

Analysis of the Life of Dandelion Based on Biophysical Models

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Abstract: As a plant with fast reproduction speed and strong survival ability, dandelion has important biological value and medicinal significance. However, its characteristics also lead to potential crowding effects on species in different regions. Therefore, quantitative modeling of the reproduction and survival of dandelions is of great research significance. **In Task1:** In order to predict the spread of dandelions over time, the **propagation dynamic model** is established considering the effect of seed inertia and vertical wind. Afterwards, through symbol calculation and numerical simulation, the probability based results of dandelion seed propagation were simulated under **different time, temperature, and climate conditions**. **In Task2:** To define the impact factor of dandelion as an invasive specie accurately, we comprehensively considered **biological characteristics, environmental factors, species interactions**, etc. Before establishing the model, we select **three observation stations** and collect the data of temperature and humidity from 2018 to 2022. **The definition of IF** is based on **Logistic model**, which considering inter-specific **interaction and drought stress**. After calculating by **Runge-Kutta Methods**, we notice that drought when precipitation should be abundant will lead to decrease of biomass obviously, and some plants can gradually return to the steady state, while some will tend to become extinct. Moreover, **two typical invasive species, three species for comparison and six physiological indicators** of each are confirmed and collected, for the use of analysis and simulation. Then, we conduct in-depth analysis of the environment again, to understand the phenomenon mentioned above better, and **pollution** from water, air are also included. In addition, **sensitivity analysis** of the impact factor of species between different ages and environments shows that, our model is **robust** and can be explained in ecology **reasonably**, which means it can be applied to complex areas. Finally, strengths and weaknesses of the model are discussed and conclusions are summarized briefly.

Keywords: Dandelion, Spread, Logistic model, Species, Interaction

1. Introduction

1.1 Problem Background

Dandelion has a distinctive “puffball” seed head. Figure 1 shows that multiple dandelion seeds are supported by inflorescence supports to give a spherical appearance. Each seed has a pappus which makes it easily carried by wind. Therefore, dandelion is a typical seed wind-borne plant, and seed dispersal is an important linking the process of plant migration, which will affect the geographical distribution pattern and growth amount of plants.

The relationship between dandelions, humans, and other plant populations is complex. Dandelions thrive in many environmental conditions, so their encroachment can crowd out native plants and threaten local biodiversity. Meanwhile, dandelion can be used as medicine and food, so it has certain social benefits. Studying the harm and benefit of invasive species at the same time is conducive to protecting biodiversity and safeguarding human interests.



Figure 1: Dandelion plants and seeds

1.2 Restatement of the Problem

To better understand the spread of dandelions in different climatic conditions and further determine the local impact of the introduction of invasive species represented by dandelions, two tasks need to be completed:

- Establish a mathematical model to predict the spread of dandelion in 1, 2, 3, 6 and 12 months, taking into account the effects of different climatic conditions on dandelion growth. Assuming that a dandelion in the expansion ball stage is located next to an open field.
- Considering the characteristics of invasive species represented by dandelion and its relationship with humans and the environment, establish a mathematical model which is capable of determining the "influencing factors" of invasive species. The model should integrate multiple variables, including the characteristics of the plant, and the nature and extent of the harm it causes to the environment.
 - Test the model by using it to compute an impact factor for dandelions.
 - Apply the model to determine the impact factor for two other plant species that are often considered invasive.

2. Assumptions and Justifications

- Assumption 1: Each dandelion plant is of the same nature and produces the same number of seeds.

Dandelion flowers develop their own seeds through a process called apomixis, so that the offspring of each plant are genetically identical to the mother. Therefore, we assume that each dandelion plant is of the same nature and produces the same number of seeds.

- Assumption 2: Simulated environment with limited space for plants to grow.

Theoretically, according to the control variables method, the irrelevant variables should be guaranteed to be certain as much as possible. However, in order to make our study relevant, a limited environmental space is set. Accordingly, the resources (e.g., solar energy) in the environment are steadily supplied, rather than absolutely sufficient.

- Assumption 3: Environmental factors not included in the analysis are considered normal and suitable for growth when not mentioned specifically.

In our model, only the deterioration of the environmental factors to be analyzed is considered in order to simplify the analysis. Theoretically, the deterioration of one environmental factor can cause a series of chain reactions. However, its timing is difficult to control and the scope of change is unknown. Focusing on the research focus, we ignore these potential changes.

3. Notations

Some important mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
$T(t)$	The temperature at time t	C.
$H(t)$	The humidity at time t	%
r_i	The inherent growth rate of species i	\
N_i	Environmental capacity of species i	\
$m_i(T, H)$	The environmental tolerance of species i	\
n_i	Number of plants in species i	\
$NR_i(t)$	Net photosynthetic rate at t	\
$RR_i(t)$	Respiration rate at t	\
$TR_i(t)$	Transpiration rate	\
$E(t)$	The ecological index of the community	\
$D(t)$	The drought index at time t	\

4. Basic Model for Analyzing the Spread of Dandelions

Seed dispersal[1] is an important stage in the spatial movement and reproduction of plants, which deeply affects the spatial and temporal pattern of plants. It is through the diffusion of seeds that dandelion can achieve high density and large-scale spatial distribution. So, analyzing seed dispersal model is very important for predicting plant migration and distribution range change.

Propagation model mainly refers to the distribution fitting function of dispersal kernel. Dispersal kernel refers to the relationship function between the probability of seed landing per unit area and the distance of seed divergence source, which reflects the propagation process and propagation ability of seed wind, and reflects the range and quantity of seed wind propagation.

In the past, many models have been developed to study the dispersal of seeds in the wind[2], which can be divided into two categories: mechanism model and phenomenon model. Phenomenal models focus on fitting, using the functions such as Gauss, negative exponents, and power laws. However, the error of that is large because it doesn't considering the mechanism. In that case, let's start with the basics and model how seeds travel with the wind from mechanism.

4.1 Propagation Dynamics Model

When dandelion seeds fall from the mother plant, they are dispersed by the wind. The distance the seed travels, that is, its propagation distance, can be represented by D. Since the propagation process is random and accidental, it is necessary to define the probability P(x) that the dispersed seed reaches the distance from the parent plant x. This is a probability density function, which is a distribution of the propagation distance and can provide information on the probability of seed arrival per unit distance.

