

Optimization Research of Smart Mine Operation Scheme Based on QUBO Model and Quantum Annealing Algorithm

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Abstract: This paper analyzes the equipment configuration of the smart mine, builds the corresponding Quadratic Unconstrained Binary Optimization (QUBO) model according to different application scenarios, and analyzes and solves the QUBO model by using the simulated annealing solver and the CIM simulator, to explore the optimal operation scheme of the smart mine under different scenarios. Firstly, under the budget constraint, the objective function is to maximize the sum of the discounted long-term profits of various excavators, and at the same time to meet the constraints such as budget limitation and type limitation, the mathematical model is established and transformed into the QUBO model, and then analytically solved to derive the optimal operation plan of the smart mine under this scenario. Subsequently, the mathematical model is reconstructed to derive the optimal operation scenario and expected profit, taking into account the factor of years of life. In terms of the solution method, *kaiwu sdk* shows good performance for small-scale problems, but it is limited by memory and less effective when facing large-scale problems. Therefore, this paper proposes the *subQUBO* method, which combines quantum and classical computing, as a potential improvement path. This study not only provides an optimization solution for the operation of smart mines, but also provides new ideas for solving similar large-scale optimization problems.

Keywords: QUBO Model, Simulated annealing solver, CIM Simulator, quantum annealing algorithm

1. Introduction

Along with the arrival of the information age, the concept of smart mine is becoming more and more known. How to do a good job in the operation process of the smart mine, the overall planning of resources, design the optimal equipment configuration and operation program is the key to improve the competitiveness of enterprises. Quantum computing is a new type of technology that follows the laws of quantum mechanics and regulates quantum units for computation, which provides new ideas for solving some complex problems. QUBO model is an optimization model based on binary variables, which has attracted a lot of attention because of its unification of the discovery of rich and diverse combinatorial optimization problems. By introducing quantum computing and the QUBO model, it can provide a more efficient computational approach when dealing with complex optimization problems in the configuration and operation of mining equipment. The quantum annealing algorithm was firstly proposed by Finnila et al. at Brown University, UK, and was mainly used to solve the minimization problem of multivariate functions [1]. In recent years, the theoretical and applied researches of quantum annealing algorithm have set off a research boom at home and abroad [2], and have made great progress. In the existing research, quantum annealing algorithm shows better optimization effect in solving some practical problems. Subsequently, Yarkoni et al. in their 2022 review article [3] described the potential and development of quantum annealing algorithms for industrial applications, emphasizing their usefulness in optimization and search problems. In addition, Sack and Serbyn proposed a quantum annealing initialization method for quantum approximation optimization algorithms in their 2021 study [4], further expanding the combined application of quantum computing and simulated annealing. Finally, Lin et al [5] and Geng et al [6] explored the application of simulated annealing algorithms in classical computing to solve the NP-hard problem and the traveling salesman problem, respectively, to provide a reference for the comparison between quantum and classical algorithms. Together, these literatures reveal the current state of research and application of quantum computing and simulated annealing algorithms in solving complex problems.

2. Model building and solving

2.1. Optimization of excavator procurement scheme based on discounted value of long-term profit

In this paper, we want to select at least three types of excavators among the four types of excavators and determine the number of purchases, which makes the smart mining enterprise to maximize the revenue under the condition of not exceeding the budget. Based on this optimization problem, a planning model is established as shown in Eq. (1) below, where a non-negative integer y_i is introduced to denote the number of purchasing the i th type of excavator ($i \in [1,4]$); and a binary variable x_i is introduced to denote whether or not the i th type of excavator has been purchased ($i \in [1,4]$); where the objective is to maximize the total profit, which is the sum of discounted value of the long term profit of various kinds of excavator is maximized. The constraints are a budget limit and a limit on the types of excavators, where b_i denotes the unit price of the purchase cost of the i th type of excavator.

