

Applications of Ordinary Differential Equations in Mathematical Modeling

Meng Liu

JiaXing NanYang Polytechnic Institute, JiaXing 563000, China

ABSTRACT. *Ordinary differential equations are the process of turning practical problems into mathematical languages. They can simplify the processing of problems and promote the solution of problems. They are an important tool for linking mathematical theory with practice. Based on a brief description of mathematical modeling, this paper proposes the method steps of establishing an ordinary differential equation model, and combines the practical exploration of the application of ordinary differential equations in mathematical modeling to provide guidance for similar research.*

KEYWORDS: *Mathematical modeling, Ordinary differential equations, Applications*

1. Overview of Mathematical Modeling

1.1 Mathematical Modeling

Mathematical modeling mainly analyzes complex phenomena, uses mathematical language to describe the relationships and laws, and gives appropriate mathematical relationships, and then uses mathematical methods to solve practical problems [1]. This process is mathematical modeling. Unlike mathematical calculations, mathematical modeling requires refining, induction, summarizing, and deductive reasoning. The key to mathematical modeling is how to convert actual problems into mathematical relationships. The process of solving actual problems is the ultimate goal of mathematical modeling. The final part of mathematical modeling is to check the results. Only when the requirements of the actual problem are met can the correct solution result be obtained.

1.2 The Role of Mathematical Modeling

(1) Mathematical modeling can enable students to experience the connection between mathematics and daily life and other disciplines. It can enable students to understand the application value of mathematics and cultivate students' mathematical consciousness, so as to stimulate students' strong desire to love

mathematics and to learn mathematics.

(2) The teaching of mathematical modeling embodies the cultivation of multiple abilities, such as the ability to express mathematical languages, the ability to use mathematics, the ability to communicate and cooperate, the ability to mathematically imagine, and the ability to create. Thirdly, mathematical modeling creates the time and space for students to participate in inquiry, allowing students to actively learn ways to acquire mathematical knowledge on their own, learn the ability to actively participate in mathematical practice, and then gain the ability to use mathematics and social activities for life. Learning subject.

(3) The purpose of quality education is to “cultivate students' creative ability and practical ability”. For the application of mathematics, we should not just think of it as a simple application of knowledge. Mold process implementation and operation. The author believes that in order to reflect the application value of mathematics, it is necessary to have the ability to build mathematical models.

2. Method Steps for Establishing an Ordinary Differential Equation Model

2.1 Establishing Ordinary Differential Equation Models Based on Known Basic Laws

In the process of establishing the ordinary differential equation model, the known theorems and laws in various disciplines are mainly used, such as Newton's second law of motion, law of universal gravitation, Fourier's law of heat conduction, Hooke's law in elastic deformation, Terry's law, Aki Mead's law, decay rates in radiological problems, biology, economics, and growth rates in population problems [2].

2.2 Definition of Derivatives

The definition of the derivative is:

$$dy - dx = \lim_{\Delta x \rightarrow 0} f(x) + \Delta x - f(x) - \Delta x = \lim_{\Delta x \rightarrow 0} \Delta x - \Delta y$$

If the function $f(x)$ is differentiable, then $\frac{\Delta x}{\Delta y}$ it can be interpreted that y is

fairly equivalent to the instantaneous rate of change of x at that point. It is mainly applied to the “rate” and “growth” that appear in biological and population research, the “decay” that appears in radiation problems, and the “margin” that appear in economics.

2.3 Differential Method to Establish Ordinary Differential Equation Model

This method is mainly to find the relationship between microelements, and

directly apply the relevant laws to the function to build a model. Suppose that the variable I in a practical problem meets the following conditions: I is a quantity related to the variation interval $[a, b]$ of an independent variable x ; I $\Delta I - i \approx f(N_i)\Delta x_i$ is additive to the interval $[a, b]$ [3]; a partial quantity, then We can consider the use of differential equations to establish ordinary differential equation models. The establishment steps are: according to the specific situation of the problem, select an independent variable x , and determine its change interval as $[a, b]$; select any one of the intervals in the interval $[a, b]$ and record it as $[x, x + dx]$, find the nearsighted value corresponding to the partial quantity ΔI in this interval, and express the approximate value of ΔI as the product of the value $f(x)$ and dx of a continuous function at x , that is, $\Delta I \approx f(x) dx$, $f(x) dx = dI$, dI is called the element of the quantity I , and the two sides of the equation can be integrated at the same time to obtain the required quantity I .

