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Monte Carlo Based Glauber Model in Pb-Pb Collisions at LHC

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Abstract: *Glauber model is a model used to describe heavy ion collision process. In this paper, we describe an implementation of a Monte Carlo based Glauber Model calculation used for the PHOBOS experiment, which simulates the collision of two heavy nuclei. The nucleon distribution in two initial Pb nuclei is parameterized with Woods-Saxon. The energy profile is* $\sqrt{S_{NN}} = 5.5 TeV$ *and inelastic cross-section area, i.e*σ_{NN}*= 72mb. Geometric quantities such as impact parameter(b), the participant elliptical and triangular eccentricities (*ε_2*,* ε_3*) are studied and presented graphically and quantitatively in this paper. The results for collisions of Pb-Pb are compared with other Glauber model calculations which agrees with the results from other studies to a great extent. In this simulation, a decline trend of* ε_2 *and* ε_3 *is observed with increasing Npart. The centralities of peripheral events (40 to 50 percent centrality) are* ε_2 *ranging from 0.5 to 0.4 and* ε_3 *being roughly 0.3. The ratio of* ε_3 *to* ε_2 *is roughly 0.5 at Npart=0 and 1 at Npart=416 (a hundred percent centrality), showing the significance of triangular flow of the area of intersection. The model can also be used to predict geometric quantities in the LHC experiments and other collision processes.*

Keywords: *Glauber Model, Monte Carlo, Eccentricity, Heavy ion collision, Pb nuclei*

1. Introduction

The study of individual particles can be rather complicated, however, the correlation between particles gives a clearer picture of the properties and geometric quantities desired. This paper studies Pb-Pb collisions to get a better understanding of matter in a Quark Gluon Plasma (QGP) state. The corresponding correlation studies measure the geometric quantities, i.e Fourier coefficients representing hydrodynamic flow of charged particles emitted from the QGP.

Quark Gluon Plasma is believed to be the initial state of the universe and has also been a condition to free the quarks. Scientists have been using heavy ion collision to create the condition of QGP. In those collisions, both nuclei are accelerated to extremely high energy and collide, freeing the nucleons from the nuclei and lead to a brief QGP state before those nucleons get together and form hadrons, which has also been believed to be the initial state of the universe. This brief QGP state has been a great interest of scientific study. Over years, experiments have found out some interesting correlations between particles being generated through the collision such as the directions in which the particles go. These correlations have a lot to do with the intersection geometry of the collision. Scientists once have thought that the initial geometry only contains elliptical flow. However, it has been recently proved that although elliptical flow is a dominant term of the angular correlation, other shapes are also present in the intersection area such as triangular flow[1]. This makes the study of initial intersection collision area essential. However, it is difficult to observe the colliding area through detectors, thus models have been built to simulate the collision process.

Glauber Models usually can be classified to two main types, as presented in ([2]), the first class is called “Optical Glauber” with randomly distributed radial coordinates and smooth density under Fermi density function. The second class referred to as Monte Carlo model is what allows us to run multiple trials to get numerical results. i.e geometric quantities presented in results section.

In this paper, we propose that there exists a strong inverse proportionality between the Number of participants of collision and the eccentricities and simulate collision based on Monte Carlo Glauber

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model. The inverse proportionality is confirmed in our simulation and other properties are also confirmed by other studies. At be- tween Npart = 100 and Npart = 200 and there is a spike at around Npart = 180. Possible explanations are discussed in discussion section. However, the general trend does agree with what [1] has presented. Another article [3] has confirmed non-monotonic behavior of MCGlauber eccentricity ratio, both at low and high collision energies of collision as well.

2. The Model

The MC Glauber model is fully based on computational algorithms. However, the model requires rigorous set-up conditions and associated assumption to achieve desired performance.

3) Nucleons in the nucleus are distributed according to 3D Woods-Saxon distribution.

4) At high energy state, the large momentum endows the nucleons the abilities to move in straight trajectory and do not change trajectory through collision.

5) No quantum mechanical interference or coherence effects are considered.

6) The probability of interaction is given by inelastic nucleon-nucleon cross-section.

7) It use random number generator to generate the initial positions of nucleons inside a nucleus.

8) In Glauber model, the distance between the centers of mass of the two colliding nuclei are called impact parameter b and the number of nucleons participate in collision is called number of participant(N_{part}). The Woods-Saxon distribution is given by[4] :

$$\rho(r) = \rho_0 \frac{1+w\left(\frac{r}{R}\right)^2}{1+\exp\left(\frac{r^2-R^2}{a^2}\right)} \qquad (1)$$

where the values of R, a and w for Lead are also given in Table I of the [4] which is shown in Fig.1.

