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Analysis of Penrose Tile Network Based on Complex Network Theory

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Abstract: *Based on graph theory and complex network theory, this paper conducts an empirical study on Penrose floor tile network. In this paper, the complex network characteristics of Penrose floor tile network is analyzed by calculating the parameters of different static indexes. By formulating a random attack strategy, the influence of different node and edge failures of the characteristics of Penrose floors tile network is studied.*

Keywords: *Complex Networks, Penrose Tile Networks, Random Attacks, Robustness*

1. Introduction

Complex networks are widely used in economic networks, traffic networks and social networks. In the study of complex networks, the average path length, degree distribution and clustering coefficient are often used to describe the properties of complex networks[1].Because the graph provides a method of describing the network of points and lines, it has been generally considered as a method of studying complex networks. People explore the topological properties of the network by studying the nodes in the complex network and the edges to the nodes. People can compare the topological properties of different networks and reveal the common characteristics of various complex networks by using graph theory.

Penrose floor tile is a kind of floor tile invented by Penrose, which has only two basic shapes but can cover the whole plane periodically. Among them, aperiodicity refers to the absence of translational symmetry. Because of such special properties, Penrose floor tiles have been widely used for artistic research, but people rarely apply the method of complex networks to Penrose floor tiles.

A large number of empirical studies have shown that average degree, network efficiency, average clustering coefficient and average path length can be used as static evaluation indexes of complex network robustness. Feng et al. Simulated and analyzed the robustness of urban rail transit network of static and dynamic evaluation systems, and proposed two ways to enhance the robustness of urban rail transit network [2]. Lai constructed a robustness evaluation system suitable for subway lines by studying subway lines [3,4], and included connectivity, cycle rate and average clustering coefficient as evaluation indexes to study the robustness of subway networks. It is considered that the average clustering coefficient is more suitable for large-scale complex networks to study the robustness of subway networks. Zheng studied the Shanghai subway networks [5]. The research shows that the average clustering coefficient of Shanghai subway network is very small, does not have the characteristics of small-world network, and the inclusion degree distribution is used as a robustness evaluation system. Wang et al. Studied complex networks [6], and believed that the assortative or disassortative robustness of networks may have a significant impact.

Studies have shown that network topology characteristics such as network efficiency, connectivity, and average path length can measure the robustness of complex networks after being attacked [6-10]. These metrics can show changes in the network after being attacked and are used to comprehensively evaluate the strength of network stability.

Therefore, based on previous studies, this paper uses a static and dynamic robustness evaluation system. Firstly, the average path length, clustering coefficient and other static indicators of Penrose tile network topology is studied. Then, according to the change trend of dynamic indicators, this paper comprehensively analyzes the impact of various indicators on the robustness of the network.

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2. Network construction

A Penrose tiles network topology can be expressed as an undirected and unweighted graph, G=(V,E),where V denotes the set of nodes in the Penrose tile network topology, including N=|V| nodes, E denotes the set of edges in the Penrose tile network topology, including E=|M| edges, if there is an edge of nodes, then the nodes are connected to each other and can be reached. Therefore, we can construct the adjacency matrix A=(aij)nxn to represent the network of Figure 1, where.

$$a_{ij}=\begin{cases} 1,(V_i,V_j)\in G \\ 0,\text{else} \end{cases} \quad (1)$$

Using the method of adjacency matrix to represent a graph, people can know whether there are edges of two nodes, and study the topological properties of the graph by matrix analysis.

3. Network robustness evaluation index system

3.1 Static evaluation system

3.1.1 Average path length

The distance dij between two nodes i and j in the network is defined as the number of edges to the shortest path connecting the two nodes. The average path length L of the network is the average distance between all node pairs.

$$L=\frac{1}{N(N-1)}\sum_{i,j\in N,i\neq j} d_{ij} \quad (2)$$

3.1.2 Average degree and degree distribution

The degree is a physical quantity that describes the importance of different nodes in the network. The degree ki of the node i am defined as the number of edges connected to the node, and the average degree of the degree ki of all the nodes i in the network are called the average degree of the network, which is defined as <k>. In this paper, by sorting the degree values of nodes in the network from small to large, the proportion of nodes with degree k to the number of nodes in the whole network is obtained. The degree distribution of nodes in the network is represented by the distribution function p(k).