$$\begin{aligned} & \max \sum_1^4 Profit_{i,x_i,y_i} \\ s.t. & \begin{cases} \max \sum_1^4 Profit_{i,x_i,y_i} \leq 2400 \\ x_1 + x_2 + x_3 + x_4 \geq 3 (y_i \in \{0,1\}) \end{cases} \end{aligned} \quad (1)$$

Further modeling QUBO, let the decision variables for this planning problem be $M = [x_1, x_1, x_1, x_4]$, then the final payoff is shown in the following equation.

$$P = \max \sum_1^4 Profit_{i,x_i,y_i} = M^T X \quad (2)$$

Since each variable to be optimized in the binary optimization problem is of Boolean type, there is $profit = profit_i^2$, which can be obtained by substituting into the above equation as Eq. (3).

$$P = \sum_1^4 Profit_i^2 x_i y_i \quad (3)$$

As a result, the optimization model of this problem can be expressed as shown in Eq. (4) as a quadratic binary constrained optimization model, which needs to be transformed into a standard QUBO model.

$$\begin{aligned} & \max \sum_1^4 Profit_i x_i \\ s.t. & \begin{cases} M = [x_1, x_2, x_3, x_4] \\ \sum_1^4 y_i = 3, y_i \in \{0,1\}, i \in [1,4] \end{cases} \end{aligned} \quad (4)$$

The penalty function is then defined according to the constraints as shown in the following equation, where α is a positive constant. The new objective function becomes equation (5)

$$J = P + H \cdot (M \cdot) = M^T M - \alpha \cdot (3 - \sum_i^4 y_i)^2 \quad (5)$$

$H(U) \leq 0$, $H(U) = 0$ if and only if the constraints are satisfied. α is chosen to be large enough so that the objective function can never be maximized when the constraints are not satisfied. Usually, α is chosen so that the penalty function can be at least 75%~150% of the original objective function. Since the constant c has no effect on the solution result, it is not considered. For convenience, the problem is expressed in the form of minimization. Therefore, the QUBO model of the problem is:

$$\begin{aligned} & \min J = -MTQM \\ & M = [x_1, x_2, x_3, x_4] \\ s.t. & \sum_1^4 y_i = 3, y_i \in \{0,1\}, i \in [1,4] \end{aligned} \quad (6)$$

Table 1: Quantum annealing parameter setting table

parameter settings	Epoch	Variable	maximum profit
initial_temperature=100 alpha=0.99	10	$x_1=1, x_3=1, x_4=7$	49000
cutoff_temperature=0.001	30	$x_2=1, x_3=2, x_4=6$	49000
solution=([0,1,1,1,1,1,1])	50	$x_1=1, x_2=2, x_3=10$	58000