Without considering complex factors[3], the trajectory model can be used to express the calculation method of seed propagation distance intuitively and succinctly:

$$D = \frac{hr \cdot \bar{u}}{Vt} \tag{1}$$

where

- hr is the height at which the seed is released.
- Vt represents the constant descent rate of the seed in still air.
- \bar{u} represents the average horizontal wind speed.

The flaw in this theory assumes that the wind has no speed in the vertical direction, which requires the wind direction remain level all the time and the resistance between the seeds and the adjacent air is infinite. The assumption seems to be contrary to common sense, so the simplest mechanism model needs to be improved.

4.1.1 The Effect of Seed Inertia

In this section, we will elaborate the relationship between seed motion and air flow, and improve the ballistic model above from the seed inertia and interaction with air flow. In order to calculate the

time change of seed velocity with the change of local air velocity, it is necessary to obtain relevant information such as the response time scale of seed velocity to air fluctuation.

So to calculate the motion trajectory of the seed, it is necessary to determine the acceleration of the seed motion. According to Newton's second law, the acceleration can be expressed as:

$$ma = mg - fD,s \tag{2}$$

where the composition of resistance is

$$fD,s = \frac{\rho C_{d,s} A}{m} |V_a - V_p| (V_a - V_p) \tag{3}$$

where fD,s is the resistance acting on the seed, m is the seed mass, V_a is the instantaneous air velocity adjacent to the seed, V_p is the seed velocity, ρ is the air density, and $C_{d,s}$ is the resistance coefficient acting on the seed surface area a .

To further determine the magnitude of the drag, a known drag coefficient and surface area are needed. When the Reynolds number of seeds is large, i.e., $Rep = \frac{\rho_p |V_a - V_p|}{\nu} > 10$, the drag coefficient can be treated as a constant, and this assumption can be applied to most larger seeds.

When the drag and gravity of the seed are equal, the acceleration of the seed is 0 according to Newton's second law. That is to say, the speed in the vertical direction will not increase and the falling speed will be constant.

Then, the terminal velocity of the seed V_t is equivalent to the vertical component of the seed velocity. Based on the basic assumption that the drag coefficient is constant and the vertical velocity of the seed as it falls is zero, the terminal velocity of the seed is:

$$V_t = w_p \Big|_{\left(\frac{dw_p}{dt}=0\right)} = \sqrt{\frac{mg}{\rho C_{d,s} A}} \tag{4}$$

In order to incorporate inertia into the formula for seed motion, a number of other aerodynamic parameters are required. However, these aerodynamic parameters are difficult to measure definitively without wind tunnel experiments.

Since the dandelion seed is large, the Lagrangian relaxation time scale for the particle can be defined as the time it takes for the seed to reach the final velocity V_t or actually reach a high proportion of V_t .

Based on the assumption that the seed starts to move from rest, it can be deduced as:

$$\tau_p = \frac{g}{V_t} \tanh^{-1}(\phi) \tag{5}$$

In summary, this section considers the inertia of the seed and the interaction between the seed and the surrounding air. Then, the effect of vertical wind will be analyzed.

4.1.2 The Effect of Vertical Wind

In this section, we will consider the case when the wind speed is not constant. In the vertical direction, the law of wind speed in the earth's atmospheric boundary layer with the ground height is mainly related to the change of ground roughness and temperature. To simplify the derivation, it can be assumed that the seed and the air have the same speed.

For a seed, its horizontal propagation distance of seeds can be obtained by integrating:

$$x = \int_{z=0}^{z=h_r} \frac{\bar{u}(z)}{V_t} dz \tag{6}$$

The wind speed u will change significantly at the height z above the ground. To calculate $u(z)$, it can be derived from the equations of mean continuity and mean conservation of momentum for a pair of two-dimensional incompressible flows with a high Reynolds number. In the long time scale, the

equation for $u(z)$ is:

$$\begin{cases} \frac{\partial \bar{u}}{\partial z} = 0 \\ \bar{u} \frac{\partial \bar{u}}{\partial z} = -\frac{\partial}{\partial z} u' w' \end{cases} \tag{7}$$

By integrating, the expression of the change of the average speed with z is obtained,

$$\bar{u}(z) = \frac{u^*}{k_v} \log \left(\frac{z - d}{z_0} \right) \tag{8}$$

where u^* is called the square of the frictional velocity, k_v is the vertical turbulent diffusion coefficient that is much larger than the fluid viscosity and can be expressed as the product of the mixing length l_m and the characteristic velocity.

Under this logarithmic profile, we have

$$D = \frac{u^*}{k_v V_t} \left((h_r - d) \ln \left(\frac{h_r - d}{e z_0} \right) + z_0 \right) \tag{9}$$

where e is Euler's number, z_0 is the momentum roughness length when $\bar{u}(z_0) = 0$.

Now, a seed wind propagation model based on the interaction between seeds and air flow and the variation of wind speed with height has been established

4.2 Results and Discussions

A dandelion in the state of "puffball" is placed on an open one-hectare plot of land, and the spread of dandelion in 1, 2, 3, 6 and 12 months was calculated respectively. The following figure shows the density distribution of seed dispersal with distance for different time lengths.

In order to describe the effect of time length on dandelion propagation, the following parameters can be defined: h_{peak} is the peak parameter, indicating the peak density; ω is the shape parameter, which is the ratio of the half-peak width to the peak density. The smaller the value, the sharper the curve. r_{peak} is the distance parameter, indicating the distance of peak density. The results of the dispersal curve are shown in Figure 2, where a, b, c, d, and e respectively represent the diffusion of dandelion seeds at 1, 2, 3, 6, and 12 months.