3.1.3 Clustering coefficient and average clustering coefficient

Clustering coefficient is a characteristic physical quantity that characterizes complex networks. Suppose that the degree of a node i in the network is k, that is, there is ki neighbor nodes directly connected with edges. When the number of edges is the largest, these neighbor nodes have ki(ki-1)/2 edges. The clustering coefficient ci is the ratio of the actual number of edges ei between all its neighbor nodes to the total maximum possible number of edges ki(ki-1)/2, that is.

$$c_i=\frac{2e_i}{k_i(k_i-1)}=\frac{\sum_{j,m} a_{ij}a_{jm}a_{mi}}{k_i(k_i-1)} \quad (3)$$

Where aij is the adjacency matrix of the network. The sum of the clustering coefficients of all nodes in the network divided by the total number of nodes is called the average clustering coefficient of the network, which is defined as C, that is.

$$C=\frac{1}{N}\sum_{i\in N} c_i \quad (4)$$

3.1.4 Network efficiency

Compared with the average path length, the global efficiency of the network can accurately reflect the change of network connectivity after the network is attacked. Global efficiency is proportional to connectivity. Therefore, if the network is attacked, the connectivity of the network will decrease, and the global efficiency will also decrease, and the robustness will become very low. The mathematical expression of global efficiency Eg is as follows:

ISSN 2616-5805 Vol. 4, Issue 2: 19-25, DOI: 10.25236/AJMS.2023.040203

$$E_g=\frac{1}{N(N-1)}\sum_{i\geq j}\frac{1}{d_{ij}}. \tag{5}$$

3.1.5 Assortativity coefficient

Assortative coefficient is used to measure the relationship between connected node pairs. There are many methods to calculate the assortative coefficient, which is solved by the degree correlation function.

We can use the difference between e_{ij} and q_iq_j to represent the degree of assortativity or disassortativity of the network, and construct the following degree correlation function:

$$<ij><i><j>=\sum_{i,j} ij\left(e_{ij}-q_iq_j\right) \tag{6}$$

Among them, qi,qj represent the degree of two nodes vi and vj connected by an edge eij, and<ij>,<i>,<j>, are the expression of average degree.

When the network reaches complete assortativity, the above equation reaches the maximum, which is the variance of the redundancy distribution qj:

$$\sigma_q^2=\sum_j j^2 q_j^2-\left[\sum_j jq_j\right]^2 \tag{7}$$

Then the assortative coefficient is obtained as follows:

$$a=\frac{1}{\sigma_q^2}\sum_{i,j} ij\left(e_{ij}-q_iq_j\right) \tag{8}$$

It is easy to know that a∈[-1,1]. If a>0, then the network is assortative; if a<0; then the network is disassortative. The size of |a| can reflect the degree of network assortative or disassortative.

3.1.6 Weight distribution difference

The number of nodes on edge is expressed as the edge weight wij, and the point weights $S_i = \sum_{j\in N_i} w_{ij}$ of the node vi. Using the reciprocal X of the weight distribution difference ri to measure local robustness.

The mathematical formula for X is as follows:

$$X=\frac{N}{\sum_{i=1}^{N} Y_i}=\frac{N}{\sum_{i=1}^{N}\sum_{j\in N_i}\left(\frac{w_{ij}}{S_i}\right)^2} \tag{9}$$

3.1.7 Connectedness

When the network is attacked, its connectivity is proportional to its resilience. Connectivity represents the ratio of the actual number of edges of the network of the theoretical maximum number of edges. Because the connectivity is oriented to the global network, rather than the domain of a node in the network, it is different from the clustering coefficient.

