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# Weighting Coefficient Optimization of Active Suspension LQR Controller Based on Whale Optimization Algorithm

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***Abstract:*** *In addressing the challenge of selecting weighting coefficients for performance indicators in the active suspension Linear Quadratic Regulator (LQR) control strategy, a Whale Optimization Algorithm (WOA) is proposed for optimizing the LQR controller's weighting coefficients. Leveraging the WOA's advantages in precision and convergence, the algorithm iteratively refines the values of the Q and R matrices in the LQR active control algorithm.A 1/4 vehicle model of the active suspension is established in the MATLAB/Simulink environment for simulation. By comparing the results with traditional LQR controllers, the designed WOA-optimized LQR controller is demonstrated to enhance the overall performance of the suspension system, significantly improving the vehicle's ride comfort and handling stability.*

***Keywords:*** *Active suspension, LQR controller, Whale Optimization Algorithm, Vehicle's ride comfort and handling stability*

## 1. Introduction

Suspension, as an elastic device connecting the wheels and the vehicle body, plays a crucial role in mitigating the impact on the vehicle. The types of suspension can be categorized into passive, semi-active, and active, with active suspension, equipped with actuators, offering the advantage of providing active force. Therefore, the utilization of active suspension can enhance the vehicle's handling stability and ride comfort[1,2]. As the effectiveness of active suspension largely depends on the magnitude and direction of the active force, research on active suspension has focused extensively on control methods and strategies. Linear Quadratic Regulator (LQR) is a classical optimal control method widely applied in the field of vehicles. However, the key to the LQR control algorithm lies in the design of matrices Q and R, determining the allocation of weight coefficients for different performance indicators. Typically, the allocation of weight coefficients can be determined through empirical methods, but such an approach is inefficient and challenging to find the optimal solution[3]. With the development of intelligent algorithms, there is a continuous emergence of designs that integrate intelligent algorithms into active suspension control, such as the work by HE et al.[4], which introduces genetic algorithms into the LQR control of suspension systems.

The Whale Optimization Algorithm (WOA) is a novel bio-inspired algorithm introduced by Seyedali Mirjalili and Andrew Lewis[5]. It is a population-based swarm intelligence algorithm rooted in the social behavior of whale populations. This algorithm is characterized by its few parameters, simple structure, high convergence accuracy, and the ability to effectively balance local and global search, making it widely applicable to various engineering optimization problems[6-8].

This paper introduces a novel approach for optimizing the weights of Linear Quadratic Regulator (LQR) control using the Whale Optimization Algorithm (WOA) in the context of active suspension. The proposed method leverages the WOA to fine-tune the weight coefficients of the LQR control. Simulation studies were conducted using a 1/4 suspension model to evaluate the performance of the designed LQR controller. A comparative analysis between the Whale Optimization Algorithm was performed to validate the feasibility and superiority of the Whale Algorithm in optimizing the weight coefficients for LQR control.

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## 2. Suspension System Model Establishment

The dynamic model of the suspension system serves as the foundation for analyzing damping characteristics. In this study, we focus on the establishment of a 1/4 active suspension model, as illustrated in Figure 1.

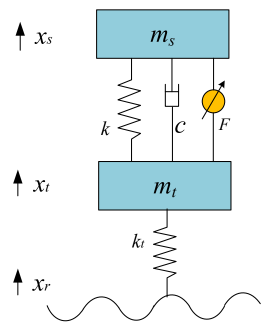


*Figure 1: 1/4 active suspension model*

According to Newton's Second Law, the motion differential equations for the 1/4 vehicle system model can be established.

$$\begin{cases} m_s\ddot{x}_s + c(\dot{x}_s - \dot{x}_t) + k(x_s - x_t) = F \\ m_t\ddot{x}_t - c(\dot{x}_s - \dot{x}_t) - k(x_s - x_t) + k_t(x_t - x_r) = -F \end{cases} \tag{1}$$

Where: $m_s$ represents the vehicle body mass; $m_t$ denotes the wheel mass; $k$ and $k_t$ are the suspension equivalent stiffness and wheel stiffness, respectively; $c$ is the suspension equivalent damping coefficient; $x_r$ is the road unevenness displacement excitation; $x_s$ and $x_t$ represent the vertical displacements of the wheel and the vehicle body, respectively; $F$ is the active control force provided by the actuator. All directions are considered positive in the upward vertical direction.

