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Application of Differential Equation in Population Stability Analysis

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Abstract: *With the development of biomathematics, population dynamics model is more and more applied to the research and protection of biological species and the quantitative development and management of biological resources. It can be used to describe and reveal the development process of biological population and regulate and control the development trend of species. This paper mainly analyzes the stability of the equilibrium point of the continuous coexistence of the predator-prey model based on logistic growth, taking the typical relationship between snow rabbit and lynx in ecology as an example, it is obtained that the equilibrium point of the sustainable coexistence of the two populations is M (18272, 19727).*

Keywords: *differential equation, stability, population prediction, population kinetic model*

1. Introduction

The population is an important level in ecological research, which is a basic unit that constitutes biological fall, species, and is of great significance for ecological research. In the development of population ecology and population dynamics, many researchers use mathematical tools to establish a mathematical model of a particular population [1], through the establishment and continuous improvement of mathematical models to describe the development mechanism of populations, and carry out breeds of breeding, migration Experimental simulation of behavior and quantity dynamics. Such models are called population kinetic models or population ecology models, is an important role in explaining natural ecological phenomena, describing the dynamics of biological populations, etc.. The study of population stability is revealed both biological plastics, biological communities, and the ecological law of the entire ecosystem, but also for human improvement, protecting the ecological environment, preventing the degradation and reconstruction of the ecosystems and the repair and reconstruction of degraded ecosystems [2]. Provide scientific ainciors.

The differential equation of one of the important branches of modern mathematics is one of the important mathematical tools to solve real problems in modern mathematics theory, and has a wide range of applications in mathematical ecology. Such as the earliest Malthus population model [3], Logistic model [4], two groups of Lotka- Volterra models [5], etc. In recent years, many researchers have made it more practical, such as two group predatory models [6] with functional reactions [7], and the like. These models explain the complex interrelationship between populations by using differential equations.

This paper mainly studies the predator-prey model in which both populations comply with logistic growth. Taking the population changes of snow rabbit and lynx in Yellowstone Park in the United States as an example, the equilibrium point and stability of population sustainable survival are analyzed. In the sense of ecology, the analysis of population stability is an important research aspect. For the ecosystem, one of its important characteristics is stability. Whether the ecosystem is stable will directly affect a series of environmental problems such as ecological habitat and biodiversity.

2. Predator-prey model

2.1. Model definition

Let $x_1(t)$, $x_2(t)$ respectively represent the population number of two populations in the model at time t, and the equations of two populations in the predation relationship are as follows:

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$$\begin{cases} \dfrac{dx_1}{dt} = a_1x_1 + b_1x_1^{\ 2} + c_1x_1x_2 \\ \dfrac{dx_2}{dt} = a_2x_2 + b_2x_2^{\ 2} + c_2x_1x_2 \end{cases} \tag{1}$$

Among them a_i, b_i, c_i ($i = 1,2$) are constant, x_1, x_2 is two populations with predation relationship; a_1, a_2 are the instantaneous growth rate of populations $x_1(t)$, $x_2(t)$, and the positive and negative growth rate is determined by the respective food sources in the living environment, b_1, c_2 are reflected in the density of the two populations, referred to as a variable coefficient. c_1, b_2 reflect the interaction factors of the two populations respectively, called the interpretation coefficient. When the interaction between two biological populations is in the form of predator-prey relationship, the population x_1 is used as a food source of population x_2, and the presence of x_1 is advantageous to increase the number of x_2, and the presence of x_2 is unfavorable to x_1, $c_1 \le 0$, $c_2 \ge 0$.

In the predation relationship, the predator population and the captured population are affected by density constraints, and there is a regulatory mechanism in the species, compliance with the Logistic growth law. Establish a population of predation and predator model according to the Lotka-Volterra equation [8]:

$$\begin{cases} \dfrac{dx}{dt} = rx(1 - \dfrac{x}{K}) - \alpha xy \\ \dfrac{dy}{dt} = hy(1 - \dfrac{y}{L}) + \beta xy \end{cases} \tag{2}$$

x, y represents the population density of prey population and predator population at time t respectively , Where r , h, K, L, α, β are constants, and r, h are the maximum instantaneous growth rate of population x and y, that is, the intrinsic growth rate. K is the maximum environmental carrying capacity of population x, and L is the maximum environmental carrying capacity of the predator population y. α, β is the interaction coefficient of two groups. $h > 0$ indicates that population y has other food sources in addition to population x. Simplified model (2) is:

$$\begin{cases} \dfrac{dx}{dt} = x(a_1 - b_1x - \alpha y) = P(x, y) \\ \dfrac{dy}{dt} = y(a_2 - b_2y + \beta x) = Q(x, y) \end{cases} \tag{3}$$

$a_1 = r$, $a_2 = h$, $b_1 = \dfrac{r}{K} > 0$, $b_2 = \dfrac{h}{L} > 0$.

2.2. Stability analysis

2.2.1. Linearization Theorem

Let the nonlinear autonomous system be

$$\begin{cases} x' = f(x, y) \\ y' = g(x, y) \end{cases} \tag{4}$$

According to the relevant definitions of differential equations [9], the constant solution $x = x_0$ of the differential equation system is called the system's balance point (singularity or stagnation point), the constant solution x^* is satisfied

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$$F(t, x_0) = 0$$

Then (x_0, y_0) can satisfy $f(x_0, y_0) = 0$ and $g(x_0, y_0) = 0$, (x_0, y_0) called (4) balance point.