The mathematical calculation formula of connectivity is as follows:

$$d=\frac{|M|}{3|N|-6}. \tag{10}$$

3.1.8 Cycle rate

Network robustness reflects the ability of the network to provide alternative paths. The number of cycles μ in the Berge definition is used to represent the available alternative paths:

$$\mu=|M|-|N|+1 \tag{11}$$

When the network scale expands, the number of cycles will increase simultaneously, but this does

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not mean that the robustness of the network will also increase. Because when the network size increases, the number of nodes will also increase, so the probability of being attacked by the network will increase. Therefore, this paper uses the cycle rate to reflect the robustness of the Penrose floor tile network.

$$\mu^T = \frac{\mu}{|N|} \tag{12}$$

3.1.9 Robustness

Robustness means that when the network is attacked, some characteristics of the network, such as connectivity, still exist, which can show that the connectivity of the network is robust to network attacks. Robustness is defined as r.

The mathematical formula for robustness is as follows:

$$r = \frac{ln(M-N+2)}{N} \tag{13}$$

3.2 Dynamic evaluation system

3.2.1 Maximum connected subgraph size

The maximum connected subgraph is a subgraph formed by connecting all nodes with the least edges. The relative size of the largest connected subgraph are the ratio of the number of nodes in the connected subgraph to the number of all nodes in the network. The calculation formula for the largest connected subgraph is as follows:

$$S = \frac{|V'|}{|V|} \tag{14}$$

Here, |V’| denotes the number of nodes in the largest connected subgraph of the Penrose tile network topology after a network attack.

4. Network robustness analysis

4.1 Static index analysis

According to the definition and description of the previous section, this paper studies the static statistical parameters of Penrose floor tile network. In this paper, 126 nodes and 227 edges in the Penrose floor tile network are selected to construct the adjacency matrix. The nine static indicators mentioned above are calculated and the degree distribution map is depicted, as shown in Figure 3. The results are shown in Table 1.

Table 1: Penrose networks static statistical parameter table

parameter	L	$<k>$	C	E_g	$\|a\|$	X	d	μ^T	r
value	6.2672	3.6111	0	0.2181	0.2617	40.427	0.6102	0.8095	0.0368

According to the calculation results of Table 1, the average path length of the network is 6.2672, which is low and has the characteristics of small world network. According to the definition of clustering coefficient, the clustering coefficient of Penrose floors tile network is 0. The assortative coefficient of the network is 0.2617, Indicating that the network is assortative. According to the number of cycles and robustness, it can be preliminarily considered that the network has good robustness.

Secondly, it can be seen from Figure 2 that the nodes with a network degree of 3 account for the main part, and the average degree is 3.6111, Indicating that the nodes can reach each other through three edges.

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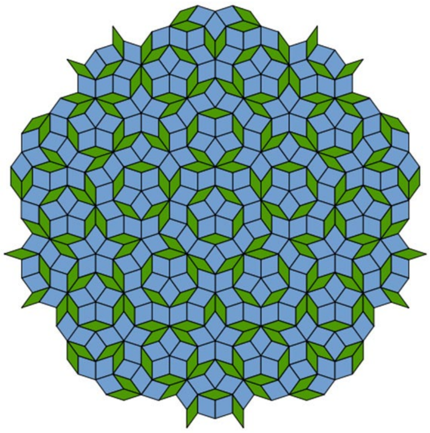

Figure 1: Penrose floor tiles

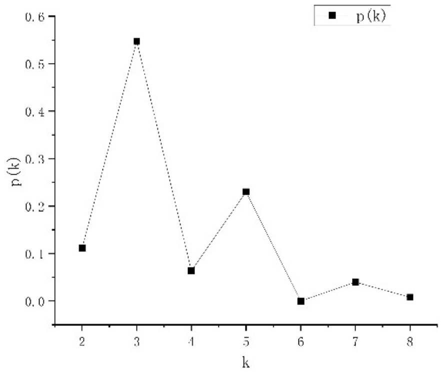

Figure 2: Degree distribution map

4.2 Network attack and analysis

4.2.1 Network attack strategy

The Pengluosi floor tile network has a large scale of nodes, a large number of edges, and a very average between nodes, with no particularly prominent key nodes. Therefore, only random attacks are performed on the network.

