline{\lambda_r})}\sum_{r=1}^{m}\phi_{kr}}{\frac{\phi_{pr}}{(j\omega-\lambda_r)(j\omega-\overline{\lambda_r})}\sum_{r=1}^{m}\phi_{kr}} = \frac{\phi_{ir}}{\phi_{pr}} \tag{11}$$

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In formula: p is a fixed reference point. So ϕ_{pr} is a fixed value at a certain natural frequency ω_r, and formula (11) can be simplified to:

$$\alpha_i(\omega) = \frac{\phi_{ir}}{\phi_{pr}} = C_2\phi_{ir} = \tilde{\phi}_{ir} \tag{12}$$

In formula(12), C_2 is constant. According to formula (12), the working mode at natural frequency ω_r can be obtained from the transfer rate curve $\alpha_i(\omega)$, and can be approximated as the *r*-order mode of the structure. The magnitude of the mode shapes depends on the value of the transmittance at the natural frequency, and the direction of the mode shape is determined by the phase of the cross power spectrum at that frequency or the sign of the real part of the transmittance.

The self-power spectrum $p_i(\omega)$ of point i and the cross-power spectrum $p_{ip}(\omega)$ of the responses of point i and point p are known respectively by classical power spectrum estimation method:

$$p_i(\omega) = \frac{1}{N_{FFT}} x_i(\omega) \cdot \mathrm{conj}\big(x_i(\omega)\big) = \frac{1}{N_{FFT}} |x_i(\omega)|^2 = \frac{C_1^2}{N_{FFT}} |h_i(\omega)|^2 \tag{13}$$

$$p_{ip}(\omega) = \frac{1}{N_{FFT}} x_i(\omega) \cdot \mathrm{conj}\left(x_p(\omega)\right) \tag{14}$$

Formula (13) shows that there is a square relationship between the self-power spectrum $p_i(\omega)$ and $|h_i(\omega)|$, so they have the same extremums, so the amplitude of the transfer function can be replaced by the self-power spectrum of the response point. Bring Formula (6) into Formula (10):

$$x_i(\omega) = f_k(\omega) \sum_{r=1}^{m} h_{ik}(\omega) = C_1 \frac{\phi_{ir}}{(j\omega-\lambda_r)(j\omega-\bar{\lambda}_r)} = \sum_{r=1}^{m} \phi_{kr} \tag{15}$$

From equation (15), we can see that the extreme value of lumped transfer function is independent of the position of the extremum.

At the same time, comparing between Formulas (13) and (14), they have similar forms, and $p_i(\omega)$ and $p_{ip}(\omega)$ have the same extremums, so the amplitude map of lumped transfer function can be replaced by the cross power spectrum between the reference point and the response point to identify the natural frequency.

Damping ratio is calculated by half power bandwidth method:

$$\xi_1 = \frac{\omega_1-\omega_2}{2\omega_i} \tag{16}$$

In formula (16), $\omega_1$and ω_2 are the intersection frequencies of $1/\sqrt{2}$ amplitude levels with peak values and curves ($\omega_2 > \omega_1$); ω_i is the i-th order peak frequencie.

So, when the measured structure satisfies the assumptions of real modal system-streightness spectral input signal and small damped structure with non-close mode, the self-power spectrum of the response point and the cross-power spectrum between the fixed reference point and the response point can replace the amplitude of the frequency response function, and then the natural frequency and damping ratio of the structure can be identified according to their peak value.

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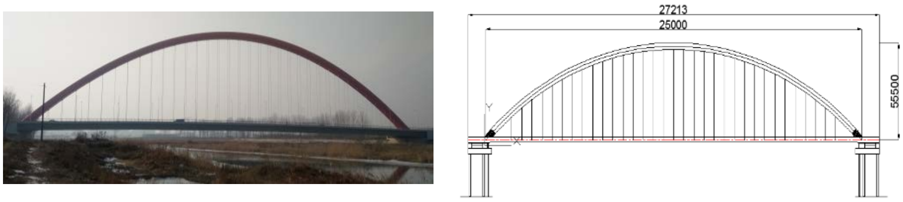
3. Modal test of an arch bridge

This test was implemented on an arch bridge in Dalian. The main span of the bridge is a through tied arch with a single supporting plane. Its total length is 272.13m and its width is 35m. The longitudinal stiffening beam of main span is a single-box five-chamber two-way prestressed concrete box structure with solid section beams at the ends. The arch rib is a steel box girder, and the connecting sections between the arch rib and the main girder are steel-concrete composite segments with the length of 6.32m. The middle section of is a steel box structure. Thespan of the arch is 250m, and the height is 55.5m, and the ratio of arch rib to span is 1/4.5.There are 33 pairs of suspenders in the whole bridge, The distance between which in longitudinal is 7m and in transverse is 3.8m. The picture and drawing of the arch bridge are as follows:



(a) Scene picture (b) Elevation drawing

Figure. 1 Picture and drawing of the Arch bridge

3.1 The scheme of the modal test

This test adopts a wireless bridge modal test and analysis system. Depending on the finite element model analysis, we focus on the frequency range of 0-4.0Hz. According to Nyquist sampling theorem [9], the sampling frequency must be more than twice the bandwidth of the sampled signal. Therefore, we set the frequency of the modal test to 20Hz, and the time of sampling is 15 minutes. Each channel collects 18,000 data. The arch rib is very high, so it is difficult to install sensors on the arch while the bridge on operation, or else specific construction facilities would be needed and traffic would be intterupted. Therefore only the beam are implemented with the velocity sensors. The layout of measuring points is shown in Fig. 2. In this modal test, the moving measuring point method is adopted, and the No. 7 measuring point is chosen as the reference point, because the No. 7 measuring point is close to the middle span and has a large amplitude.

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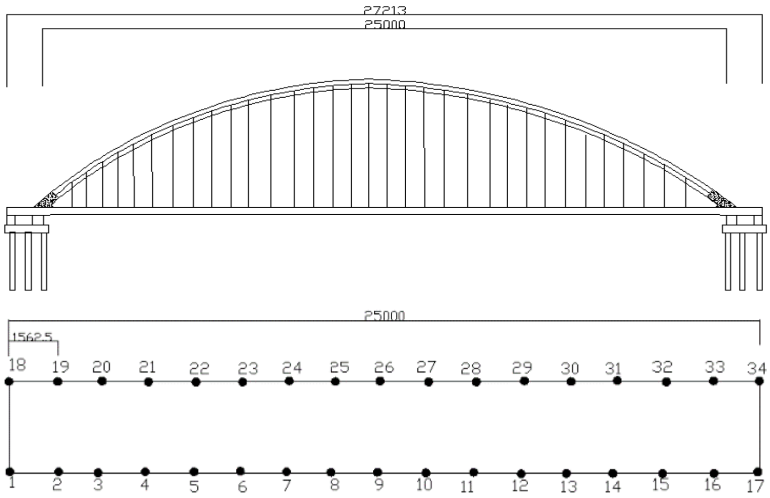

Figure. 2 Layout of measuring points (unit: cm)

3.2 Modal analysis

The velocity time histories of two measured points are shown in Fig. 3. The abscissa is time, the unit is s, the longitudinal axis is velocity amplitude, and the unit is mm/s.

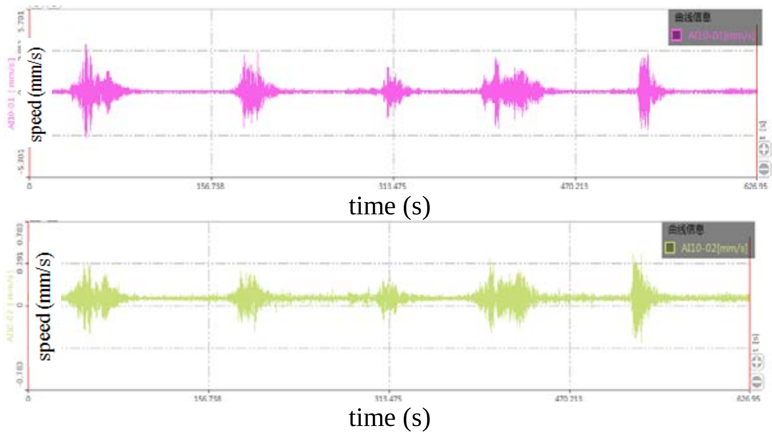

Figure. 3 Velocity time history curve of two measured points

Firstly, the time-domain data are filtered in order to eliminate the noise in the signal, which will make the spectrum more smooth and identify the peak more

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accurately. Generally, the filtering methods include windowing, averaging, overlapping and so on. In this experiment, the testing time is long enough, so it is not nessesary to use overlapping, and only the rectangular window is used to filter the waves. The velocity data are analyized by self-cross spectrum method, and the spectrum diagram is obtained as shown in Figure 4. Then first six natural frequencies and damping ratios of bridges are obtained according to the spectrum diagram, which are listed in Table 1. The first six modes shapes are shown in figs. 4 to 9.