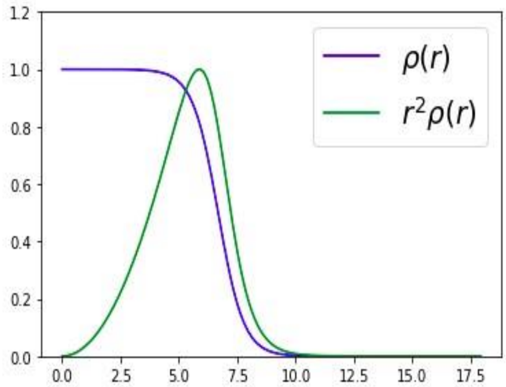

Fig. 1: Woods-Saxon distribution [5].

The green curve represents the actual probability distribution function from which 208 radii are drawn.

The diameter of the cross section, which also represents the probability of a collision just like cross section area, is called ball diameter, which is represented by [4]:

$$D = \sqrt{\frac{\sigma_{NN}}{\pi}} \qquad (2)$$

where σ_{NN} is the cross section, which only depends on the collision energy. Participants eccentricity ε_2 and ε_3 describe the extent of the plots of collision different from regular circle. ε_2 and ε_3 are driven from the equations below [1]

$$\varepsilon_2 = \frac{\sqrt{\left(\sigma_y^2-\sigma_x^2\right)^2+4\left(\sigma_{xy}\right)^2}}{\sigma_y^2+\sigma_x^2} \qquad (3)$$

where σ_x and σ_y are the standard deviations of the x and y coordinates of the participants in the center of mass coordinate and σ_{xy} is the co-variance. This in other patterns can be written as

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$$\varepsilon_2 = \frac{\sqrt{\left(r^2\cos(2\phi_{part})\right)^2+\left(r^2\sin(2\phi_{part})\right)^2}}{r^2} \quad (4)$$

and

$$\varepsilon_3 = \frac{\sqrt{\left(r^2\cos(3\phi_{part})\right)^2+\left(r^2\sin(3\phi_{part})\right)^2}}{r^2} \quad (5)$$

Where r, φ_{part} describes the location of the participants in the center of mass coordinate.

3. Method

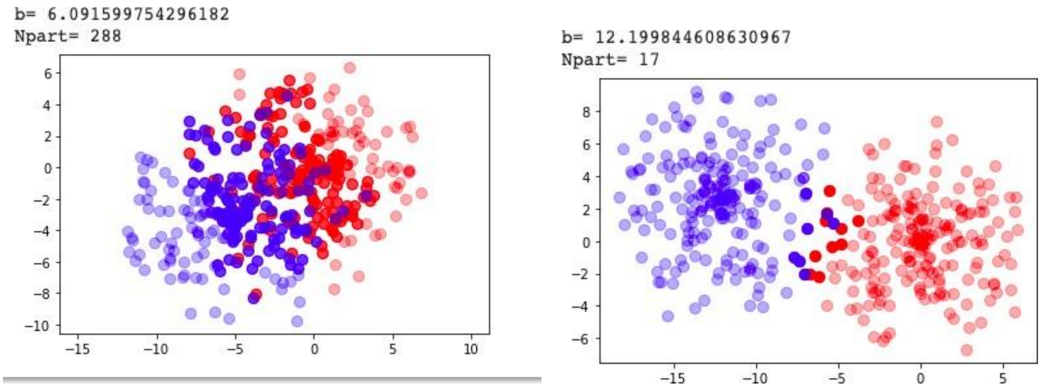
3.1 Set up for the collision

We used Python to simulate the collision process of two Pb nuclei[6]. The collision process is simulated based on Glauber Monte Carlo model. To determine the position of the nuclei, the center of one of the nuclei is set at (0, 0). The center of the second nucleus is randomly set within a 2D box with width 2 times 8 (we assume the radius of the nucleus is 8fm) centered at (0, 0). The 2D box represent an area from the transverse plane within which the two particles may collie. To make sure that every event represents an actual collision, all the cases that the distance between the centers of nuclei is bigger than 16 is discarded. Then, 208 nucleons are built around each center of the nuclei based on Fermi distribution. To do this, 208 r ranging from 0 to 8 based on Fermi distribution are randomly picked, with r representing the distance between each nucleon and the center of the nucleus. Then for each r, a nucleon with random spherical coordinate θ and φ is created. Although the nuclei are built in 3D, only the transverse plane is plotted.