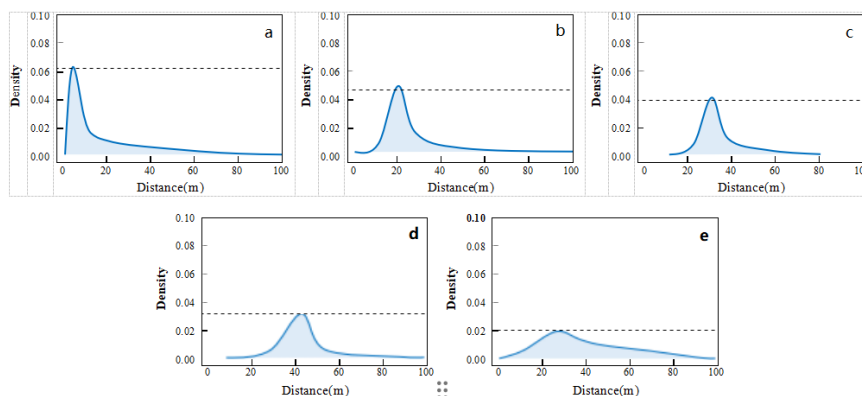


Figure 2: Dispersal curve with time

As shown in Figure 2, when the time is shorter, r_{peak} is smaller, indicating that the distance of dandelion seed spread by wind is smaller, and the probability of spreading to the far distance is lower. With the increase of the time scale, r_{peak} moves to the right, indicating that the probability of seed

spreading far away is higher at this time.

Further considering the influence of environmental factors on the seed propagation curve of dandelion, temperature was firstly taken as the variable. Figure 3 shows the seed wind dispersal curves at suitable, too low and too high temperatures respectively.

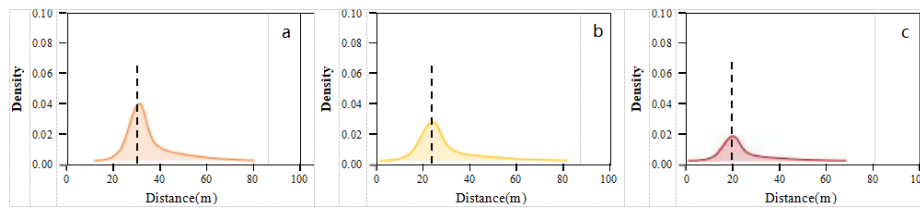


Figure 3: Dispersal curve with temperature

At the optimum temperature, r_{peak} is the largest, h_{peak} is the largest, and ω is the smallest. These parameters indicate that when the temperature is suitable, the probability of seed spreading to a longer distance is higher and more concentrated. The inactivation probability of seeds at higher temperature is greater, which makes it difficult for seeds to spread, and the probability of spreading close distance is higher. At lower temperatures, the density of the air increases, causing the wind speed to decrease, thus reducing both h_{peak} and r_{peak} .

Secondly, the change of the dispersal curve with the degree of dryness is considered. Figure 4 shows the seed dispersal curves under conditions of moderate humidity, dry and extremely dry respectively.

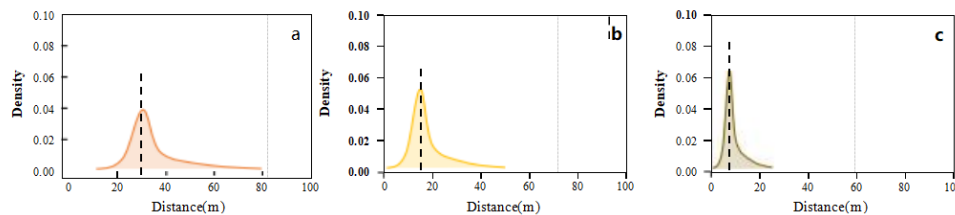


Figure 4: Dispersal curve with drought

When the humidity is appropriate, the propagation curve is consistent with the above description. However, with the increase of drought degree, h_{peak} increases, r_{peak} decreases, and ω increases, then the sharpness of the curve increases, and the peak value shifts to the left. The above trend indicates that the probability of seed dispersal to long distance decreases when it is dry, and most seeds are difficult to spread further.

In summary, we analyze the influence of different environmental conditions on seed propagation, and the suitable temperature and humidity were more suitable for seed propagation.

5. Ensemble Model for Evaluating an Invasive Species

As the problem notes, the relationship between dandelions, humans, and other flora is complicated. Therefore, when defining the impact factor of dandelion, we first describe the mutual relationships between species quantitatively.

5.1 Multi-Factor Logistic Model

We begin with analyzing the change in the community as it is exposed to various irregular weather cycles. Considering community consists of species, we calculate the changes of individuals in each population over time first.

In order to describe the interactions between different species as well as restrictions from the environment, we refer to Logistic model[4](Growth-Retardation model) to build our basic model. Let t as time indicator, $T(t)$ as temperature at t and $H(t)$ as humidity at t , the mathematical model for the prediction of the number of different species over time is:

$$\begin{cases} \frac{dx_i}{dt} = r_i \cdot x_i \cdot C_i \cdot B_i \cdot S_i \\ C_i = 1 - c_{i1} \frac{x_1}{N_1} - c_{i2} \frac{x_2}{N_2} - \dots - c_{in} \frac{x_n}{N_n}, i = 1, 2, 3, \dots, n \\ B_i = 1 - \frac{N_1}{x_1 + N_1} - \frac{N_2}{x_2 + N_2} - \dots - \frac{N_n}{x_n + N_n}, i = 1, 2, 3, \dots, n \\ S_i = 2 \times \frac{(T_{max} - T_{min})^2 + (H_{max} - H_{min})^2}{(T_{max} - T_{min})^2 + (H_{max} - H_{min})^2 + (T_t - T_i^b)^2 + (H_t - H_i^b)^2} \end{cases} \quad (10)$$

Where

- r_i : The inherent growth rate of species i , only related to the species itself.
- C_i, B_i, S_i : Competition coefficient, beneficial coefficient of species i in the population and stress coefficient of species i suffered from the environment.
- $N_i(T, H)$: Environmental capacity of species i , related to temperature and humidity.
- $T_{max}, T_{min}, H_{max}, H_{min}$: Maximum and minimum values of temperature and humidity.
- c_{ij} : The relative competitiveness of species i . More precisely, we have

$$C = (c_{ij})_{n \times n} = \begin{pmatrix} 1 & c_2(T, H) & \dots & c_n(T, H) \\ c_1(T, H) & 1 & \dots & c_n(T, H) \\ \dots & \dots & \dots & \dots \\ c_1(T, H) & c_2(T, H) & \dots & 1 \end{pmatrix} \quad (11)$$

- $c_i(T, H) = \frac{m_i(T, H)}{\sum_{i=1}^n m_n(T, H)}$: The inherent competitive coefficient of species i .
- $m_i(T, H)$: The resistance index of species i .

The model above measures the population state by the number of individuals on the basis of taking the interaction between species into account. Differential equation (set) will be used to describe the change of each state quantity over time. After getting the environmental parameters, the model will be solved by Runge-Kuka method.

