

Analysis of High School Mathematics Teaching under Heuristic Teaching Model—Taking “Trigonometric Functions” as an Example

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Abstract: *With the continuous deepening of the new curriculum reform, developing students' mathematical core literacy has become an important goal of middle school mathematics teaching. Systematically cultivating middle school students' mathematical abilities and improving their ability to solve practical problems is a new challenge facing current middle school mathematics teachers. In the mathematics classroom teaching of middle school, the proportion of heuristic teaching is small and the heuristic teaching environment is lacking. However, high school trigonometric functions contain rich mathematical ideas and problem-solving methods. If only “transmissive” learning is used, it is impossible to organically integrate the heuristic teaching model with high school mathematics, which also has a great impact on the improvement of students' comprehensive abilities.*

Keywords: *Heuristic; High School Mathematics; Trigonometric Functions; Case Analysis; Translation into English*

1. Questions raised

Problem Statement The new mathematics curriculum standard for compulsory education in China proposes that teachers should guide students to recognize the connections between mathematical knowledge points, mathematics and other subjects and life, stimulate thinking interests, and inspire thinking directions through mathematics learning, so as to improve students' mathematical core literacy. Secondly, "heuristic teaching" is also the primary link in a lesson. A good start is half the battle, and it inspires us. The intrinsic purpose of heuristic teaching is to arouse students' attention, stimulate learning interest, quickly feel the mathematical knowledge points, and establish connections between mathematical knowledge. Its performance purpose is to enable teachers to enter the teaching role in a good state, and students to enter the classroom learning situation in a good state, thereby making the entire learning atmosphere positive and harmonious.

The concept of trigonometric function is relatively abstract, the difficulty coefficient is big, the formula reasoning is too complex, the memory and recites the content are many, this causes the student to have the resistance psychology easily to it, most of the students' understanding of the concept of trigonometric function is only superficial, even just rote memorization. Their understanding of the process of trigonometric function derivation is also quite vague, and their knowledge points are not clear enough, so a few days do not review will forget. In addition, from the current situation of high school mathematics, students for the use of trigonometric functions are deficient, it is difficult to relate knowledge points, encounter problems, it is difficult to immediately think of the use of trigonometric functions to solve problems; Most teachers use the “Sea of exercises” training as a means, but the long-term boring and monotonous problems will greatly hit the students' enthusiasm for mathematics, so we need to develop new teaching methods, and then optimize the teaching classroom, it is also the significance of this paper to explore and research heuristic teaching[1-3].

2. The heuristic teaching mode

2.1 The connotation of heuristic teaching

Article thirty-five of our compulsory education law clearly states that the state encourages schools and teachers to adopt heuristic teaching methods. Heuristic teaching means that teachers make use of the leading role according to the educational aim, the learning task and the learning law, the students

cognitive law, the knowledge base and the physical and mental development, using various vivid and flexible methods to stimulate students' intrinsic motivation to study and improve their ability of independent exploration and active research, encourage students to think, question, imagine, explore, argue and innovate, so that students can not only acquire knowledge but also develop their intelligence and discover, explore and solve mathematical problems, master problem-solving strategies, learn to use the mathematical knowledge and methods to solve practical problems, experience full of exploration and creative mathematical activities brought about by happiness and enthusiasm.

2.2 A method of heuristic teaching

There are four types of heuristic teaching: inductive heuristic, deductive heuristic, analogical heuristic, and experimental heuristic. Inductive heuristic is a way of teaching that emphasizes summarizing and generalizing, starting from specific examples or concrete situations. Learners can use inductive heuristic to intuitively summarize the common properties of multiple examples, conditions, skills, and problem-solving methods, and to induce new knowledge systems. It is widely applicable in mathematical principles, concepts, and formulas. In the classroom learning process of trigonometric functions, teachers can use inductive heuristic teaching to generalize the derivation of formulas, and then explain the generalization content.