3.2 The collision process

For any pair of nucleons in the collision, with one from one nucleus and one from another nucleus, both nucleons will be marked as participants if the distance between the centers of the two nucleons is smaller than the ball diameter. σ_{NN}=72mb [1] in our simulation, which means that the ball diameter is around 1.5fm. The model loops through each pair of nucleons and marks all the participants. The locations of all participants are also stored during the process.

Fig.2 shows the simulation of the two colliding nuclei at b = 12.20 and b = 6.09. The nucleon with a deeper color represents participants. Fig.3 shows the number of participants verses the impact parameter b. The number of participants decreases as the distance between the two nuclei increases, which matches our expectation.



(a)Collision simulation at b = 2. Nucleons with a deeper color represent participants.

(b)Collision simulation at b = 12. Nucleons with a deeper color represent participants.

Fig. 2: Simulation graph of the Collision

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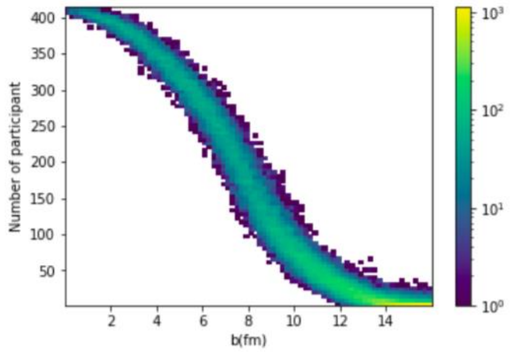

Fig. 3: Npart versus b, plotted from 100000 events.

3.3 Studying the participating area

With all the location information of the participants, the geometry of the participating area is calculated. For each event, the coordinate is shifted to the participants' center of mass coordinate and the corresponding r and φ for each nucleon are calculated. ε_2 and ε_3 are then calculated through equation 4 and equation 5. ε_2 and ε_3 verses the number of participants in each event are plot- ted, with the number of events equals to 100000, which is shown in Fig.3 and Fig.4. To investigate this more, ε_2 and ε_3 of the 40 – 50 percentage Npart (number of participant) are also plotted.

4. Results

Figure 4 shows distribution of geometric quantities of 100, 000 events along with the corresponding average values. It can be observed that the average of ε_2 and ε_3 decrease as number of participants increases. Also, the eccentricities are more wide-spread across the plane if there were fewer participants, which represents more fluctuation at smaller number of participants.

Figure 5 shows that 40 – 50 percent centrality falls in the range where Npart is between 84 and 102, which agrees with the result from other studies[7]. Within this range, ε_2 goes down from roughly 0.5 to 0.4 while ε_3 stays at roughly 0.3.

Fig.6 has shown that the ratio of ε_3 over ε_2 is roughly 1 at Npart=416 and roughly 0.5 as Npart approaches zero (the smallest number of Npart is 2 in this case).

5. Discussion

The distribution of ε_2 and ε_3 vs Npart looks quite similar to the result in the study done by B.Alver and G.Roland.[1], which also meets our expectations. The more the number of participants is, the closer the two nuclei are. Elliptical centrality ε_2 is determined by both the shape of the intersecting area as well as the fluctuation, while the triangular eccentricity ε_3 is determined only by fluctuation of the particles. The decrease of elliptical eccentricity ε_2 as Npart increases is because of the more circle-like shape of the intersection area as two nuclei become closer to each other. The decrease of triangular eccentricity as Npart increases shows that less participant causes more fluctuation. This agrees with our expectation since fewer participant will generate more uncertainty with the shape of the intersection area and thus higher fluctuation. This is also confirmed by the ε_2 vs Npart plot—there is a larger deviation at smaller Npart.

The intersection area is mostly a circle when Npart = 416, which means that all the nucleon collide with each other. Since the eccentricity of a circle is zero, at Npart = 416 all the elliptical eccentricity ε_2 comes from fluctuations, thus it should be equal to ε_3. While Npart is 2, the elliptical eccentricity should be 1 since the geometry in this case is a line. However, the fluctuation of the shape is large in this case, which impacts both ε_2 and ε_3, thus the ratio of ε_2 and ε_3 is more complicated. From Fig.4 we can see that the spike at roughly Npart = 180 is due to a slightly lower ε_2 and higher ε_3 at that spot, which requires further investigation.

