A Case of Reverse Instructional Design Based on Ubd Theory: Take "Set" as an Example

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Abstract: *The study of set mainly includes definition, characteristic, relation, classification, common number set and representation, is the basis of modern mathematics, is also the basis of high school mathematics learning. After the proposal of core literacy, in order to make students' core literacy in the classroom landing root, the innovation of teaching methods has become an indispensable part of high school teaching. The teaching method based on UbD theory is the innovation and orderly reconstruction of the traditional teaching method. It emphasizes students' understanding of knowledge, and can be fully implemented in the classroom, so that the cultivation of core literacy can really take root. This study takes the collection as an example and makes a case analysis of teaching design based on the UbD theory, aiming at providing a new way of thinking for middle school teachers' teaching.*

Keywords: *UbD; Instructional design case; Collections*

1. Introduction

After the concept of "literacy" was proposed, some western countries began to study the connotation and framework of core literacy. Their existing thinking provided valuable experience for the research of the core literacy in our country. Subsequently, Opinions on Comprehensively Deepening Curriculum Reform and Implementing the Fundamental Task of Cultivating Virtues and Educating People published by the Ministry of Education in 2014 put forward the concept of "core literacy". After the concept was put forward, education researchers and front-line teachers attached great importance to it. Then the Curriculum Standard of Mathematics in Senior High School (2017 edition) analyzed and answered the question of what is the core literacy of mathematics from six aspects. However, how to implement the basic requirements of the core literacy? The key is classroom practice. In the past, more teachers' teaching was placed in the dominant position, reflecting students' passive learning, but this way of teaching is not conducive to the realization of students' real development, does not meet the requirements of students' future development. Therefore, the innovation of teaching methods has become an indispensable part of senior high school teaching.

Students' active understanding and construction is the key to the implementation of core literacy in the classroom, as well as the key to reflect the student-oriented teaching. Different from traditional teaching methods, this comprehension-based teaching method should first analyze students and teaching objectives, and make clear what knowledge students should learn and what expected results they should achieve. Then, appropriate evaluation methods and standards are formulated to test whether students can transfer knowledge based on understanding, so as to master the knowledge. Finally, classroom teaching is implemented based on the above two stages, which has achieved the effect of optimizing teaching. The teaching design based on UbD theory is just in line with this teaching method, and can be fully implemented in the classroom, so as to make the development of students' core literacy take root, and provide reference for the teaching of front-line teachers[1].

"Set" belongs to the content of the first chapter of Compulsory Mathematics of High School A Edition of People's Education and belongs to the big concept category of "Mathematical language". It provides a language tool for the research object, and also lays a foundation for the learning of functions and other knowledge in the future. It is the foundation of modern mathematics and the foundation of the whole high school mathematics learning. Learning set can improve the rigor and accuracy of students to deal with problems, and is conducive to the implementation of students' core quality of mathematics abstraction, so the study of set is essential. The theory of UbD is just suitable for the chapter of sets. The

research takes the concept of sets as an example, and based on the theory of UbD, the teaching design case analysis of the content of this section is carried out.

2. UbD Theory

2.1. Overview of the Theory

Since 1998, when Grant Wiggins co-authored the Understanding by Design series with Jay Mc Tighe, the model has been continuously updated and is simply known as the UbD model[2]. From the perspective of lexical meaning understanding, UbD model emphasizes "understanding" and "design". Later scholars (such as Lai Lizhen, Hou Qiuling, etc.) translated UbD as "curriculum design emphasizing understanding", which is more in line with the original meaning. However, when scholars (such as Yan Hanbing, Sheng Qunli, etc.) translate UbD into "instructional design that seeks understanding" and "instructional design that puts understanding first", they also emphasize understanding and design and apply them in teaching. Different from traditional teaching methods and ways of thinking, UBD-based instructional design is a kind of reverse design, emphasizing students' understanding and evaluation of students in many aspects. The instructional design idea based on this theory is: if the desired outcome is learner understanding (big concepts) and thoughtful thinking (core issues), then the instructional process design needs to demonstrate the learner's ability (6 levels of understanding) from the seven core principles (WHERETO), and evaluation needs some visual evidence (performance tasks)[3].