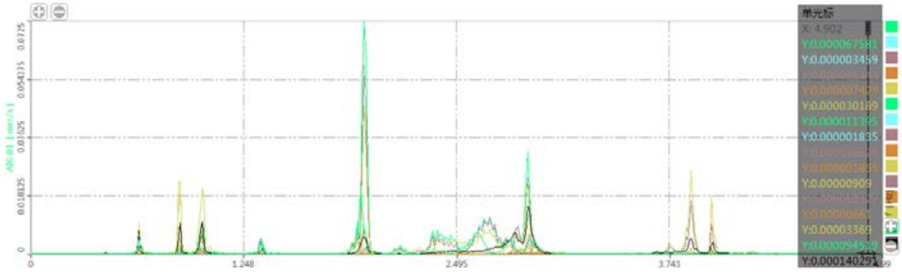

Figure. 4 Velocity Spectrum

3.3 Dynamic analysis of the finite element model and resulsts comparision with the modal test

The finite element model of the arch bridge is established by the finite element analysis software ANSYS. According to the structure characteristics of each part of the arch bridge, BEAM188 element is used for arch ribs and longitudinal beams, and LINK10 element is used for suspender simulation. In addition, the MASS21 elements are distributed on the arch ribs to attach the mass of the diaphragms here. There are 279 elements and 428 nodes in the finite element model of the whole bridge. The arch bridge constraints are considered as full constraints. In this paper, the subspace method is used to calculate the modes of the finite element model, and the first six order modes corresponding to the measured ones are extracted.

The calculated and measured frequencies corresponding to the first three vertical modes and the first three torsional modes are listed in Table 1, Table 2, together with the errors between them. The errors are less than 5%. The calculated results are in good agreement with the measured results. It shows that although there are errors between the initial finite element model and the actual bridge structure, the errors are relatively small. The comparison of the finite element analysis mode and the measured mode is shown in fig. 5 to fig. 10. In the figures, (a) are the measured mode shapes of the bridge deck, as there were no sensors on the arch, so the measurement shapes includes no arch component; (b) are mode shapes of the finite element model of the whole arch bridge.

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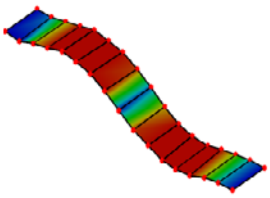
Table 1 Comparison of measurement and computational frequencies of the first three order vertical modes

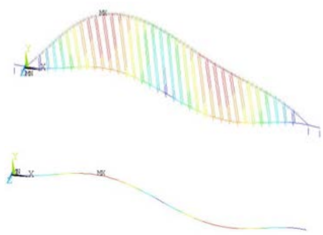
Number	Mode order	Measured frequency (Hz)	Calculated Frequency (Hz)	Error (%)	Damping ratio (%)
f_1	First -order vertical bending	0.635	0.66556	4.8125	0.003
f_2	Second -order vertical bending	0.874	0.86913	-0.5572	0.015
f_3	Third-order vertical bending	1.353	1.3996	3.4441	0.006

Table 2 Comparisons of measured and calculated frequencies of the first three-order torsion modes

Number	Mode order	Measured frequency (Hz)	Calculated Frequency (Hz)	Error (%)	Damping ratio (%)
f_1	first-order torsion	1.001	1.0311	3.0069	0.007
f_2	Second-order torsion	1.948	2.0246	3.9322	0.020
f_3	Third-order torsion	2.91	3.02016	3.7869	0.022

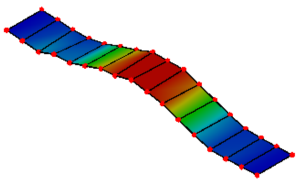


(a) By measurement

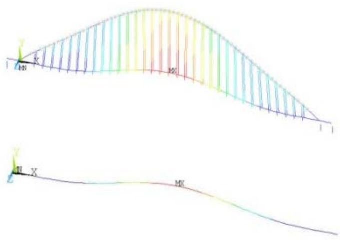


(b) By FEM

Figure. 5 The shapes of first order of vertical modes



(a) By measurement



(b) By FEM

Figure. 6 The shapes of second order of vertical modes

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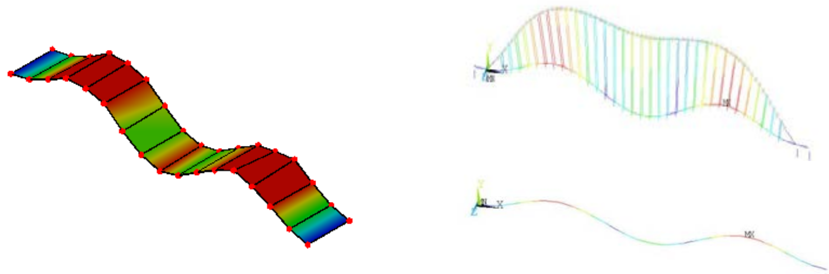
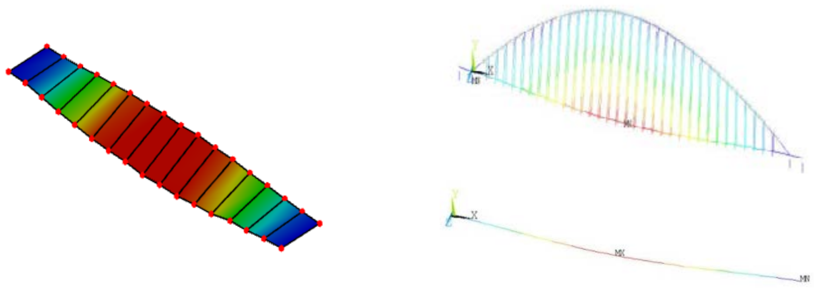