Compared with inductive heuristic, deductive heuristic is a completely opposite way of teaching. It is a process from general to specific, from ordinary to special. In deductive heuristic, students are required to abstract concepts from intuitive content through logic. Compared with inductive heuristic, deductive heuristic is more suitable for deriving new theorems through definitions and axioms. Students need more time to understand, and guidance and hints from teachers are needed through class discussions.

Analogical heuristic is slightly different from deductive heuristic in its way of generalizing from specific to specific. Analogical heuristic requires learners to have a pair of eyes that are good at discovering similarities and differences between different objects, and then to use analogical thinking for inspiration. For teachers, they should first guide the objects to be compared, and then let students compare them to stimulate their initiative, so that students can explore and discover the characteristics of the objects to be compared, make hypotheses, reason, and finally verify the results.

Experimental heuristic requires students to verify the theorems and concepts in trigonometric functions by themselves, and to stimulate their experimental exploration ability by making abstract concepts concrete. With the help of computer technology, students can feel the changes in graphics well, and make abstract concepts concrete, thereby deepening their understanding of the theorems. Abstract mathematical concepts themselves are intangible, and students find it difficult to understand them. Therefore, for teachers, they should maximize their subjective initiative and lead students to make abstract concepts concrete, so that students can easily enter the world of mathematics[4-5].

3. Teaching analysis of trigonometric function in senior high school

Trigonometric functions are an important topic for high school students to learn and analyze. They are also the first periodic models that students encounter in high school, and as one of the basic elementary functions, they occupy an important position. They are also a necessary foundation for students to continue their studies in higher mathematics. However, in teaching practice, the numerous formulas and abstract definitions make it difficult for students to remember and easily confuse similar content, which makes it difficult for teachers to teach and the content becomes dull. Students find it difficult to understand the mathematical essence and lack initiative in learning. Under the new curriculum reform, using heuristic teaching methods to teach high school trigonometric functions has theoretical significance for improving the quality of high school mathematics education. Trigonometric functions are initially learned in junior high school, and in high school mathematics classes, the content of trigonometric functions is more in-depth, complex, and rich, and the knowledge points are more difficult to grasp, requiring higher learning thinking and problem-solving methods.

Trigonometric functions have three main characteristics: there are many relationships between formulas, and the transformation formulas are also complex. There are more than ten basic formulas alone, and due to the complexity of the relationships between formulas, memorization and recitation alone are not effective, which becomes a major bottleneck for students to use formulas for derivation. Secondly, trigonometric functions are the first many-to-one corresponding function and the first

periodic function involved in high school. Their teaching content contains rich mathematical ideas and problem-solving methods. The in-depth content of trigonometric functions is not only a challenge for students, but also a huge challenge for teachers to connect the content of junior and senior high school and transform students' learning methods. Thirdly, the definition of trigonometric functions plays a role in linking the past and the future in the high school curriculum. The mastery of trigonometric function knowledge not only deepens students' understanding of the connotation of functions, but also lays a foundation for the study of plane vectors, analytic geometry, and other content in the future.

4. The research of trigonometric function under the heuristic teaching mode

4.1 The existing problems at this stage

The first issue is that the heuristic teaching method is too singular, and the teaching atmosphere is not enthusiastic enough. Nowadays, many teachers lack innovative ideas, and schools pursue scores, so teachers adopt the tactic of drilling students with questions and stick to the original teaching methods, resulting in a single teaching method that is difficult to combine with heuristic teaching. On the one hand, it cannot achieve the effect of heuristic teaching, and on the other hand, it reduces students' enthusiasm for active learning. The heavy teaching tasks are also a reason for the singularity of heuristic teaching. The explanation of concept examples and the consolidation exercises of homework greatly reduce the class time, leaving limited thinking time for students, making it difficult to carry out heuristic teaching in the classroom[6-8].