The limitation and randomness of Monte Carlo method is the major source of systematic error. For instance, the function used to determine the location of each nucleon is numpy. random. However, the

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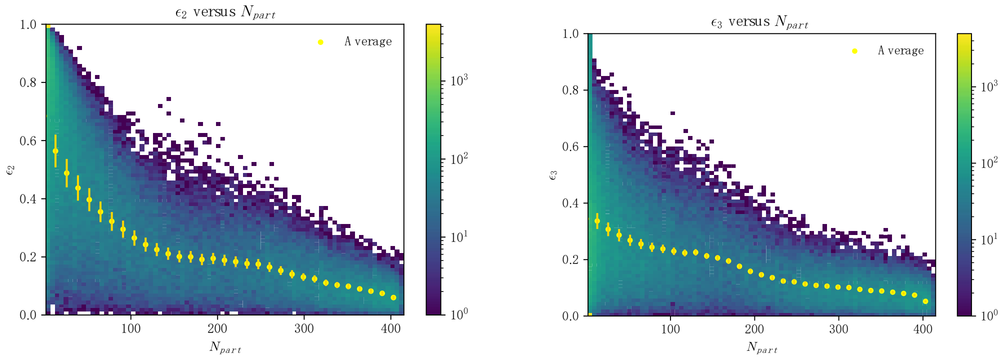
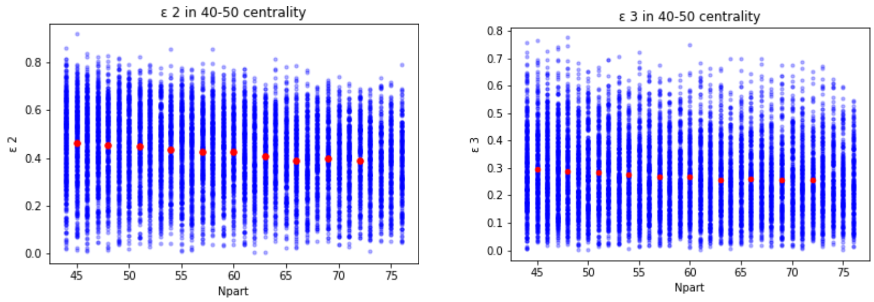
general distributions of ε_2 and ε_3 do not change significantly.



(a) ε_2 vs Npart for 100000 events, showing the distribution of impact parameter with respect to number of participants. The yellow dots represent the average eccentricities of 4 consecutive ε, error bars indicate statistical and systematic errors

(b) ε_3 vs Npart for 100000 events, showing the distribution of impact parameter with respect to number of participants. The yellow dots represent the average eccentricities of 4 consecutive ε, error bars indicate statistical and systematic errors

Fig. 4: Eccentricities versus Npart for 100000 events.



(a) ε_2 40-50 percentage centrality. Red dot is the average of ε_2 for three consecutive values of Npart

(b) ε_3 40-50 percentage centrality. Red dot is the average of ε_3 for three consecutive values of Npart

Fig. 5: ε_2 and ε_3 40-50 percentage centrality

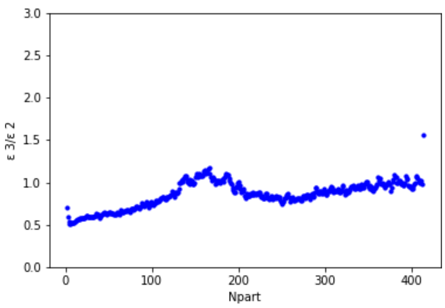


Fig. 6: ε3 to ε2 ratio vs the number of participants, showing the non-monotonic trend of eccentricity ratio

6. Conclusion

This paper presents two particle correlation studies of Pb-Pb collisions. The Monte Carlo method is implemented to obtain geometric quantities of two Pb nuclei i.e eccentricities and impact parameter. The eccentricities of the newcreated matter, belongs to collision overlap- ping region dictate collective flow anisotropy, which can be represented numerically by the detailed study of initial eccentricities. By graphical and numerical analysis of 10,000 events, we concluded that the eccentricities ε_2 and ε3 are related to the number of collision participants significantly with inverse proportionality. Furthermore, the trend agrees with the physical interpretation of ecliptic flow anisotropy, the shape of the tiled collision

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plane becomes more circular where there exist more participating nucleons.

We have also found that our implementation of the classical Glauber model behaves differently between Npart = 100 and Npart = 200, which requires further scientific investigation beyond the algorithm artifact. Our studies further validate the effectiveness of Monte Carlo implementation on two-particle correlation studies and also suggests a possible new pattern of non- monotonic eccentricity ratio providing a better under- standing of the initial states of lead nuclei. The result indicates the relative great fluctuation of elliptic characterization of initial collision geometry and collective expansion dynamics in heavy-ion collisions. The instability of the elliptic flow characterization can be improved by introducing new geometry such as triangular flow geometry as presented in [1].

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