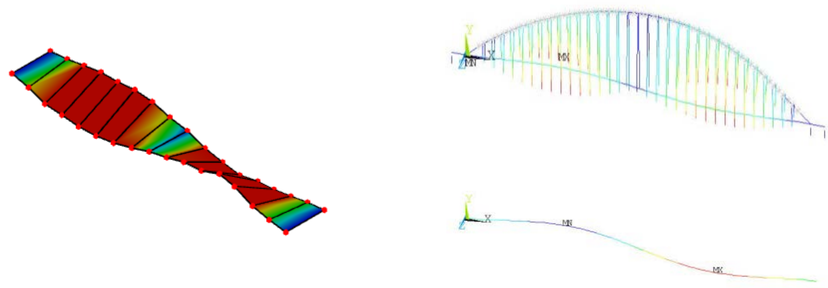
(a) By measurement (b) By FEM

Figure. 7 The shapes of third order of vertical modes



(a) By measurement (b) By FEM

Figure. 8 The shapes of first order of torsional mode



(a) By measurement (b) By FEM

Figure. 9 The shapes of second order of torsional mode

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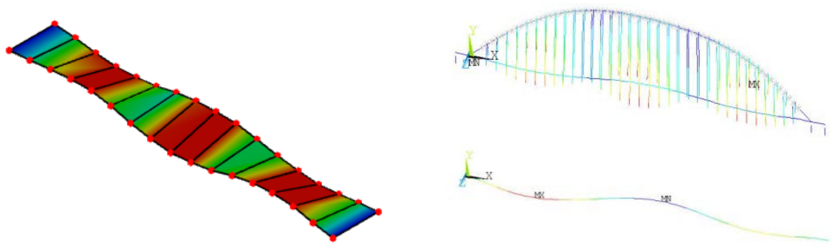

(a) By measurement (b) By FEM

Figure. 10 The shapes of third order of torsional mode

4. Conclusion

Comparing the modal parameters of arch bridge test with the results of finite element model analysis, the results show that they are basically consistent, which means that the identification of bridge modal parameters based on self-cross spectrum density method is feasible. By the locomotive measure point method, the natural mode frequencies, damping ratios and shapes of structures can be identified accurately by only two sensors.

Acknowledgments

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References

[1] L.P. Hu, D.J. Han and Z.T. Yu (2005). Modal test of long-span bridges based on environmental excitation. Journal of Guangdong University of Technology, no.01, p.100-104.
[2] Z.H. Zong, B. Jaishi, J.P. Ge,et al (2005). Dynamic analysis of a half-through concrete-filled steel tubular arch bridge. Engineering Structures, vol.27, no.1, p.3-15.
[3] B. Jaishi, H. Kim, M.K. Kim, et al (2007). Finite element model updating of concrete-filled steel tubular arch bridge under operational condition using modal flexibility. Mechanical System and Signal Processing , vol.21, p.2406-2426.
[4] T. Peng, Z.C. Tian and T.Y. Jiang (2009). Research on modal test method of long-span composite arch bridge.Journal of China & Foreign Highway vol.29, no.3, p.128-132.
[5] C. Deng (2004). Modal test analysis of large-span structures under environmental excitation. Southeast University, p.30-32.

ISSN 2616-5767 Vol. 2, Issue 1: 29-40, DOI: 10.25236/AJETS.020012

[6] J.S. Bendat and A.G. Piersol (1980). Engineering Applications of Correlation and Spectral Analysis.New York: John Wiley, p.167-179.
[7] J.G. Sun (2006). Research on Experimental Modal Analysis Method Based on Environmental Vibration. Harbin: Harbin University of Technology, p.57-59.
[8] D.B. Li (1989). Vibration modal analysis and application. Beijing: Aerospace Press, p.16-21.
[9] S. Zhang, Y.F. Ren and Z. Yao (2015). Application of anti-aliasing filter design in data acquisition system. Computer measurement and control, vol.23, no.1, p.243-246.