5.2 Multi Population Related Factors

The survival of a community in a harsh environment is affected by many factors[5]. The number of species in the community is a very important factor. Before analyzing it, we need to define quantifiable indicators among communities to reflect the viability of communities.

The analysis above is based on population density, i.e. the number of species per unit space, but which obviously cannot be applied to community analysis. In general, the population density of plants with large volume and long growth cycle is generally small.

We define another indicator of population $y_i(t)$ according to the following formula:

$$y_i(t) = n_i(t) \times \left(\frac{NR_i(t)}{NR_i(t)} + \frac{RR_i(t)}{RR_i(t)} \right) \quad (12)$$

where

- $n_i(t)$: Number of plants in the population.
- $NR_i(t), RR_i(t)$: Net photosynthetic rate and Respiration rate at time t .
- $\overline{NR_i(t)}, \overline{RR_i(t)}$: Net photosynthetic rate and Respiration rate, only relevant to species.

Furthermore, since a community contains multiple populations, we have

$$Y(t) = \sum_{k=1}^n y_i(t) \tag{13}$$

$Y(t)$ is a reflection of the community state and a indicator closely related to energy. However, in real-world environments, the locations of species are often space-constrained and resource-limited. In addition to the continuous and slow supply of external energy such as solar energy, it is difficult for plant communities to obtain other energy sources from the outside world.

5.3 Other Environment Factors

In addition, on the other hand, we can also describe the environment more specifically.

Define drought index $D(t)$, which is

$$D(t) = \text{Norm}\left[\left(\frac{T(t)}{\bar{T}_b} + \frac{H(t)}{\bar{H}_b}\right) \times \left(\frac{|T(t) - 25|}{100} + 100\% - H(t)\right)\right] \tag{14}$$

where

- $T(t), H(t)$: The temperature and humidity of a region at time t .
- \bar{T}_b, \bar{H}_b : The average temperature and humidity of the region over the past period.
- *Norm*: Normalize based on historical data to make sure the value ranges in $[0, 1]$.

According to the definition of the above equation, $D(t)$ gives a quantitative description of the degree of drought and can be obtained by converting temperature and humidity.

5.4 Definition of Impact Factor for Invasive Species

Therefore, from a long-term perspective, the energy that one species gets is a stable value, which we denote as IF. After that, we define the Impact Factor IF (t) of the community:

$$IF(t) = \frac{Y(t)}{\max Y} \tag{15}$$

When one species is at an advantage, the impact factor of it should be close to 1.

So far, we can use the impact factor to reflect the impact of the invasive species, and the population density to reflect the ecological situation of specific species.

6. Model Application to Dandelions and Two Other Species

6.1 Data Collection, Preparation and Statistics

The problem doesn't provide us with data directly, so we need to consider which data to collect for model building. Generally, there are mainly two parts of data to be collected, environmental data and plants' data. The former provides a test scenario for the model, while the latter is necessary for solving specific problems.

6.1.1 Typical Regions

To make our model more scientific and reasonable, we choose three typical climates and select one typical region for each. Specifically, the climate conditions of the three states selected are different, which is conducive to the subsequent comparative analysis. As shown in Figure 5, the selected climate and corresponding regions are Nevada in the arid region, Wyoming in the semi-arid region, and Mississippi in the humid region.



Figure 5: Three regions selected with temperature(line) and precipitation(histogram) in a year

Moreover, we collect the temperature and humidity data of the three regions monthly from 2018 to 2022, as shown in Figure 6. The solid line represents the average value monthly, and the range of the light part indicates the highest and lowest value.

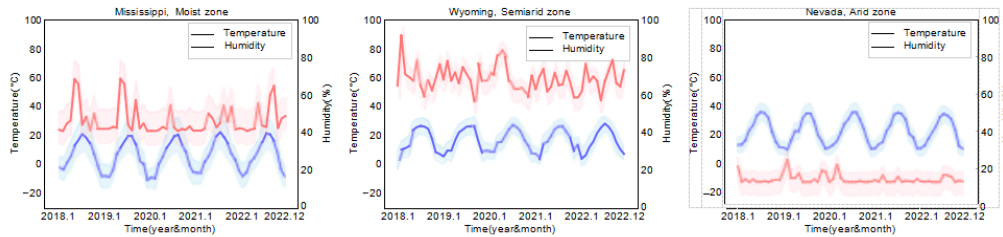


Figure 6: Temperature and humidity data of three observation points.

6.1.2 Two typical invasive species and three native species

To ensure the generalizability and analyzability of the model, we select 2 typical invasive plants[6], 3 native species for comparison and collected six physiological indexes of each.



Apocynum pictum Schrenk: Oleander family, upright semi-shrub, up to 2.5 meters high, generally about 1 meter high. It is mainly wild along saline lakes, wastelands, desert edges, rivers and channels. It is often used as a companion species to form a community with salt-tolerant mesophytes.



Allium mongolicum Regel: Allium plants of Liliaceae, 10-30 cm high, grows in deserts, sandy lands and strong permeability under wild conditions with an altitude of 800-2800 meters, belongs to long-sunshine photophile plants, and has the ability of drought resistance, cold resistance and barren resistance.



Ammopiptanthus mongolicus (Maxim. ex Kom.) Cheng f.: Leguminous evergreen shrub with a height of up to 2 meters and can grow in harsh natural environment. It has thick cuticle, dense surface fur, strong drought resistance, salt resistance, barren resistance, and can grow under extreme water shortage.



Gymnocarpus przewalskii Maxim.: Caryophyllaceae, gymnospermum, up to 100 cm high, which grow at an altitude of 1000-2500 meters and is resistant to drought, cold and barren soil, with strong wind resistance. It grows on dry desert soil and forms monodominant species communities in surface runoff areas.



Launaea polydichotoma (Ostenfeld) Amin ex N. Kilian: Perennial herb, about 15-40 cm high. It is born in the sand, the edge of the sand, the low land between the sand dunes, and the edge of the sand field, with an altitude of -142-1800 meters. Its rhizome has multi-level bifurcated branches which can fix sand.