2.2. Theoretical Advantages

In recent years, in order to pursue the implementation of core classroom literacy, UbD theory has been paid more and more attention and in-depth study by scholars. UbD theory emphasizes that students can truly understand knowledge and transfer what they have learned, including the transfer of knowledge and skills, as well as the transfer of thinking methods and strategies. UbD theory also emphasizes that evaluation precedes activity design, and adopts continuous tracking evaluation to form a long-term, unstructured, real situation and complex continuous evaluation system, including informal examination, observation and dialogue of understanding, in-class tests and examinations, question and answer questions, and expressive tasks[4]. Expressive tasks are the characteristics of UbD theory, the so-called expressive tasks. It refers to a task that tests students' knowledge and ability in a situation very similar to the real world situation. This theory can be used in the design of a lesson, a unit or a book, reflecting the integration of teaching evaluation.

3. Instructional design cases

3.1. Design Concept

"Common High School Mathematics Curriculum Standards (2017 edition, 2020 revision)" points out that "mathematics is not only a tool of calculation and reasoning, but also a language of expression and communication". Through the process of mathematical abstraction, "mathematics becomes a highly generalized, accurate expression, general conclusion, ordered and multi-level system"[5].

For the section "The Concept of Set", based on the UbD theory, the expected result is first determined, and the teaching objectives and basic problems are designed from the expected result.

3.2. Analysis of teaching content

This lesson is the content of the first section of the first chapter of Compulsory A version of Human teaching. As a language tool, set is the preparatory knowledge for high school mathematics learning, and it is an important knowledge that reflects mathematics abstraction. From the knowledge point of view, set is located in the first chapter of high school mathematics, throughout the whole high school mathematics, lays the foundation for the later learning, plays a leading role in the arrangement of textbooks; From the perspective of method, in the concept of set, the classification thought is the basis for the generation of definition, and the invariance and regularity contained in the phenomenon are abstracted through real life examples. Mathematical abstraction is the core basic idea of the whole set definition. Understanding concept teaching through mathematical abstraction, with the help of the examples of the use of the set in life, the teaching infiltrates the thought and method of classification, so

as to develop the core quality of mathematical abstraction of students. The new curriculum standard does not specify clear requirements for this lesson. But I think the requirements of this class are: master the definition and letter representation of elements and sets; To understand the relationship between elements and sets; Understand the properties of the elements in the set; Understand the classification of sets and common sets of numbers and their notation; Master the three methods of representation of sets. Based on the analysis of the above content, the teaching focus of this lesson is determined to master the concept of elements and sets, and master the three representation methods of sets.

3.3. Analysis of learning situation

The object of this lesson is senior one students. Students have been exposed to some sets and mastered some classification methods in junior middle school, but they have not systematically learned the language and tools to describe the mathematics content. At this stage, students have active thinking, strong desire for knowledge and performance, and can actively participate in class discussions. However, they lack abstractness in thinking, so they need to be guided by teachers.

According to the analysis of the school situation, the teaching difficulties of this lesson are determined as follows: teaching difficulties: the relationship between elements and sets; Three ways to express the set.

3.4. Teaching Strategies

The teaching of the new curriculum is student-centered, advocates students' independent learning and exploratory learning, requires teachers to become the guide, organizer, collaborator and facilitator of students' learning, and makes the teaching process a process of teacher-student communication, active interaction and common development. This lesson carefully targeted at the key and difficult points, adopting the teaching methods of allowing students to explore independently, cooperate and communicate with each other as well as teachers' inspiration and guidance, and assisted teaching with multimedia means, enabling students to experience the basic process of rational thinking such as observation, analysis, abstraction, induction, analogy, communication and reflection, so as to effectively improve students' learning methods and make them truly become the master of learning. The specific practices are as follows:

1) In order to fully mobilize the enthusiasm of students in learning mathematics, take the two controversial issues that students have learned as the introduction, take the practical problems as the guidance, inspire and induce learning to think positively and participate in mathematical activities; 2) Make use of modern educational means, PPT to present teaching content and improve teaching efficiency; 3) After learning the knowledge of this section, set performance tasks to test whether the students' understanding is profound; 4) Introduce the origin of set theory, let students feel the greatness of mathematicians and mathematics culture extensive and profound, stimulate students to continue to study and research mathematics with high motivation, as shown in Figure 1.
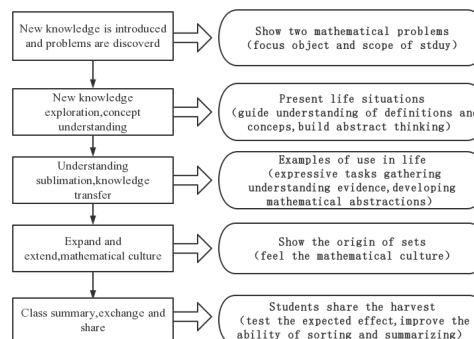
3.5. Basic teaching ideas

Figure 1: The basic idea of "set" teaching

3.6. Teaching Process

Table 1: Teaching Process

Section One: Introduce new knowledge and find problems
Why do we need to learn collections? Let's look at two questions first: Question 1: Is there a solution to $x^2=2$? Question 2: What is the graph formed by points whose distance from a fixed point is equal to a fixed length? Through discussion between teachers and students, it is found that different research scopes will lead to different research results. Why the different results? We need to use today's set to explain. Set is a language tool, in order to succinctly and accurately express mathematical objects and mathematical scope, is the foundation of high school learning, and lays the foundation for the learning of later chapters. So what is a set? Let's start today's lesson: the concept of a set. Design intention: Through two questions, arouse students' interest in continuing to explore, at the same time show the rigor of mathematics, and then introduce a new topic "the concept of set", play the role of "attracting, focusing and pointing".
Section two: New knowledge exploration, concept understanding
Problem situation: A store went to purchase some of the following products, including cookies, bread, rulers, pencils, potato chips and erasers. Question 1: How do I put them on different shelves? Class activities: Students think actively and answer questions. Follow-up question 1: Why: Class activity: Students express their ideas. Teacher's guidance: We have the same idea. Now we name the two shelves as the stationery collection of the food collection. The objects inside are cookies, bread, potato chips and ruler, pencil and eraser respectively, and call each object element. Then reveal the definition of the element and the set and its letter representation, and explain the concept. Question 2: Can a set be formed just by giving some elements? Class activity: Go back to the example of buying goods in the store and discuss together. Then give the example of the tall boy. Then we can get the three characteristics of the elements in the set and the relationship between the elements and the set. Question 3: Putting store-bought items into different shelves is a familiar method of sorting. What does classification have to do with collections? Class activity: Students discuss the relationship between categorization and collection. Teacher's note: Classification is classified according to the type of objects, and a set is the totality of some objects. The idea of classification is needed in the collection, and it is representative to explain the collection by means of classification. Student exercise, teacher evaluation: Judge whether the following can constitute a set? 1. The fat boys in our class; 2. The solution of x^2=4; 3. Natural numbers less than 6; 4. All squares; 5. Numbers greater than 5 and less than 3. Question 3: From the above five questions, think about what kinds of sets can be divided into? Classroom activities: teachers and students can find rules from these 5 questions. And then get the three categories of the set. Teacher guidance: in mathematics common number set and its representation, guide students to memorize. Q4: Besides expressing sets in the above natural language, what other methods can be used to express them? Class activities: Introduce enumeration method and description method to students through the examples in the book, analyze the definition and work out four exercises for students to do by themselves to consolidate and deepen their understanding of knowledge. Follow-up question 2: Can all sets be represented in these three ways? Classroom activities: Teachers and students will verify the above questions through the sample questions in this lesson. Question 3: What is the set of features that each representation is used for? How do you choose three representations to make them more useful in different situations? Classroom activities: Teachers and students discuss and illustrate the characteristics of the three expressions together. Design intention: Firstly, starting from the examples in life, the abstract mathematical knowledge into life, students can understand the concept more easily, and use the classification thought to discuss the practical problems, so as to get the definition; Then continue to ask questions, the purpose is to help students understand the knowledge rather than memorizing. It cultivates the students' mathematical thought of classification and develops the students' core quality of mathematical abstraction.
Section three: understanding sublimation, knowledge transfer
Class activity 1: Each group gives an example of a set in life and explains the elements and sets in it, as well as the classification of the set, and represents the elements in the set with appropriate methods. Class Activity 2: If you were a teacher, how would you explain the three ways to represent a set to your students? Ask students to brainstorm ideas in a group and agree on a teaching process. Design intention: Set performance tasks to test whether students have a deep understanding of this lesson and promote the transfer of students' knowledge in life; Intra-group cooperation is conducive to the collision of students' thinking and produce more novel ideas.
Section Four: Expansion and extension, mathematics culture
Class Activity: Show the origins of set theory Design intention: Students feel the great creation of mathematicians and the extensive and profound mathematical culture, which can further deepen students' memory and stimulate students' interest in exploring the history of mathematics and mathematical culture.
Section Five: Class summary, exchange and share
Summary: (1) Ask the students to share their gains in this class and talk about their feelings; (2) The teacher leads us to sort out the knowledge points of this lesson again. Design intention: to cultivate students' ability to summarize and show their own ability, to make students further familiar with knowledge and help them sort out knowledge through joint recall of knowledge.