The second issue is the lack of students' thinking construction and the neglect of students' subjectivity. In mathematics teaching, although there is the teacher's instruction and guidance, most of the time in the classroom is the teacher's analysis and explanation, and the interaction between students is poor, which overall lacks students' own thinking construction and cannot deeply grasp the intersection of algebra and geometry to improve mathematical ability. Heuristic teaching emphasizes that students are the main body and strengthens their initiative. However, in actual classrooms, many teachers still play the role of lecturers, and students find it difficult to participate, only playing the role of listeners. The "role exchange" between teachers and students is difficult to carry out in the classroom, and the "status" of some teachers in the classroom is still difficult to shake, ignoring the fact that students are the main body in the classroom.

The concept of trigonometric functions is relatively abstract and difficult. Many teachers only introduce the definition of trigonometric functions through simple explanations in teaching. Such teaching can only make students understand the whole knowledge system construction process of trigonometric functions in terms of concepts, which is an abstract process, but in fact, students lack personal understanding and discovery of the knowledge framework process of trigonometric functions.

The teacher's teaching lacks flexibility and maneuverability. In teaching, many mathematics teachers have the same problem, their language expression is very stiff and lacks affinity. The rigor of mathematical logic can make the thinking of long-term teaching teachers rigid, which in turn causes the language expression of teaching teachers to be stiff, coupled with the abstraction of mathematical knowledge, further reducing students' interest in learning. If teachers do not pay attention to the artistic and expressive aspects of their language, students' learning enthusiasm will gradually disappear. The charm of the teacher's language and appropriate guidance on the generalization of knowledge points in the classroom are very important, which can not only make students focus on what they are learning but also deepen their perception ability of activities. Teachers should not stop their thinking at the teacher's "teaching" and students' "listening," but ignore the fact that one of the major characteristics of heuristic teaching is students' autonomous learning and cooperative learning. Before teaching, teachers should guide students into the course of study, be a good guide, and discuss and explore with students during the learning process. Teachers should timely change their concepts, constantly explore ways to improve their teaching tasks, improve their professional quality, enhance their innovative spirit, and liven up the classroom atmosphere.

4.2 Teaching strategies

Based on the problems in heuristic teaching mentioned above, the following four teaching strategies are summarized:

Stimulate students' interest in learning and cultivate innovation spirit. Maintaining great interest

before, during, and after class will make students learn more efficiently. As mentioned earlier, the knowledge points of trigonometric functions are numerous and difficult, which leads to some high school students having a negative attitude towards trigonometric functions. Interest is a powerful internal driving force for students' learning. Activating students' learning interest is undoubtedly the key to improving the efficiency of trigonometric function teaching. Before class, students can be encouraged to find answers with curiosity and questions through "problem strings". The teacher can grasp the students' thinking focus through the setting of heuristic problem strings and the use of rich and varied body language, and the application of modern multimedia teaching methods can make heuristic teaching full of vitality, forming a variety of classroom teaching modes. It can not only stimulate students' learning interest, strengthen mathematical thinking, and broaden learning ability, but also guide and inspire students' autonomous and cooperative learning, which has an imperceptible effect on cultivating students' innovation spirit.

Guide students to learn autonomously and cooperatively. The main idea of the new curriculum concept is to guide students to learn autonomously and develop the habit of autonomous learning. The passive teaching method of "filling the classroom" is no longer suitable for modern middle school students. This not only limits students' thinking by the teacher's "teaching", but also does not give students the opportunity to express their thinking process, which greatly affects their thinking interest. For example, in the teaching design of the first section of the "double angle formula" in the trigonometric function chapter, students can be guided to review the sine, cosine, and tangent formulas of two angles and sums before thinking about the relationship between the double angle formula and the sum angle formula. In classroom learning, teachers should guide students' thinking and methods more, rather than "imparting" knowledge, and guide students to consciously think and digest knowledge. When an individual's power is limited, encouraging students to apply collective wisdom and brainstorming in discussions can help them learn and understand, and make progress in cooperation. Therefore, learning in cooperation and cooperating in learning will broaden students' thinking and improve learning efficiency significantly.