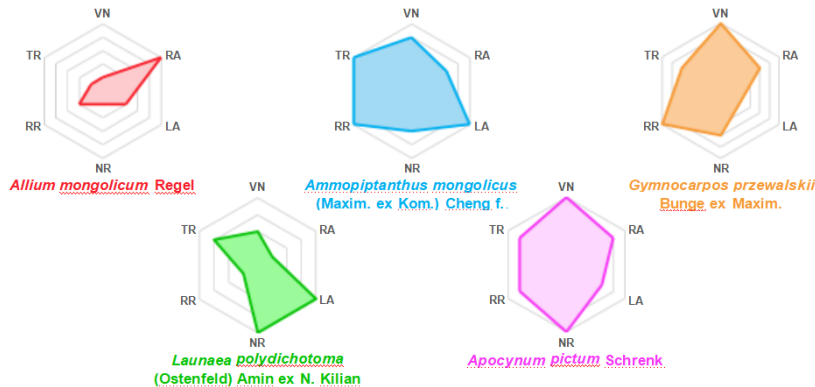


Figure 7: Relative indexes of 5 typical plants. (VN: Vertical niche, RA: Root absorption area, LA: Leaf cover area, NR: Net photosynthetic rate, RR: Respiratory rate, TR: Transpiration rate)

Figure 7 shows the six typical physiological indicators of these five typical plants.

6.2 Solution and Results: Dandelion

As mentioned in the problem, community’s change is a complex process. Here, we specify most of the parameters directly, and use specific species collocation to introduce the numerical calculation method and give a solution. More details will be discussed in the following part.

6.2.1 Runge-Kutta methods

Runge-Kutta method is a high-precision single-step algorithm for the numerical solution of differential equations, which has the advantages of high accuracy and wide application. Due to the long time span of the problem, the implicit method is used to solve it for numerical stability.

Most programming software has built-in this method (eg, ode45), so we do not repeat it again.

6.2.2 Parameter Setting

The value of some important parameters used is listed in Table 2.

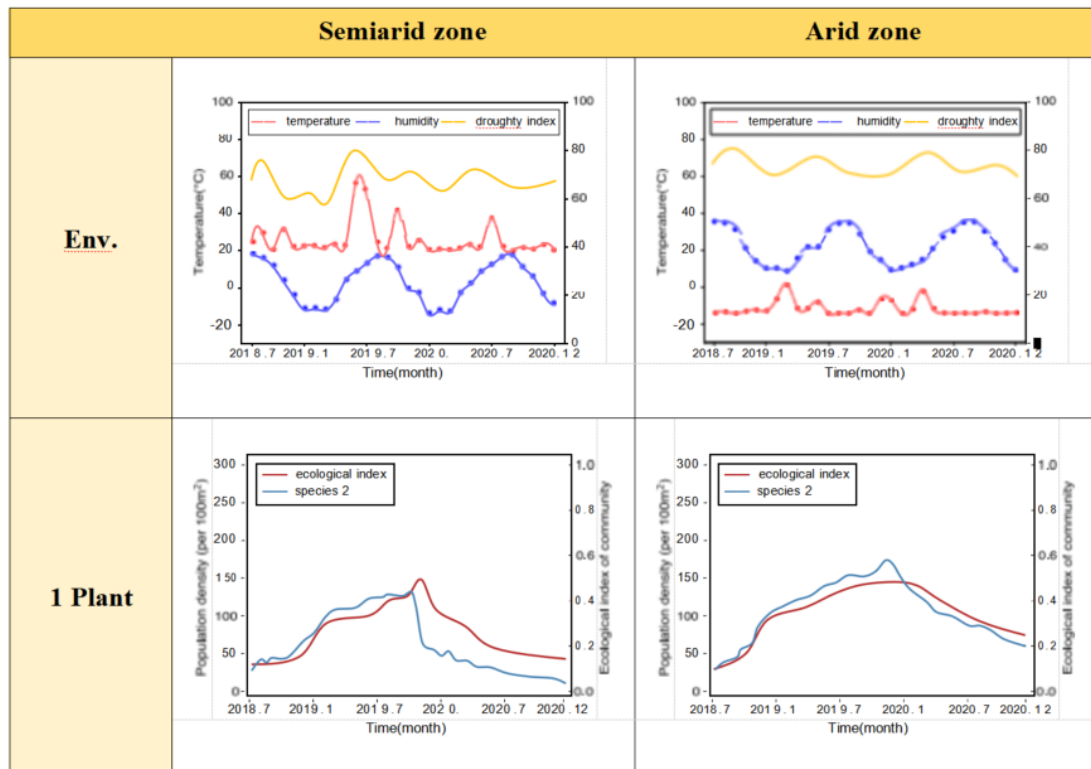
Table 2: Value of some important parameters

Parameter	Value
Environmental capacity of species 1 to 5	[200, 300, 300, 300, 200]
Inherent growth rate of species 1 to 5	[0.8, 1.2, 1.1, 1.0, 1.0]
Relative competitiveness of species 1 to 5	[1.0, 2.5, 1.2, 1.5, 1.0]

6.2.3 Results

Analyze according to the parameters in Table 2, the results are shown in Figure 8. In the figure, we show that in two typical climates, dandelion is an invasive species. The impact factors are calculated in a community with only dandelion, a community with dandelion and two native species, and a community with dandelion and four native species.

In the figure, since dandelion is an invasive species, the number of dandelions is small at the initial moment, but with the increase of time, the number of dandelions increases sharply, occupying the resources of the native species, making the number of native species decreased. In this process, the influence factor of dandelion is small at the initial moment, and it has not caused any impact on the environment. But over time, the impact factors increased, suggesting that dandelion invasion had a serious impact on the environment and native species.