3. Ordinary Differential Equations Applied to Mathematical Modeling

3.1 Application of Ordinary Differential Equations to the Model for Forecasting Corruption

In the current search and arrest of a large number of corrupt officials involved in the crime, ordinary differential equations can be used for mathematical modeling. To this end, ordinary differential equations can be used for mathematical modeling and innovation. The number of involved persons is used to predict the total number of persons involved, and a new model for predicting the number of corrupt people is established, which includes the following three steps. .

(1) Hypothetical stage. Let t be the time, $x(t)$ be the function of the total number of persons involved in the corrupt group involved in t , X_0 be the total number of persons involved in the corrupt group at time $t = 0$, and $r(x)$ be the involved The growth rate of the elements involved, r represents the growth rate of the number of persons involved in the case at time x_0 , also known as the inherent growth rate, x_m represents the maximum number of people that may be involved in this corruption event, μ indicates the resistance coefficient generated during the tracing, $i(t)$ indicates the proportion of the number of people involved in this corruption event among the total number of people, λ indicates the proportion of the number of people involved in this corruption event at $t = 0$, λ indicates The average number of confessed members of each corrupted person caught within a month[4].

(2) Analysis stage. If the number of corrupt elements already involved shows an increasing trend, it means that the number of potential corrupt elements is gradually decreasing. $x(t)$ represents the functional relationship between the number of people involved in this corruption event and time t , $x(t)$ is a continuous function related to t , one of which is x_m , and the growth rate $r(x)$ corresponding to the number of people There is also a specific functional relationship with $x(t)$. From the previous hypothesis, $r(x)$ is a linear function of x , $r(x) = r - kx$ (k represents the slope, $k > 0$).

When $x = x_m$, the growth rate of the number of people involved is 0, $r(x_m) = 0$, so that $k = r / x_m$ can be determined, then the growth rate function of the number of people involved can be used: $r(x) = r(1 - x / x_m)$.

(3) Calculation phase. Without considering the intensity of the reconnaissance and the difficulty of the reconnaissance, which may affect the results of the reconnaissance, the following differential equations can be established[5]:

$$dx / dt = r(1 - x / x_m) x$$

$$x(0) = x_0$$

$$\text{The solution is } x(t) = \frac{x_m}{1 + \left(\frac{x_m}{x_0} - 1\right)e^{-rt}}$$

Considering that the difficulty of investigation may affect the investigation results, the coefficient of resistance can be set to establish the following differential equation:

$$di/dt = \lambda i(1-i) - \mu i$$

$$i(0) = i_0 \quad (\lambda \neq \mu)$$

$$\text{The solution } i(t) = \frac{1}{\frac{\lambda}{\lambda - \mu} + \left(i_0 - \frac{\lambda}{\lambda - \mu}\right)e^{-\lambda t}}$$

This mathematical model can be used as a method for anti-corruption departments in China to predict the number of corrupt members involved in future anti-corruption work. It is not difficult to find that the theoretically calculated number of corrupt persons and the number of corrupt persons found in actual work has very similar error ranges.

3.2 Ordinary Differential Equations Applied to Population Prediction Models

If all the factors are taken into account at the initial stage, then building the model is naturally impossible. Therefore, the problem can be simplified first, and a rough mathematical model can be established and gradually modified until a perfect mathematical model is obtained. Weirhurst introduced the constant N_m into mathematical modeling and used it to indicate the maximum population that the unnatural environment can tolerate [6]. Generally speaking, the higher the level of industrialization in a country, the larger the living space in that country, [7] and the larger the N_m . Weirhurst assumes that the growth rate can be expressed as $r(1 - N_t / N_m)$, and the net growth rate will gradually decrease as N_t increases. When N_t gradually approaches N_m , the net growth rate will be gradually approach zero, using this assumption can build a population prediction model. Therefore, we can use Weirhurst's theory to innovate and build a new population prediction model.

$$dN / dt = r(1 - N / N_m) N$$

$$N(t_0) = N_0$$

The mathematical model established by this ordinary differential equation is a logical model, which can be processed with separated variables and the solution is:

$$N(t) = \frac{Nm}{1 + \left(\frac{NM}{N_0} - 1\right)e}$$

Based on this population forecasting model, combined with Welhurst's related theory, a reasonable forecast of population growth can be made.

4. Conclusion

In summary, the article mainly analyzes the application of ordinary differential equations in mathematical modeling in detail, improves some previous mathematical models, and uses the ordinary differential equations to creatively design some new mathematics. The model is used in the research of different fields. In the future, more in-depth research will be carried out to create more mathematical models to solve some complex issues in society.

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