3.7. Teaching evaluation

(1) Based on the perspective of student learning

Like learning a tool, students can know the set as a rigorous and accurate mathematical language tool, and realize that it is more concise to use the set to express the objects in life.

In the activities of "problem finding -- concept understanding -- knowledge transfer -- mathematical culture -- classroom summary", students can identify the research object, understand the concept from the real life situation, and finally apply their understanding to the real life situation, so as to simplify and contextualize the research problem. Under the guidance of teachers, students can clarify the relationship between elements and sets. And can analyze the three expression methods of the set, so as to obtain the learning methods of abstract problems, experience the process of acquiring knowledge from life, and apply knowledge to life, and develop the core quality of mathematical abstraction, as shown in Table 1.

(2) Based on the perspective of teacher teaching

The essence of the teaching based on the set concept designed for the pursuit of students' understanding is to show the complete process of knowledge formation as far as possible, including the raising of problems, understanding and analysis, and sorting out the results. In the teaching, due to the specific situation of students' foundation and time, challenging performance tasks may be time-consuming for students to complete in class. However, teachers must understand the formation process of knowledge in the previous teaching, and design smooth teaching links based on students' existing knowledge and experience, so as to ensure students' understanding of knowledge, so that meaningful learning can take place, and students can develop their thinking while acquiring knowledge.

To grasp the set of teaching as a whole, and can not scatter them, scattered teaching is not conducive to the establishment of students' knowledge framework. The definition and letter representation of set and element, the relationship between element and set, the characteristics of elements in set, the classification of set, the common number set and the three expressions of set should be explained together like a string of beads, in order to pursue the continuity of exploration process and the depth of knowledge understanding. Finally, in our practice, Teachers should adjust and improve them flexibly according to the specific situation of students.

4. Conclusion

Only by learning in pursuit of understanding can students really incorporate knowledge points into their own cognitive structure. The ensemble instructional design case based on UbD theory provides a new way of thinking for middle school teachers. Teachers should pay attention to students' subjectivity in the teaching process, reflect students' real understanding and transfer of knowledge based on six aspects of understanding, and promote the generation of meaningful learning. And cultivate students' core literacy of mathematics. At the same time, according to the evaluation standards set, teachers should carry out continuous observation on students' learning, judge whether the expected goals have been achieved, what part of the design of teaching activities is unreasonable, and constantly adjust teaching activities, optimize teaching and improve teaching efficiency[4] based on specific feedback.

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