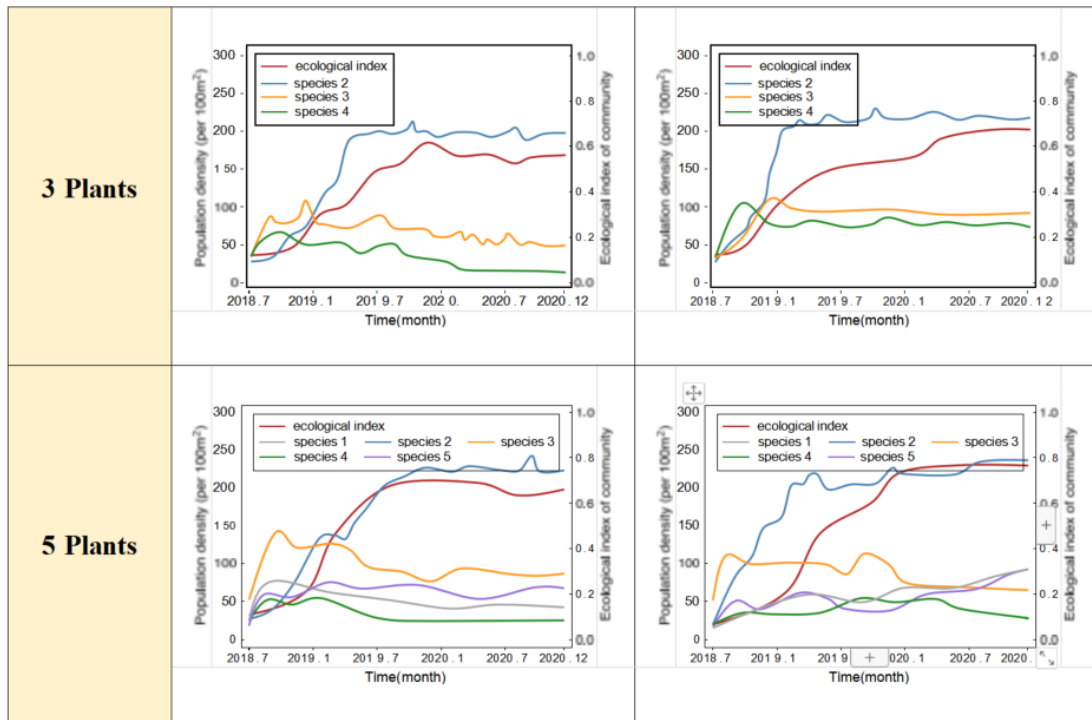


Figure 8: Impact factor with different number of native species and dandelions

6.3 Solution and Results: Two Invasive Species

In the previous section, we analyzed the characteristics of a single invasive species, dandelion, and its impact on the environment. In this section, we need to further consider species- to-species interactions, so we chose two other invasive species for our analysis. Information on invasive species is given above.[7]

In order to measure the interaction between species more conveniently[8], we construct the matrix of mutually beneficial symbiotic interaction matrix A, competition matrix B, resistance to stress matrix C, and integrated capacity matrix D. Their same form is as below:

$$M = (m_{ij})_{n \times n} = \begin{pmatrix} 1 & m_1/m_2 & \cdots & m_1/m_n \\ m_2/m_1 & 1 & \cdots & m_2/m_n \\ \cdots & \cdots & \cdots & \cdots \\ m_n/m_1 & m_n/m_2 & \cdots & 1 \end{pmatrix} \quad (16)$$

where M refers to A or B or C or D.

In this section, we will consider the following strategies:

- Principle 1: Invasive species 1 and two native species.
 - Collocation: Species 1,3,4. Species 1,3,5. Species 1,4,5.
- Principle 2: Invasive species 2 and two native species.
 - Collocation: Species 2,3,4. Species 2,3,5. Species 2,4,5.
- Principle 3: Invasive species 1,2 and one native species.
 - Collocation: Species 1,2,3. Species 1,2,4. Species 1,2,5.

In the simulation calculation, it is not necessary to do the simulation of nine schemes. For each strategy, we select only the first two options, which are odd and even. It is worth noting that when calculating the impact factor that contains two invasive species at the same time, we take the average as the impact factor of invasive species.

The environment information used for simulation is shown in Figure 9.

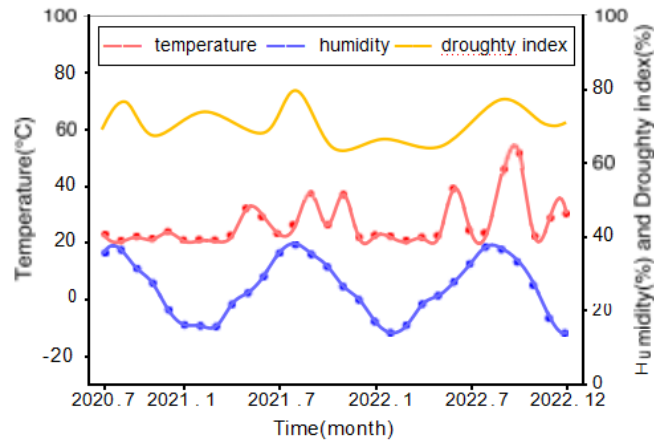


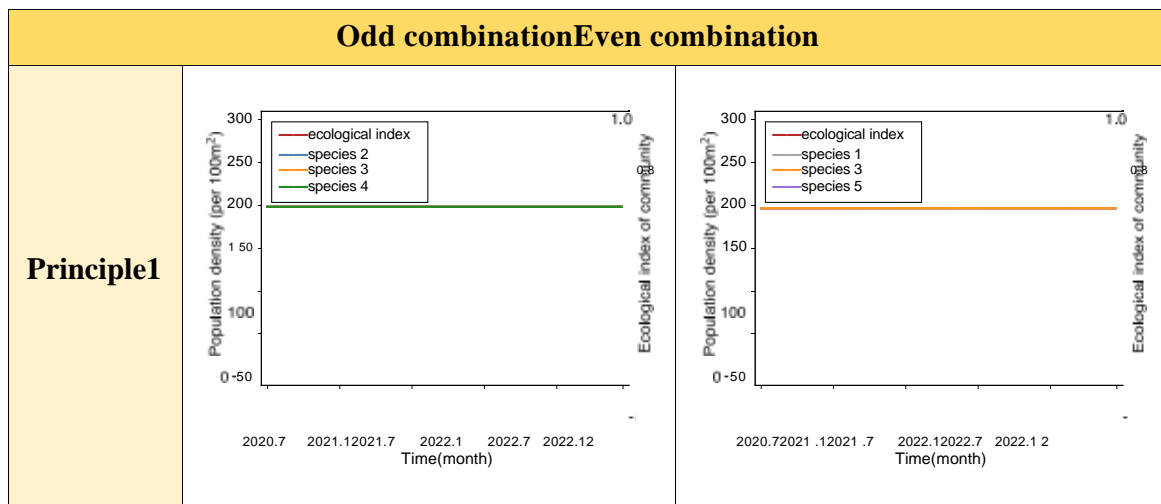
Figure 9: Environmental information used to simulate different species combinations.