Before the network attack, make the following settings:

a. In order to observe the change process of each index of Penrose network more specifically, this paper attacks the nodes and edges of the network respectively, and only attacks one node or one edge at a time.

b. 50 attacks on the network.

4.2.2 Dynamic indicator analysis

In this paper, the nodes and edges of Penrose floor tile network is randomly attacked, and the robustness of Penrose floors tile network is comprehensively evaluated.

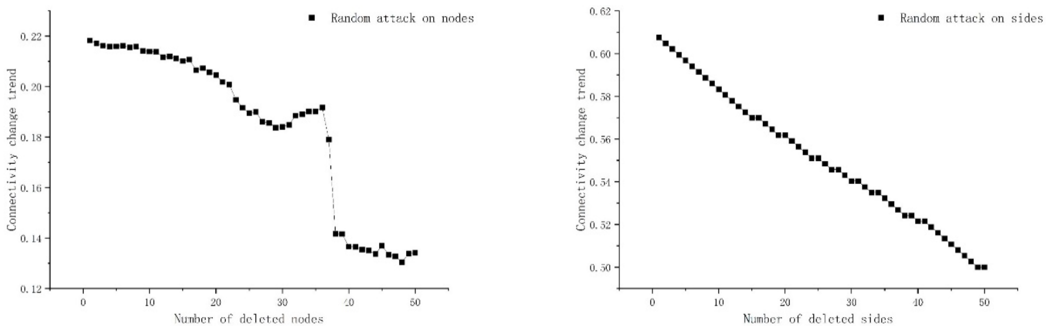

Figure 3: Connectivity changes curve under random attack

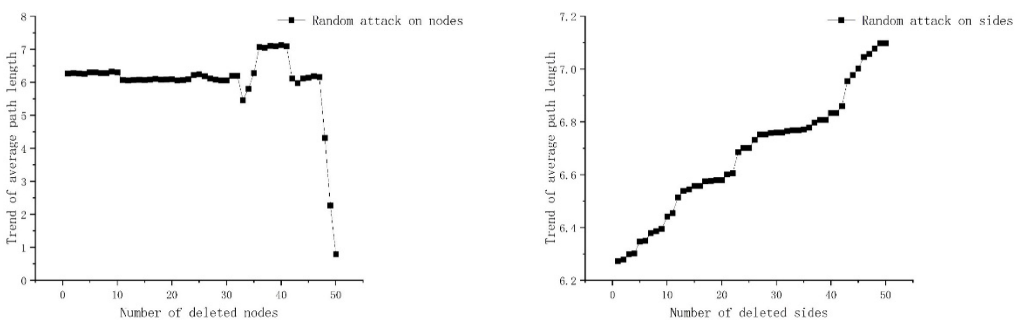

Figure 4: Average path length curves under random attack

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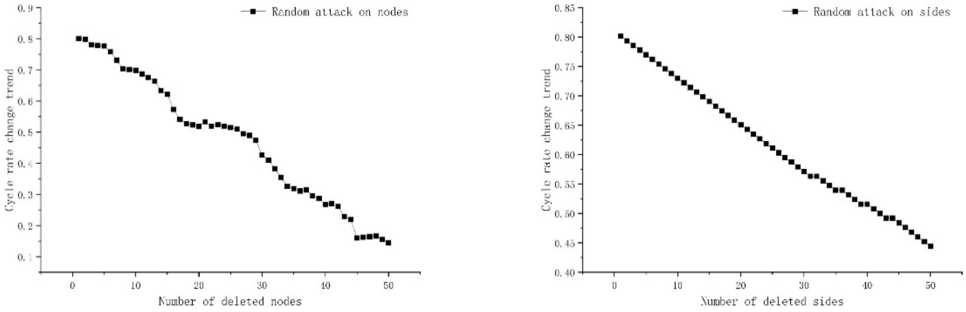