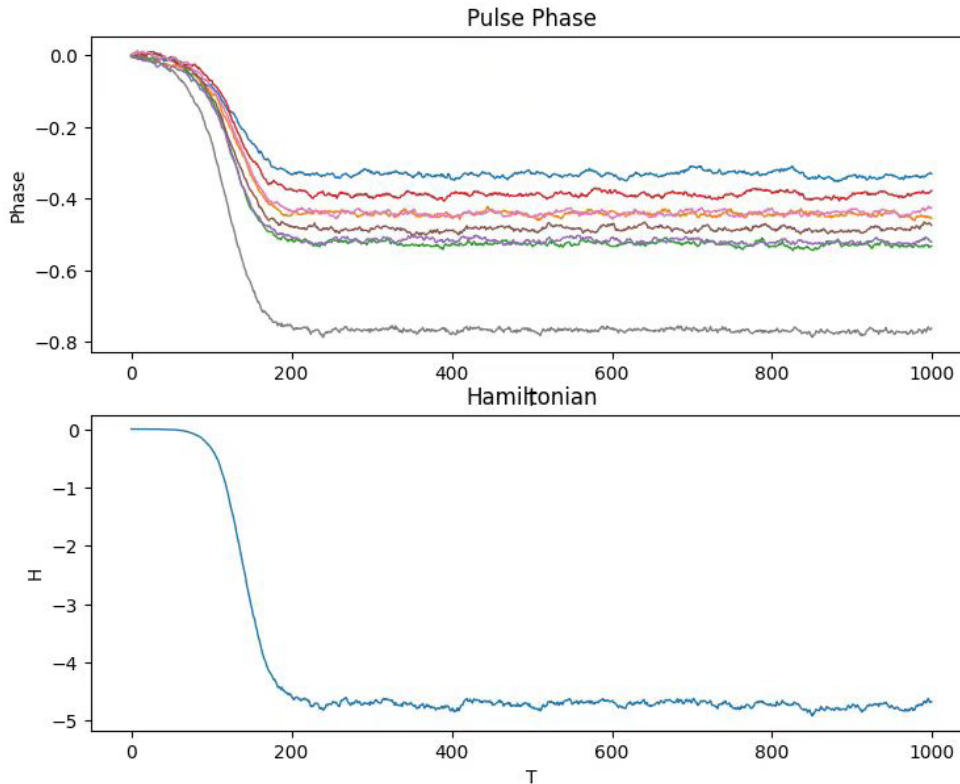


Figure 1: CIM simulator run results.

Input objective function level constraints use `kaiwu.qubo.cim_ising_model` function to convert QUBO matrix to CIM Ising model; use simulated annealing solver in `kaiwu.classical` package to solve CIM Ising model with multiple parameter settings to solve it. As can be seen from Table 1, under the constraints and the corresponding parameter configurations, it can be known that the objective function value, i.e., the maximum total profit, is the largest, and the maximum total profit is 58,000,000 Yuan when there are 1 excavator of the first type, 2 excavators of the second type, 10 excavators of the third type, and 0 excavators of the fourth type. Using the CIM simulator in the `kaiwu.cim` package to solve the problem, the results are shown in Figure 1 below, which shows that the data obtained from the CIM simulator is a sharp decrease and then a smooth progression.

2.2. QUBO Model Optimization of Excavator and Mine Truck Sourcing and Matching Relationships

We want to plan the model and number of excavators to be purchased among the four types of excavators to maximize the total profit of the smart mining enterprise within 5 years without exceeding the budget (assuming the service life of excavators and mining trucks is 5 years). Based on this optimization problem, a planning model is established as shown in Eq.(12) below, where a non-negative integer variable x_i is introduced to denote the number of purchasing the i th type of excavator ($i \in [1,4]$); and a non-negative integer variable y_{ij} is introduced to denote the number of the i th type of excavator matched with the j th type of mining truck ($i \in [1,4], j \in [1,3]$); where the objective is to maximize the total profit within 5 years, i.e., the difference between the revenue in 5 years and the cost difference. Where the total profit, i.e., the profit brought by the excavator, needs to consider the price of ore, work efficiency and service life, etc.; the total cost in addition to consider the various types of costs of excavators and mining trucks, as well as the matching relationship between the two, and the cost of labor, etc.; therefore, the objective function, the constraints constitute the planning model can be expressed as the following equation. The final return can be expressed as eq(7), where β is a constant term.

$$\begin{aligned}
 P' &= Revenue_i - cost_i (i \in \{1, 2, 3, 4\}) \\
 M &= [x_1, x_2, x_3, x_4] \\
 s.t. &\begin{cases} \sum_{i=1}^4 x_i y_i P e_i \leq 2400 \\ \sum_1^n a_{i,j} x_i y_i = x_j y_j (n = 1, 2, 3, \dots) \\ y_{i,j} \in N_0, \forall i, j \end{cases}
 \end{aligned} \tag{7}$$

The QUBO model is also analyzed and solved using kaiwu sdk simulated annealing solver and CIM simulator as shown in Table 2. However, in order to prevent the number of model bits from exceeding the SDK limit, the matrix in the model is changed to be stored in a dictionary to improve the computational efficiency. The optimal operation plan for the smart mine in this scenario is the selection of the 1st, 2nd, and 3rd type of excavators, with the number of vehicles being 7,5,4 respectively, and the maximum profit of 1216.799 million yuan.