From Figure 10, we can find that for communities with a single invasive species, with the increase of time, a single invasive species gradually dominated the community competition, and the impact factors on the environment increased for odd combination.

However, for even combination, although for a period of time the competition of invasive species was dominant and their numbers increased; while as the two native species became extinct, the number of invasive species declined over time. This maybe because invasive species need to take advantage of the resources and reciprocal benefits of native species.

For communities with two invasive species, in communities with two invasive species at the same time, the number of both invasive species remained consistently low, and the number of native species increased. This suggests that native species are not competing with invasive species, but rather two invasive species competing with each other. The impact factor remained at a higher level over a longer period of time.

In even combination, consistent with the previous speculation, native species may need to take advantage of the resources or mutually beneficial effects of the two invasive species. So when two invasive species compete, the number of native species increases[9]. But as time went on, the resources available to native species decreased and they were no longer subject to reciprocity, so the number of native species decreased.



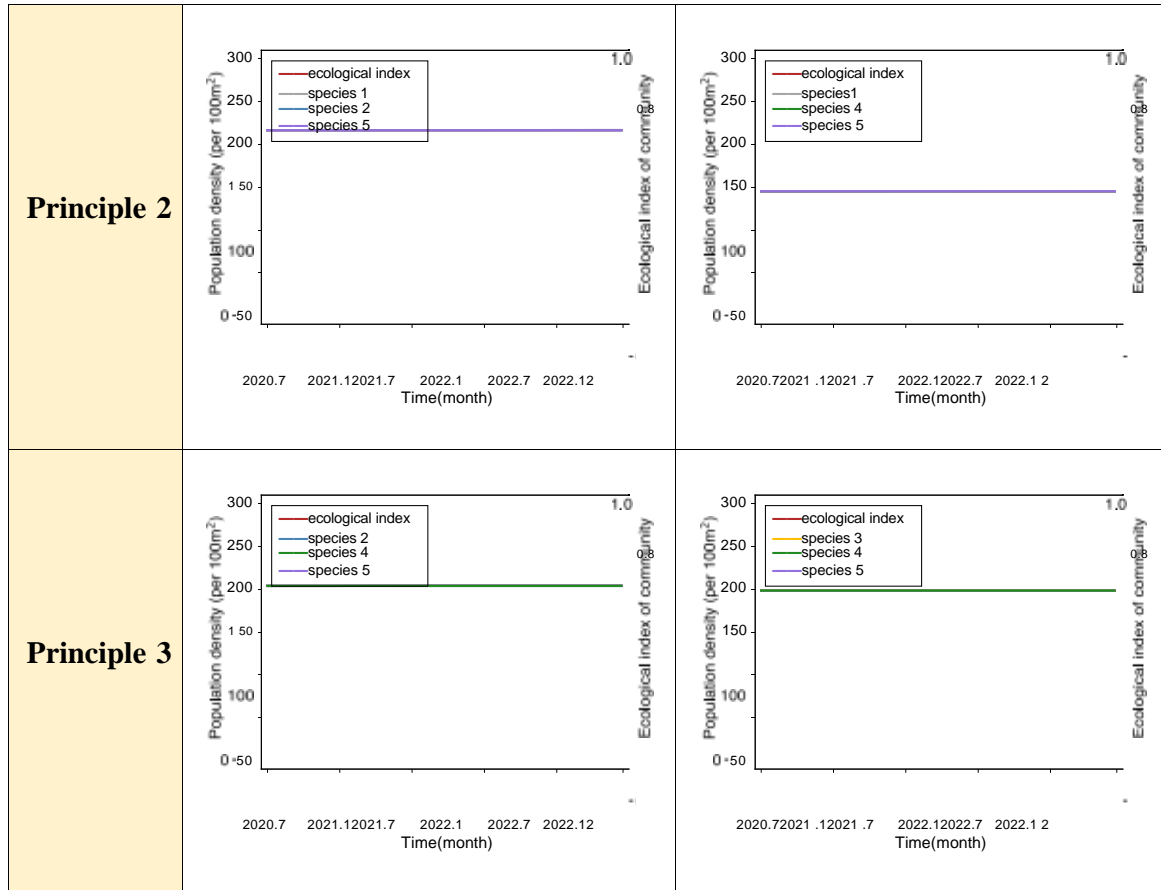


Figure 10: Community growth status with different type of species

6.4 Impact Factor in Extreme Weather

The above two parts analyze the impact factor of invasive species in environment we set. However, in real-world scenarios, such predictions are often inaccurate, although we make predictions about the environment. Influenced by sudden environmental changes, human factors, and other majeure, the real changes in the environment often deviate from our expectations. Therefore, we should conduct a broader study on the resilience of plants, that is, analyze their adaptability in more frequent and wider environmental changes.

We are to analyze the impact factor of species suffering from more frequent and more severe environment. For this purpose, we select one typical environment and choose Principle 2 as an example, i.e., invasive species 2 and native species 3 and 4. Afterwards, we take the environmental data of Missouri state, from 2018.7 to 2020.12 as the benchmark and add perturbation to it artificially, as shown in Figure 11.

The data transformation formula is :

$$T(t) = \alpha T T_0(t), H(t) = \alpha H H_0(t) \tag{17}$$

where

- $T_0(t), H_0(t)$: Temperature and humidity at time t before transformation.
- $T(t), H(t)$: Temperature and humidity at time t after transformation.

And, coefficient αT and αH is

$$\begin{cases} \alpha_T = \text{Smooth} < \text{Interpolation}[50], [t_1, t_2, \dots, t_5], [\text{Rand}(0.75, 1.25)] > \\ \alpha_H = \text{Smooth} < \text{Interpolation}[100], [t_1, t_2, \dots, t_10], [\text{Rand}(0.8, 1.2)] > \end{cases} \tag{18}$$

where:

- *Smooth* $\langle \dots \rangle$: Data smoothing, using smooth curves to connect discrete points.
- *Interpolation* $[a], [b], [c]$: Data interpolation, interpolate $|b|$ number (corresponding to c) to $|a|$.

Use the formula to transform the data between July 2019 and December 2020, and use the transformed data to calculate the transformed drought index. Further more, we stipulate that :

- The environment is considered arid when drought index is more than 0.75.
- The environment is considered arid severely when drought index is more than 0.85.