Consider the suspension dynamic travel, vehicle body vertical velocity $x_s - x_t$, tire dynamic deformation $\dot{x}_s$, and wheel axle vertical velocity $\dot{x}_t$ as the system state vector $X$. The vehicle body vertical acceleration $\ddot{x}_s$, suspension dynamic travel $x_s - x_t$, and tire dynamic deformation $x_t - x_r$ constitute the system output $Y$. The system control variable is denoted as $U$=$F$, and $W$= $\dot{x}_r$ represents the system disturbance. The system state equation can then be expressed as:

$$\begin{cases} \dot{X} = AX + BU + LW \\ Y = CX + DU \end{cases} \tag{2}$$

where, $A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k}{m_s} & -\frac{c}{m_s} & 0 & \frac{c}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_t} & \frac{c}{m_t} & -\frac{k_t}{m_t} & -\frac{c}{m_t} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ \frac{1}{m_t} \end{bmatrix}$, $L = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$,

$C = \begin{bmatrix} -\frac{k}{m_s} & -\frac{c}{m_s} & 0 & \frac{c}{m_s} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} \frac{1}{m_s} \\ 0 \\ 0 \end{bmatrix}$

## 3. LQR Control Algorithm

The essence of the LQR control algorithm is to achieve the best control performance for a system by minimizing the cost. In the case of LQR control for suspension systems, the key performance indicators that need improvement are primarily the vehicle body acceleration, suspension dynamic travel, and tire dynamic displacement. Simultaneously, it is essential to ensure that the actuator's output force remains within an acceptable threshold. While optimizing these performance indicators, efforts are made to minimize the magnitude of the output force, thereby reducing the power consumption of the active suspension.

In accordance with the above considerations, the performance evaluation functional for active suspension LQR control can be expressed as:

$$J = \lim_{T\to\infty}\frac{1}{T}\int_0^T [\rho_1 \dot{x}_s^2 + \rho_2 (x_s - x_t)^2 + \rho_3 (x_t - x_r)^2 + rF^2]dt \tag{3}$$

Where: : $\rho_1$ , $\rho_2$ , $\rho_3$ , $r$ are the weighting coefficients for vehicle body acceleration, suspension dynamic travel, tire dynamic displacement, and active control force, respectively. By increasing the values of these weighting coefficients, the performance of the corresponding indicators can be enhanced.

Let $Q_0 = \begin{bmatrix} \rho_1 & & \\ & \rho_2 & \\ & & \rho_3 \end{bmatrix}$, then the aforementioned performance evaluation functional can be expressed as:

$$\begin{aligned} &J = \lim_{T\to\infty}\frac{1}{T}\int_0^T [Y^T Q_0 Y + rF^2]dt = \\ &\lim_{T\to\infty}\frac{1}{T}\int_0^T [(CX+DU)^T Q_0 (CX+DU) + F^T rF^2]dt = \\ &\lim_{T\to\infty}\frac{1}{T}\int_0^T [X^T C^T Q_0 CX + 2X^T C^T Q_0 DF + F^T (D^T Q_0 D + r) + F^T rF^2]dt \end{aligned} \tag{4}$$

According to Equation (4), with $Q = C^T Q_0 C$ ; $N = C^T Q_0 D$ ; $R = D^T Q_0 D + r$ , the feedback gain matrix K can be expressed as:

$$K = R^{-1}(B^T P + N^T) \tag{5}$$

Where the matrix *P* can be determined by solving the Riccati equation.

$$PA + A^T P - (PB + N)R^{-1}(B^T P + N^T) + Q = 0 \tag{6}$$

The optimal control force for the suspension system output is given by:

$$F = -KX \tag{7}$$

## 4. Whale Algorithm Optimization of Weighting Coefficients

### *4.1. LQR Weighting Coefficient Parameters*

In the design of LQR controllers, the design of matrices Q and R is crucial, as it directly affects the control effectiveness of the active suspension. The key to designing matrices Q and R lies in the selection of weighting coefficients $\rho_1$ , $\rho_2$ , $\rho_3$ , $r$ . When the magnitudes of the corresponding performance indicators are close, larger values of the weighting coefficients within a certain range result in better optimization of the corresponding performance. However, simultaneously, the optimization level of other system performances may decrease or even be negatively impacted. The selection of appropriate weighting coefficients to determine the optimal control parameters has been an ongoing discussion in LQR control.