Figure 5: Change curve of cycle rate of random attack

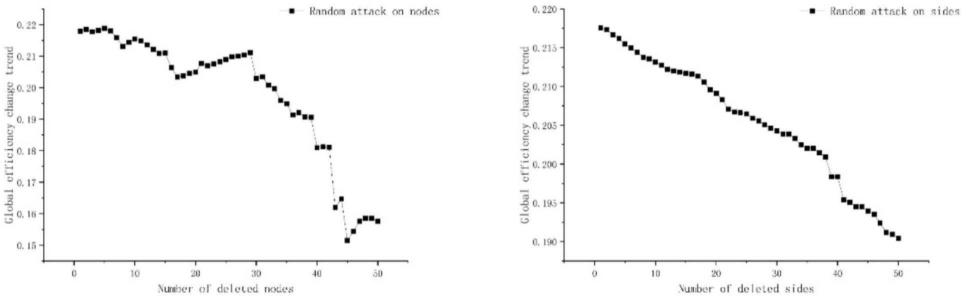

Figure 6: Global efficiency changes curve under random attack

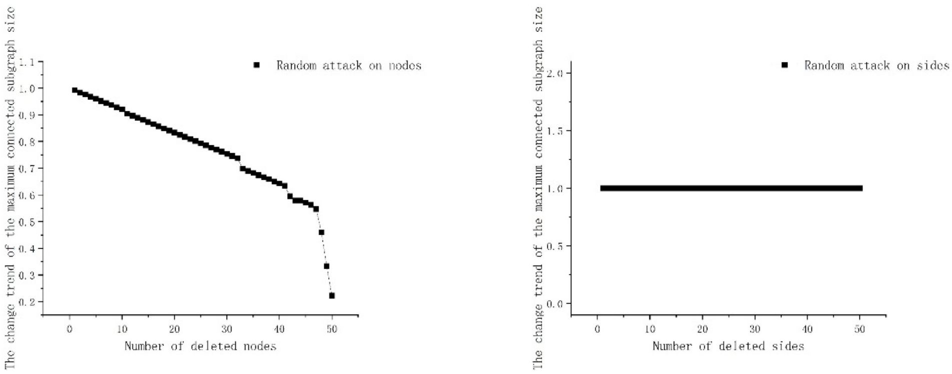

Figure 7: The maximum connected relative size change curve under random attack

It can be seen from Figure 3 that after the Penrose floor tile network is attacked by random nodes and random edges respectively, the connectivity shows a different curve. With the edge attack, the connectivity of the network shows a linear downward trend. Under the node attack, after deleting 3X nodes, the connectivity suddenly drops to a certain low point. At this time, the Penrose brick network is close to collapse, indicating that the network is very robust in the early stage of the attack.

From Figure 4, It can be seen that the random attack of the edge makes the average path length of nodes gradually increase, but does not show a downward trend, which indicates that the nodes are still interconnected. However, the attack destroys the shorter connection route, indicating that the original network has a certain small-world nature. Since the average path length value increases first and then decreases, it shows that most nodes in the network are disconnected after deleting 50 nodes.

From Figure 5, it can be seen that both random edge and node attacks show a gradual downward trend, but the impact on obvious node attacks on the overall network is greater than that of edge attacks. As the attack intensifies, the robustness of the network gradually decreases.

It can be seen from Figure 6 that the random attack of the edge gradually reduces the global efficiency, but after the random attack on the node, the global efficiency shows a partial upward trend after the decline, indicating that as the number of deleted nodes increases, the connectivity of the network can be improved in a small range.

It can be seen from Figure 7 that the relative size of the maximum connected subgraph of the edge attack are always 1, Indicating that the deletion of 50 edges will not affect the connectivity of the network, and the nodes are still interconnected. The random node attack makes.

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5. Conclusion

From the perspective of complex network, this paper makes an empirical study on Penrose floor tile network. In this paper, the robustness of Penrose network is evaluated by using statistical parameters such as degree distribution, average path length, average clustering coefficient and relative size of maximum connected subgraph. The results show that the Penrose floor tile network has low clustering coefficient and long average path length, that is, there is no small world effect, and the degree distribution obeys the law of piecewise distribution. In this paper, node attacks and edge to attack are carried out on the network respectively. It is found that Penrose network has strong resistance to edge to attack, and connectivity is not easily affected. However, it has weak resistance to node attack, and many nodes have no connected edges.

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