$$\begin{aligned}
 P' &= \sum_1^4 D_i aM(y_i Eoe_i * Orp_i)^2 - \sum_1^4 (Cost_i)^2 - \sum_1^4 yM(y_i P e_i)^2 \\
 &\quad - \sum_1^4 aM(y_i Fc_i)^2 - \sum_1^4 aM(y_i Lc_i + Mc_i)^2 + \beta
 \end{aligned} \tag{8}$$

Table 2: Quantum annealing parameter setting table

parameter settings	Epoch	Variable	maximum profit
initial_temperature=100 alpha=0.99 cutoff_temperature=0.001 solution=([0,1,1,1,1,1,1])	80	$x_1=1, x_2=1, x_3=7$	10686.8
	100	$x_2=1, x_3=2, x_4=6$	9623.7
	150	$x_1=7, x_2=5, x_3=4$	12167.9

2.3. Capital Constrained Excavator and Mine Truck Sourcing and Matching Optimization Problems

Building on the above problem and further considering more constraints, when solving this problem, not only do you need to build a QUBO model, but you also need to consider how to maximize profits with limited funds. When the number of bits in the established QUBO model is high, you can try to use methods such as subQUBO to solve the problem. This method gradually approaches the optimal solution of the original problem by extracting a sub-problem of QUBO (i.e., subQUBO) for solving each time and updating the solution of the original problem according to the solution result, and by solving subQUBO for many times. As a result, the objective function is obtained as shown in equation (9).

Let the decision variable be $M = [x_1, x_2, \dots, x_9, x_{10}]$, then the final return is shown in the following equation. Since the variables to be optimized are all of Boolean type:

$$\begin{aligned}
 P'' &= \sum_1^{10} D_i aM(y_i Eoe_i * Orp_i)^2 - \sum_1^{10} (Cost_i)^2 - \sum_1^{10} yM(y_i P e_i)^2 \\
 &\quad - \sum_1^{10} aM(y_i Fc_i)^2 - \sum_1^{10} aM(y_i Lc_i + Mc_i)^2 + B'
 \end{aligned} \tag{9}$$

Since the constant has little effect on the model solution, it is rounded off to obtain a chi-squared equation. Therefore, a new planning model is established as shown in Eq. (10) below, and more constraints are considered, such as the constraint of the number of excavators, the total number of excavators purchased is not more than 10; the constraint of the number of excavator models, the types of excavators cannot be less than 5; the constraint of the matching of excavators and mining trucks: each type of excavator needs to satisfy a certain condition of the matching of mining trucks. These constraints can be expressed as a series of inequalities; budget constraints, i.e., the total cost of purchasing the excavators does not exceed 40 million dollars.

$$\begin{aligned} \min J' &= -P'' \\ M &= [x_1, x_2, \dots, x_n] \end{aligned} \tag{10}$$

$$s.t. \begin{cases} \sum_{i=1}^4 x_i y_i P e_i \leq 4000 \\ \sum_1^n a_{i,j} x_i y_i = x_j y_j \\ y_{i,j} \in N_0, \forall i, j \end{cases}$$

The optional combination of excavators, i.e., a set of other conditions, was increased from the problem above, substantially increasing the difficulty of solving the problem. The model constructed has increased in size, and the constraints and objective functions are basically similar to those of Problem 2, and its form of QUBO transformation is also similar to the above. The QUBO model is also solved using the kaiwu sdk simulated annealing solver and the CIM simulator, where Revenue_i and Cost_i are the expected total revenue and total cost of each excavator of model digging i in 5 years, respectively.