In the case of the above perturbation, the environment changes are shown in Figure 11 :

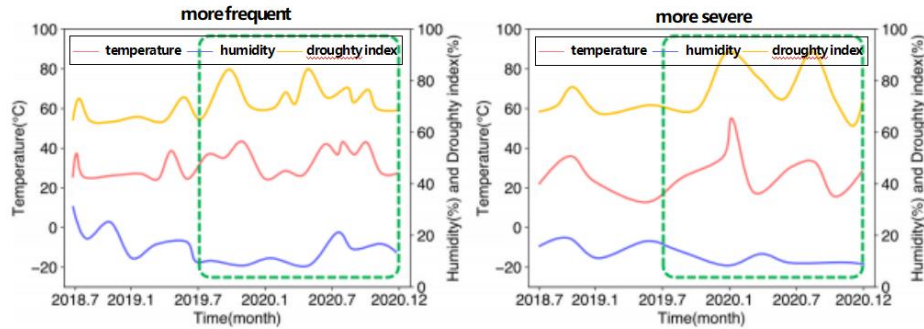


Figure 11: Environmental state after transformation.

After calculating, when droughts become more frequent and the environment becomes more severe, the simulated states of the two aforementioned communities are:

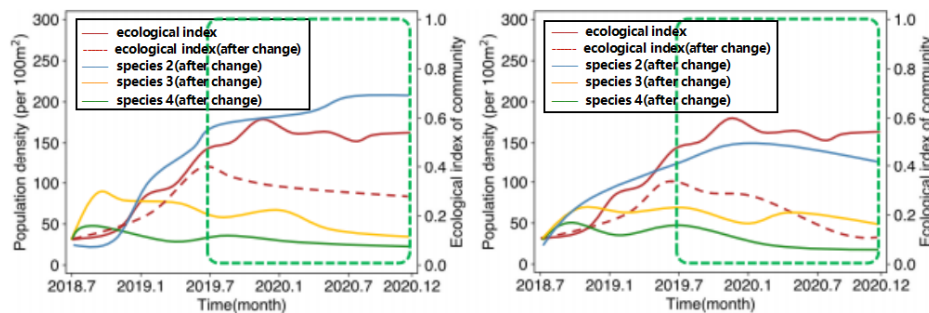


Figure 12: Physiological state of plants under more frequent (left) and severe (right) drought

From Figure 12, it can be observed that:

- In a sense, plants have some ability to adapt to their environment. However, the impact factors are significantly reduced by the more severe weather.
- It takes time for plants to adapt to the environment, and if droughts occur frequently, plant communities are more likely to be destroyed. The impact factor also decrease.

7. Sensitivity Analysis

Many parameters are introduced in our model above. Some of them have objective values, while others are specified directly according to history data or just by our experience. Therefore, it is necessary for us to experiment their sensitivity to them.

In this section, we mainly analyze the physiological index of species. In the previous article, the physiological indexes $P_i = [VN, RA, LA, NR, RR, TR]$ of the same species are regarded as constants, but the index must be different for plants with different ages and different growing environments. However, if we treat it as a variable, we will put forward higher requirements for the accuracy of the solution, and the computational complexity will increase significantly.

For analysis purposes, we also introduce an intermediate group (between the groups of constant combinatorial variables), that is

$$\begin{cases} f_1 = P_i(t) + P_i - P_i(t) \\ f_2 = P_i(t) + 0.5 \times (P_i - P_i(t)) \\ f_3 = P_i(t) \end{cases} \quad (19)$$

Here, we select plant combinations of species 2, 3,5 (one invasive species and two native species) and species 1,2,4(two invasive species and one native species) and analyze the change of the impact factor based on the data of x region and x time. The results are shown below.

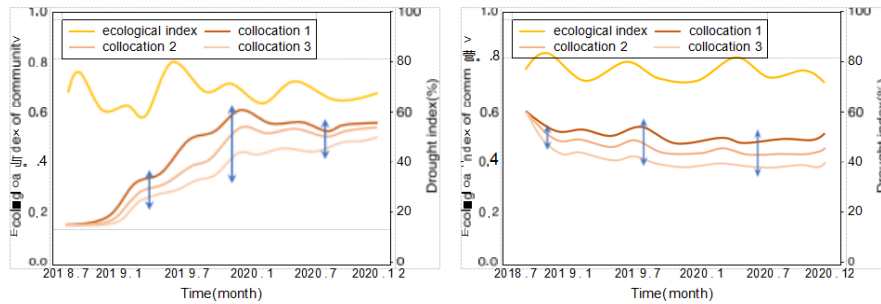


Figure 13: Sensitivity Analysis

From Figure 13, we can see that the trend of impact factor is identical in the different groups. Moreover, after a period of time, the distance between the different groups all tend to stabilize, especially the results in the right panel. This means that our previous model simplification is reasonable and the treatment is correct. For a more specific analysis, the left panel shows the state of the nascent community. At the beginning, the values remain consistent across the groups. Gradually, the parameter settings lead to an increasing butterfly effect, resulting in an increase in the difference between the data. However, as each group tends to mature, the interpolation of the data will finally decrease to a stable, but not equal level. And for the right figure, the baseline state is a steady state, just as we predicted.

8. Model Evaluation and Discussion

8.1 Strengths

- Professional with ecological knowledge: Our model refers to ecological research methods, practical factors of the problem, and creates multiple quantitative indicators to measure the ecological status of environmental tolerance of species better.
- Universal as well as representative: In studies of invasive species, the definition of variables is appropriate and innovative, and the selection of environment is representative.
- Logic and easy to extend: The whole model adopts the structure of “one foundation, four extensions and one summary”, with simple principle and wide range of scenarios.
- Relatively complete factors and controllable error make model easy to promote and use.
- High accuracy of calculation: For the differential equations of the model, Runge - Kuka method is used to solve. Although the amount of calculation will increase, but this method can effectively avoid the instability of the ordinary difference method, so as to avoid the “butterfly effect” of small values from affecting the results.

8.2 Weaknesses

- Simple indicator of species' indexes: When the model is established, the six indicators of plants are discretized according to the relative values. This greatly simplifies the calculation, but will lose some accuracy.

▪ Continuous processing of time series data: The time series is continuous data. We obtain data in months and days, but we still carry out mixed precision calculation and introduced interpolation method in order to avoid accidental events (human factors).

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