Typically, in the design of LQR controllers, an empirical approach can be employed to determine the magnitudes of the various weighting coefficients. As shown in Equations (8) and (9), one can control the

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magnitudes of different performance indicators to be approximately consistent. By doing so, a balanced LQR controller with optimized performance can be obtained. However, this empirical method struggles to achieve the optimal control parameters and often results in a LQR controller with performance optimization that tends to be more balanced. This approach poses challenges in designing LQR controllers to meet diverse requirements.

$$Q_0 = \begin{bmatrix} \rho_1 & & \\ & \rho_2 & \\ & & \rho_3 \end{bmatrix} = \begin{bmatrix} 1/BA_{\max}^2 & & \\ & 1/SWS_{\max}^2 & \\ & & 1/DTD_{\max}^2 \end{bmatrix} \tag{8}$$

$$r = \frac{1}{F_{\max}^2} \tag{9}$$

### *4.2. Whale Algorithm Optimization Process*

The Whale Optimization Algorithm (WOA) is a metaheuristic algorithm that employs the hunting patterns of whales to seek optimal solutions. In the whale hunting process, the main target is a school of small fish swimming near the water surface, which the whale preys upon. Whales create bubbles by shrinking their circles, forming a path resembling the number 9.

The following outlines the process of the Whale Algorithm, which can be divided into exploration and exploitation phases. The exploration phase involves a random strategy to search for prey, while in the exploitation phase, whales surround and attack the prey using a spiral bubble net.

Exploitation Phase:

To capture prey, whales must first locate and encircle them. The mathematical model of their behavior is represented by Equations (10) and (11):

$$\vec{X}(i+1) = \vec{X}^*(i) - \vec{A} \cdot \vec{D} \tag{10}$$

$$\vec{D} = \left| \vec{C} \cdot \vec{X}^*(i) - \vec{X}(i) \right| \tag{11}$$

Where: $i$ represents the iteration count, $\vec{X}(i)$ indicating the $i$-th iteration when the whale is at its best position. $\vec{X}(i+1)$ is the current best position. $\vec{D}$ is the distance from the whale to the prey. $\vec{A}$、$\vec{C}$ is the coefficient vector, calculated as follows:

$$\vec{A} = 2 \cdot \vec{a} \cdot \vec{r} + \vec{a} \tag{12}$$

$$\vec{C} = 2 \cdot \vec{r} \tag{13}$$

where: The value of $\vec{a}$ starts with an initial value linearly decreasing from [0,2] until it becomes 0 at the end of the iteration. $\vec{r}$ is a variable and represents a random number in the range [0,1].

The region where whales approach prey can be controlled by the values of the vectors $\vec{A}$、$\vec{C}$. By assigning values to $\vec{A}$ within the range [-1, 1], a new position for the search agent can be identified between the whale's current position and its best position.

Whales use a spiral bubble to surround prey, and the position update is achieved through the equation (14), which models the spiral movement of the whale. This equation is also used to calculate the distance between the best position and the current position.

$$\vec{X}(i+1) = e^{bk} \cdot \cos(2\pi k) \cdot \vec{D}^* + \vec{X}^*(i) \tag{14}$$

$$\vec{D}^* = \left|\vec{X}^*(i) - \vec{X}(i)\right| \tag{15}$$

where: $\vec{D}^*$ represents the optimal distance between the whale and the prey during the iteration process. $b$ is a constant value represented in a spiral form. $k$ is a random value in the range [0, 1].

In reality, both the behavior of whales surrounding prey and emitting a spiral bubble to encircle prey may occur. To simulate this behavior, the choice of whale position update is expressed by Equation (16).

$$\vec{X}(i+1) = \begin{cases} \vec{X}^* - \vec{A} \cdot \vec{D} & \text{if } p < 0.5 \\ e^{bk} \cdot \cos(2\pi k) \cdot \vec{D}^* + \vec{X}^*(i) & \text{if } p \geq 0.5 \end{cases} \tag{16}$$

where: $p$ is a random quantity within the range [0,1].

Exploration Phase:

During this phase, whales update their positions through random searching. By controlling whales to move away from their best positions, this type of search relies on randomly chosen whale position vectors. Therefore, it represents a global search for prey. This approach helps address issues related to local optimization.

$$\vec{X}(i+1) = \vec{X}_{\text{rand}} - \vec{A} \cdot \vec{D} \tag{17}$$

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_{\text{rand}} - \vec{X}\right| \tag{18}$$

where: $\vec{X}_{\text{rand}}$ represents the position of a randomly chosen whale from the whales.