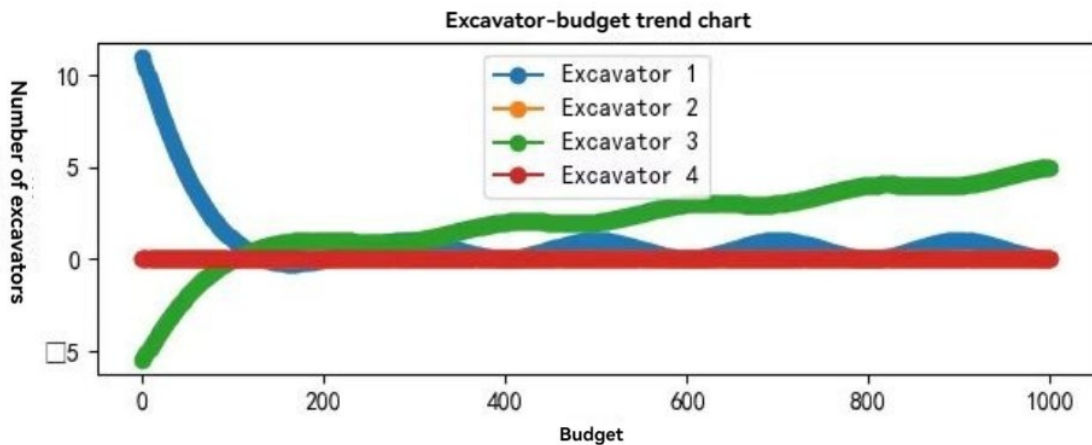


Figure 2: CIM simulator run results.

From Fig. 2, as the budget grows, the number of excavators of the first type first drops sharply and then rises and falls a little, but the ups and downs are not big, the number of excavators of the second type rises and then drops with the budget growth, the number of excavators of the third type rises in general, and the fourth type tends to be flat all the time. After calculating and analyzing, the best operation plan for the smart mine in this scenario is to choose the number of excavators of the 1st, 2nd, 3rd, 5th, 9th and 10th kinds of excavators to be 2, 3, 1, 5, 1, 3, respectively, and the maximum profit will be 229161600 RMB.

3. Conclusions

In this paper, we have successfully constructed several QUBO models for different application scenarios through in-depth research on the equipment configuration problems of smart mines. These models are effectively analyzed and solved, and the optimal operation scheme for smart mines in different scenarios is derived. This study not only provides specific optimization solutions for mine operations, but also provides solution ideas for similar resource allocation problems. Subsequently, the impact of the yearly factor on smart mine operation was considered. With the objective of maximizing the difference between revenue and cost within 5 years, the new optimal operation scheme and expected profit are obtained by re-modeling the solution. In this paper, excellent algorithms in kaiwu sdk are used to construct the QUBO model, and different algorithms are used to highlight the advantages of kaiwu. the QUBO model can be used to solve a variety of discrete optimization problems, and the mathematical form is relatively simple and can be used to solve a variety of discrete optimization problems. This research not only improves the long-term benefits of mine operations, but also provides a strong support for the sustainable development of mines.

References

- [1] Simon D R. *On the power of quantum computation*[J]. *SIAM journal on computing*, 1997, 26(5): 1474-1483.
- [2] Harrow A W, Hassidim A, Lloyd S. *Quantum algorithm for linear systems of equations*[J]. *Physical review letters*, 2009, 103(15): 150502.
- [3] Yarkoni S, Raponi E, Bäck T, et al. *Quantum annealing for industry applications: Introduction and review*[J]. *Reports on Progress in Physics*, 2022, 85(10): 104001.
- [4] Sack S H, Serbyn M. *Quantum annealing initialization of the quantum approximate optimization algorithm*[J]. *quantum*, 2021, 5: 491.
- [5] Lin F T, Kao C Y, Hsu C C. *Applying the genetic approach to simulated annealing in solving some NP-hard problems*[J]. *IEEE Transactions on systems, man, and cybernetics*, 1993, 23(6): 1752-1767.
- [6] Geng X, Chen Z, Yang W, et al. *Solving the traveling salesman problem based on an adaptive simulated annealing algorithm with greedy search*[J]. *Applied Soft Computing*, 2011, 11(4): 3680-3689.