Improve teachers' teaching level and create personalized teaching. Teachers' teaching level is also reflected in their personalized teaching language. Concise and vivid teaching language always has endless appeal to students. When teaching, the teacher's language inflexibility will grasp students' desire to listen, and strengthening language expression ability is a basic ability that every teacher must practice. Mathematical concepts are the logical starting point of mathematics and the focus of mathematical thinking. Therefore, the teacher's clear logical thinking and strong summarization ability will inspire students to better and faster grasp knowledge points and find suitable learning thinking. Secondly, teachers should constantly innovate their own teaching methods to enrich teaching methods. For example, the content and formula of trigonometric functions are varied, and the types of questions are numerous. Students generally find it difficult to memorize each formula and are prone to confusion, which leads to incomplete understanding of concepts and difficulty in connecting content. By designing unique teaching cases and diverse teaching methods, teaching can be tailored to each student's individual characteristics, which can not only effectively improve teaching level but also create personalized teaching[9-10].

Optimize the teaching classroom and create a passionate teaching atmosphere. Creating a classroom connected to the outside world truly reflects that mathematical knowledge comes from real life and can serve reality well. Optimizing the teaching classroom and creating a passionate teaching atmosphere can tightly grasp students' thinking and exploration interests through appropriate topic introductions, giving students the opportunity to combine theory with practice, and setting carefully designed problem backgrounds, which will make the teacher's heuristic guidance more efficient. For example, when teaching "the graph and properties of sine curve", a "ECG" picture and a "clock's trajectory map" can be shown to students through multimedia courseware. The clock's motion change graph can give students a preliminary and intuitive understanding of the periodicity of the sine curve, but the irregular "ECG" can arouse students' curiosity and exploration of the "graph and properties of sine curve", creating a positive classroom exploration atmosphere. At the same time, through this lesson, students can further understand how to solve practical problems through mathematical modeling.

Therefore, inspiring students to observe and think about practical problems in life with a mathematical perspective, and analyzing and solving practical problems with mathematical knowledge, requires bringing life into the classroom. Teachers should use heuristic questions to give students enough time to think, create a warm teaching atmosphere, stimulate students' active participation in classroom activities, constantly inspire students' thinking, guide students at the appropriate time, use interactive classroom activities to make students learn to cooperate and share learning results, and

improve the efficiency of the entire classroom teaching. Heuristic teaching can guide students to participate actively, thus more effectively cultivating students' learning literacy and comprehensive abilities, and providing good conditions for the implementation of quality education.

5. Conclusion

This paper has conducted a preliminary study and exploration on heuristic teaching. Taking high school trigonometric functions as an example, it has considered and summarized some problems in the current heuristic teaching, providing theoretical concepts for the design of subsequent heuristic teaching models, and summarizing a set of feasible teaching strategies. Through the study of high school trigonometric functions under the heuristic teaching mode, the following two conclusions are drawn:

Firstly, the impact of heuristic teaching mode on students: it can strengthen students' mathematical thinking and improve their overall learning quality. Heuristic teaching can expand students' mathematical thinking. The use of heuristic teaching can stimulate students' learning interest and cultivate their innovative learning ability. It encourages students to become learners, practitioners, and researchers. Through well-designed heuristic teaching, students can truly become learners, practitioners, and researchers. In practice, teachers can create a vivid and rich problem scenario by digging deep into the textbook, revealing the knowledge background, and presenting new problems, allowing students to feel it firsthand and participate as a true subject. Through exploration, discovery, and hands-on experimentation, students can experience the hardships and repetitions of life, the atmosphere and essence of research, and practice true knowledge. Through repeated observation and experimentation, they can ultimately construct a new knowledge system and become true researchers. In the learning process, heuristic teaching can guide students to learn autonomously and cooperatively.

Secondly, the impact of heuristic teaching mode on teaching research: teachers' educational concepts have been updated and transformed. Under heuristic teaching, teachers can analyze and compare the advantages and disadvantages of different heuristic teaching methods, learn from their outstanding methods, and improve them according to the students' situation, integrating others' teaching experience and ultimately forming their own teaching methods that are suitable for their teaching practice. Teachers' teaching level is constantly improving, and their teaching skills are gradually maturing, forming a personalized and innovative teaching style.

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