### *4.3. Establishment of the Objective Function and Parameter Optimization*

The purpose of establishing the objective function is to leverage the optimization capabilities of the algorithm to different extents in optimizing various performance indicators of the system. The suspension system has four performance indicators: vehicle body acceleration representing ride comfort and handling stability, suspension dynamic travel, wheel dynamic displacement, and the active control force representing the power consumption performance of the system. Due to the different magnitudes of these indicators, it is necessary to dimensionless them when establishing the objective function. This involves taking the ratio of the values of the three indicators for both active and passive suspensions under the same conditions. The objective function is then constructed using the ratio of the active control force to the maximum active control force.

$$\min \begin{cases} f(\vec{x}) = w_1 \dfrac{RMS\_BA_a(\vec{x})}{RMS\_BA_p} + w_2 \dfrac{RMS\_SWS_a(\vec{x})}{RMS\_SWS_p} + w_3 \dfrac{RMS\_DTD_a(\vec{x})}{RMS\_DTD_p} + w_4 \dfrac{\left|F(\vec{x})\right|}{F_{\max}} \\ RMS\_BA_a(\vec{x}) < RMS\_BA_p \\ RMS\_SWS_a(\vec{x}) < RMS\_SWS_p \\ RMS\_DTD_a(\vec{x}) < RMS\_DTD_p \\ \left|F(\vec{x})\right| < F_{\max} \end{cases} \tag{19}$$

where: RMS represents the root mean square value, $\vec{x} = [\rho_1 \quad \rho_2 \quad \rho_3 \quad r]$ which serves as the weighting coefficients for each performance indicator. $w_1$, $w_2$, $w_3$, $w_4$ are the weights for each performance indicator, and $w_1 + w_2 + w_3 + w_4 = 1$.

The weights can be adjusted based on different performance requirements.

The optimization steps of the Whale Algorithm for tuning the weighting coefficients in the Linear Quadratic Regulator (LQR) control are outlined as follows:

**Step 1:** Generate the initial whale population by setting the initial positions, maximum iteration count, and other algorithm parameters for the whale population.

**Step 2:** Utilize Equation (19) to calculate the fitness function value for each individual whale, identifying the position of the optimal whale.

**Step 3:** Update the values of $\vec{r}$, $\vec{a}$, k, l. Evaluate whether $p$ is less than 0.5; if greater, the whale individual updates its position according to Equation (14) using a spiral bubble net mechanism. If $p$ is less than 0.5 and |A| is greater than 1, the whale updates its position randomly in a global search manner using Equation (11); otherwise, it updates its position through Equation (10) employing a surrounding mechanism.

**Step 4:** Compute the fitness function value after the whale positions are updated, update the position of the currently optimal whale, and calculate the fitness function value for the optimal whale individual.

**Step 5:** Check whether the termination condition for optimization is met. If satisfied, conclude the optimization, retaining the optimal solution; if not, proceed to Step 3.

## 5. Simulation Analysis

### *5.1. Simulation Conditions*

The simulation employed MATLAB/Simulink to construct passive and active suspension models. Comparative analyses were conducted on the suspension performance indicators for passive suspension, empirical LQR, and Whale LQR. The control performance of the Whale LQR was specifically analyzed. Table 1 outlines the parameters for the suspension system.

*Table 1: Suspension System Parameters*

| Name | Symbols | Value | Unit |
|---|---|---|---|
| Vehicle body mass | ms | 340 | kg |
| wheel mass | mt | 59 | kg |
| suspension stiffness | k | 28000 | N/m |
| wheel stiffness | kt | 263000 | N/m |
| suspension damping | c | 600 | N·s/m |
| Maximum control force | F | 500 | N |

The road excitation employs a random road surface, and in Simulink, a stochastic road surface time-domain model is constructed using a filtered white noise method. The specific mathematical model is derived from Equation (20).

$$\dot{z}_g(t) = -2\pi n_1 u z_g(t) + 2\pi n_0 \sqrt{G_q(n_0)u}\, w(t) \tag{20}$$