Set $u = x - x_0$, $v = y - y_0$ Move the balance point to the origin, do a first-order Taylor to get

$$\begin{cases} f(x_0 + u, y_0 + v) \approx \left[\frac{\partial f}{\partial x}(x_0, y_0)\right]u + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right]v \\ g(x_0 + u, y_0 + v) \approx \left[\frac{\partial g}{\partial x}(x_0, y_0)\right]u + \left[\frac{\partial g}{\partial y}(x_0, y_0)\right]v \end{cases}$$

Get (4) linearization equation in (x_0, y_0) is

$$\frac{dY}{dt} = J \cdot Y$$

$Y = (u, v)^T$, J is (4) Jacoby matrix,

$$J = \begin{pmatrix} \frac{\partial f}{\partial x}(x_0, y_0) & \frac{\partial f}{\partial y}(x_0, y_0) \\ \frac{\partial g}{\partial x}(x_0, y_0) & \frac{\partial g}{\partial y}(x_0, y_0) \end{pmatrix}$$

If all eigenvalues of Jacoby matrix are negative real numbers or complex numbers with negative real parts (traces T <0, rowing> 0), the balance point (x_0, y_0) gradually stabilizes.

2.2.2. Balance point stability

Ask the square $\begin{cases} P(x, y) = 0 \\ Q(x, y) = 0 \end{cases}$, to get four possible balance points: $O(0,0)$, $P(0, \frac{a_2}{b_2})$, $Q(\frac{a_1}{b_1}, 0)$, $M(\frac{a_1 b_2 - a_2\alpha}{b_1 b_2 + \alpha\beta}, \frac{a_2 b_2 + a_1\beta}{b_1 b_2 + \alpha\beta})$.

From the biological significance of the predator-prey relationship model of biological population, only the stability of the equilibrium point of the continuous coexistence of two populations is discussed [10].

When two populations coexist continuously, the equilibrium point $M(x_0, y_0)$ is of practical significance in the first image limit .The Jacobian matrix at equilibrium point $M(x_0, y_0)$ of system (2) is:

$$J_M = \begin{bmatrix} a_1 - 2b_1 x_0 - \alpha y_0 & -\alpha x_0 \\ \beta y_0 & a_2 - 2b_2 y_0 + \beta x_0 \end{bmatrix}$$

Get into the coordinate of M point

$$J_M = \begin{bmatrix} a_1 - 2b_1 \frac{a_1 b_2 - a_2\alpha}{b_1 b_2 + \alpha\beta} - \alpha \frac{a_2 b_2 + a_1\beta}{b_1 b_2 + \alpha\beta} & -\alpha \frac{a_1 b_2 - a_2\alpha}{b_1 b_2 + \alpha\beta} \\ \beta \frac{a_2 b_2 + a_1\beta}{b_1 b_2 + \alpha\beta} & a_2 - 2b_2 \frac{a_2 b_2 + a_1\beta}{b_1 b_2 + \alpha\beta} + \beta \frac{a_1 b_2 - a_2\alpha}{b_1 b_2 + \alpha\beta} \end{bmatrix}$$

The characteristic equation is

$$\lambda^2+\frac{a_1A+b_2B}{b_1b_2+\alpha\beta}\lambda+\frac{AB}{b_1b_2+\alpha\beta}=0$$

$A=a_1b_2-a_2\alpha>0$, $B=a_2b_1+a_1\beta>0$. According to the Weeda theorem

$$\begin{cases}\lambda_1+\lambda_2=-\dfrac{a_1A+b_2B}{b_1b_2+\alpha\beta}<0\\ \lambda_1\lambda_2=\dfrac{AB}{b_1b_2+\alpha\beta}>0\end{cases}$$

The positive balance point $M(x_0, y_0)$ is stable.

2.3. Example analysis

In the wild, lynxes mainly live on snow rabbits as their main prey. Ecologists have compiled a large amount of data to draw the curve of the changes in the numbers of the two. The periodic changes are very obvious; and the numbers of the two are one after another, and there is a close correlation.

According to the recording data of the snow rabbit and Bamboo of the US Yellowstone National Park and system (3), using MATLAB software to estimate the parameters, the positive equilibrium point M of the two populations is obtained.

Establish a model based on the predation and predation relationship between lynx and snow hare. Let x_1 (k) and x_2 (k) denote the population of snow hare and lynx, respectively.

Parameter estimation of predation and prey model [11] is as follows:

Table 1: Parameter estimate of two groups of snow rabbits and Bamboo in predation prey model

Parameter Description	Population x	Predator population y
Intrinsic growth rate parameter value	0.1854	0.0079
Coefficient parameter value	0.0266	0.0075

According to the analysis and calculation in 2.3, it is estimated that the stable balance point of the two groups of snow rabbits and bamboo is M(18272, 19727). That is, two groups can continue to coexist, with natural stability.

3. Conclusion

In the analysis of the predator-prey model with logistic growth, the stability of the equilibrium point of the model is analyzed. The model parameters are estimated through the examples of snow rabbit and lynx, and the stable equilibrium point is M(18272,19727), and the two groups have natural stability. The study of predator-prey model is of great significance and role in the stability of biodiversity, the protection of endangered wild animals, the prevention and control of biological invasion, the biological control of disease insects and so on. Revealing the biological law of interspecific relationship of species also plays an important role in agriculture, forestry, breeding and animal husbandry.

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