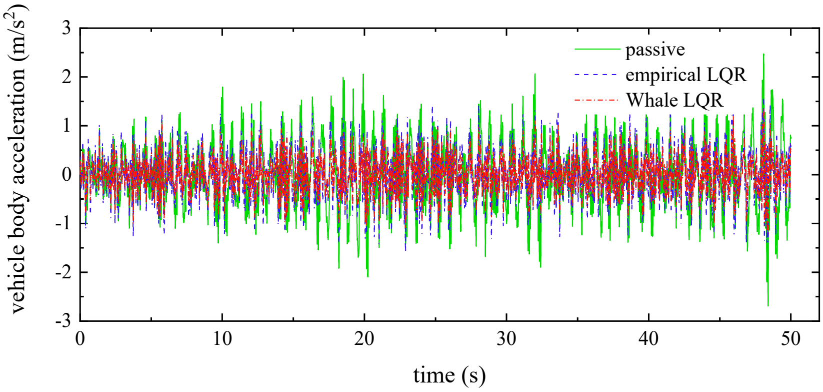
Where: $z_g(t)$ represents the time-domain road surface, $G_q(n_0)$ denotes the road roughness coefficient, $n_1$ is the lower cut-off spatial frequency set at 0.011 $m^{-1}$, and $w(t)$ represents a unit white noise.

The B-class road surface model with a vehicle speed of 10 m/s is established in Simulink using Equation (20). This road surface serves as the simulation condition for conducting a comprehensive analysis of the suspension system.

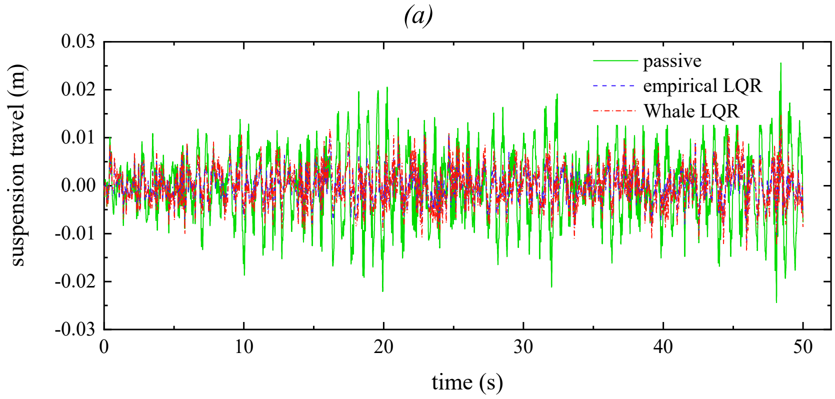
In addition to the aforementioned simulation conditions, the active suspension in this simulation prioritizes ride comfort. Specifically, the primary optimization goal is to reduce vehicle body acceleration while concurrently considering the optimization objective of suspension dynamic travel, aiming to avoid impact on suspension bump stops. Hence, when employing the Whale Optimization Algorithm for optimization, the weightings $w_1$, $w_2$, $w_3$, and $w_4$ in the objective function are set to 0.5, 0.3, 0.1, and 0.1, respectively. Under these conditions, the design of the LQR controller yields empirical LQR control and Whale LQR control.

### *5.2. Simulation results*

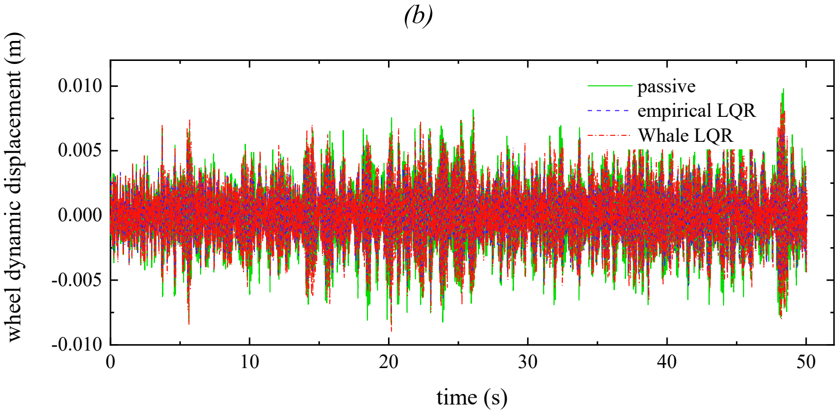
Under the simulation conditions described in Section 4.1, an analysis is conducted on the passive suspension as well as the active suspensions employing empirical LQR and Whale LQR. The graphical results of the simulation analysis are presented in Figure 2.




*(a)*




*(b)*




*(c)*

*Figure 2 Compare the results*

*Table 2: Control effect analysis*

| Control algorithm | vehicle body acceleration ($m/s^2$) | | suspension dynamic travel (m) | | wheel dynamic displacement (m) | |
|---|---|---|---|---|---|---|
| | RMS | control effect | RMS | control effect | RMS | control effect |
| passive | 0.6516 | | 0.007046 | | 0.00227 | |
| empirical LQR | 0.4593 | 29.5% | 0.003311 | 53% | 0.00173 | 24% |
| Whale LQR | 0.3524 | 46% | 0.003934 | 44% | 0.00214 | 6% |

According to Table 2, empirical LQR demonstrates optimization improvements of 29.5%, 53%, and 24% for vehicle body acceleration, suspension dynamic travel, and wheel dynamic displacement , respectively. On the other hand, Whale LQR exhibits optimization improvements of 46%, 44%, and 6% for these three indicators. A comprehensive analysis, as depicted in Figure 2, reveals that both empirical LQR and Whale LQR contribute to varying degrees of enhancement in the vehicle's damping performance. Notably, empirical LQR shows the most pronounced improvement in suspension dynamic travel optimization, while the optimization effects on vehicle body acceleration and wheel dynamic displacement are moderate. In contrast, Whale LQR primarily optimizes vehicle body acceleration and

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suspension dynamic travel, sacrificing some of the optimization in wheel dynamic displacement .

In the context of this simulation, the primary objective for active suspension is to enhance ride comfort while avoiding impacts on suspension bump stops. Consequently, the emphasis is placed on improving damping performance, particularly in vehicle body acceleration and suspension dynamic travel. Through the comparative simulations presented above, Whale LQR control successfully achieves the objectives set for this optimization design. This confirms that Whale LQR control is capable of meeting the varied performance requirements of active suspension LQR controller designs.

In conclusion, when addressing the optimization of weighting coefficients in active suspension LQR control under the same conditions, the Whale Algorithm outperforms the traditional empirical method in optimization results. This underscores the superiority of the Whale Algorithm in handling such problems.

## 6. Conclusions

In addressing the challenge of determining optimal weighting coefficients in the Linear Quadratic Regulator (LQR) control method, this paper introduces an approach for optimizing the weighting values in active suspension LQR control based on the Whale Optimization Algorithm. Initially, a set of weighting coefficients is determined using an empirical method. Subsequently, the Whale Algorithm is employed to identify the optimal weighting coefficients that meet performance requirements. Through the establishment of a 1/4 vehicle model and simulation analysis, the feasibility of the Whale Algorithm in optimizing weighting coefficients for active suspension LQR control is validated. Furthermore, a comparative analysis with the traditional empirical method highlights the superiority of the Whale Algorithm in addressing the optimization problem of weighting coefficients in active suspension LQR control.

In conclusion, based on the Whale Optimization Algorithm, the proposed method for active suspension LQR control effectively enhances the vibration performance of the suspension system, achieving a balance between the vehicle's comprehensive demands for different performance aspects.

## References

*[1] Hrovat D. Survey of advanced suspension developments and related optimal control applications[J]. Automatica, 1997, 33(10): 1781-1817.*

*[2] Hoogterp F B, Eiler M K, Mackie W J. Active suspension in the automotive industry and the military[J]. SAE transactions, 1996: 96-101.*

*[3] Ebrahimi B, Bolandhemmat H, Khamesee M B, et al. A hybrid electromagnetic shock absorber for active vehicle suspension systems[J]. Vehicle System Dynamics, 2011, 49(1-2): 311-332.*

*[4] He Y, McPhee J. Multidisciplinary design optimization of mechatronic vehicles with active suspensions[J]. Journal of Sound and Vibration, 2005, 283(1-2): 217-241.*

*[5] Mirjalili S, Lewis A. The whale optimization algorithm[J]. Advances in engineering software, 2016, 95: 51-67.*

*[6] Oliva D, Abd El Aziz M, Hassanien A E. Parameter estimation of photovoltaic cells using an improved chaotic whale optimization algorithm[J]. Applied energy, 2017, 200: 141-154.*

*[7] Aljarah I, Faris H, Mirjalili S. Optimizing connection weights in neural networks using the whale optimization algorithm[J]. Soft Computing, 2018, 22: 1-15.*

*[8] El Aziz M A, Ewees A A, Hassanien A E, et al. Multi-objective whale optimization algorithm for multilevel thresholding segmentation[J]. Advances in soft computing and machine learning in image processing, 2018: 23-39.*