

# Simulation of lake system based on multi-objective optimization algorithm and system dynamics model

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**Abstract:** The Great Lakes of the United States and Canada are the largest freshwater lakes in the world, and people face changing dynamics and stakeholder conflicts when it comes to lake issues. The purpose of this study was to investigate the influence of multi-objective programming and system dynamics models on the optimal water level results in the Great Lakes. A multi-objective programming model was constructed to maximize benefits and minimize costs, and multiple factors affecting water level change were considered. The genetic algorithm was used to solve the model to obtain the global optimal solution. By establishing a system dynamics model to simulate the fluctuation of water level in the Great Lakes, considering the influence of climate and human activities on the water level, and further refining the water level control, the results show that the model can effectively simulate the water level change, provide an important basis for relevant decision-making, and provide an important reference for the optimal control of the water level in the Great Lakes. In addition, the results of this study can help to provide new ideas for water level control of the same type of large lakes.

**Keywords:** Multi-objective Programming, Genetic Algorithm, System Dynamics

## 1. Introduction

The Great Lakes of the United States and Canada are the largest group of freshwater lakes in the world which contains many large areas with different climates in both countries. Water in lakes is used for a variety of purposes, such as shipping, daily life, maintaining ecosystems, etc. The problem of regulating water levels in a way that fully satisfies the interests of all parties is challenging due to the presence of anthropogenic and non-anthropogenic influences. Factors influencing water levels have been studied extensively. Woolway R I et.al proposed that decreases in winter ice cover and increases in lake surface temperature modify lake mixing regimes and accelerate lake evaporation [1]. Senlin Zhu et.al concluded Mathematical models are powerful tools to model and forecast water level fluctuations in lake systems [2]. Inland lakes are sensitive to the regional climate and environmental change, and human activities [3]. Forecasting water level is an extremely important task as it allows to mitigate the effects of floods, reduce and prevent disasters [4]. Changes in lake levels are affected by many factors such as precipitation, direct and indirect runoff from adjacent basins, evaporation from the free surface of the lake, and interactions between the lake and the groundwater table [5]. In recent decades, with the development of artificial intelligence, data-driven adaptive models were developed and have been widely applied to lake water level simulations [6]. This paper focuses on water level regulation in the Great Lakes region of the United States and Canada to address the complexity of lake water level change and stakeholder conflicts. This paper proposed a new method for the regulation of water level in the Great Lakes region, which combined multi-objective programming, genetic algorithm and system dynamics model, and provided a new idea and method for the regulation of water level in the Great Lakes region. A dynamic change model of lake water level considering multiple factors is established. The model considers the influence of climate, human activities and lake characteristics on water level, and can simulate the dynamic change process of water level more comprehensively. It can provide reference for the regulation of water level in other large lakes, and the research results in this paper can provide reference and reference for the regulation of water level in other large lakes.

## 2. Optimal water level and lake simulation model

### 2.1 Multi-objective programming model

In cost, we constructed new indicators based on the data: traffic loss rate, environmental loss rate, and flooding incidence rate, which refers to the indicators of losses caused by the change in water level. To fully consider the indicators that affect the water level, the formula is as follows.

$$TL_{it} = \frac{iceberg_{it}}{Day_t} \quad (1)$$

$$EL_{it} = \frac{Nd_{it}}{Day_t} \quad (2)$$

$$FR_{it} = \left( \frac{C_{outflow(it)}}{Day_t} + \frac{C_{evaporation(it)}}{Day_t} \right) \quad (3)$$

where  $TL_{it}$  is traffic loss rate of the lake monthly,  $EL_{it}$  is environmental loss rate of the lake monthly,  $FR_{it}$  is flood incidence of the lake monthly,  $Nd_{it}$  is days at non-seasonal discrete points monthly,  $C_{outflow(it)}$  is number of days when lake water flows below the critical level monthly,  $C_{evaporation(it)}$  is days with evaporation rates below criticality monthly,  $iceberg_{it}$  is percentage of ice plugs,  $Day_t$  represents the number of days in the t month.

In order to fully consider the interests of all parties, it is necessary to build bridges between water levels and benefit costs and to start from the maximum benefit and the minimum cost to obtain the optimal water level.

Since there are many factors affecting the water level, we are inspired by the cluster analysis algorithm, and in benefits, we group multiple factors into three categories: daily life, safety, and recreation, which refer to the indicators of the benefits that will be brought by changes in the water level.

Safety mainly refers to flood risk management. Daily water use mainly refers to drinking water supply stability. Recreation primarily refers to Water-based activities. Figure 1 illustrates the Benefit indicator.



Figure 1: Benefit indicator

After obtaining sufficient metrics, we build the following multi-objective planning model:

$$\text{Max Profit}_i = \sum_{t=1}^{12} \omega_1 E(x_{it}) + \omega_2 S(x_{it}) + \omega_3 D(x_{it}) \quad (4)$$

$$\text{Min Cost}_i = \sum_{t=1}^{12} \theta_1 TC(x_{it}) + \theta_2 EC(x_{it}) + \theta_3 FC(x_{it}) \quad (5)$$

where E is Entertainment earnings, S is Safety earnings, D is Daily water usage earnings, TC is traffic loss, EC is environmental loss, FC is flood incidence loss.

According to the needs of various stakeholders, we constrain the lake water level. For shipping companies, the water level needs to meet the minimum water level required for vessel navigation; For daily water use, industrial water use, etc., the water level needs to meet the amount of daily domestic water used; At the same time, it is necessary to prevent dam collapse, as the water level of the lake needs to be lower than the minimum height of the dam. After preliminary consideration, the formula is as follows:

$$\begin{cases} x_{it} \geq x_{min-boat(i)} \\ x_{it} \geq x_{min-dl(i)} \\ x_{it} \leq H_{min(r)} \end{cases} \quad (6)$$

So the final constraint conditions are as follows:

$$\text{Max}\{x_{min-boat(i)}, x_{min-dl(i)}\} \leq x_{it} \leq H_{min(r)} \quad (7)$$

where  $x_{min-boat(i)}$  is the lowest design navigable water level,  $x_{min-dl(i)}$  is Water level required for daily life water use,  $H_{min(r)}$  is the minimum height of the dam.

To make the optimal water level results more accurate, we choose the genetic algorithm based on multi-objective planning to seek the global optimal solution.

Genetic algorithm: In the process of genetic algorithm operation, there exists a set of parameters that have a significant impact on its performance, mainly including the length of chromosome strings, population size, crossover probability, and probability of mutation [7].

The Genetic Algorithm (GA) is an optimization algorithm that emulates the evolutionary process found in nature [8]. The fundamental steps of a genetic algorithm typically involve the following stages:

1) Initialization: A set number of individuals are randomly generated to form the initial population. Each individual represents a solution, often referred to as a chromosome.

2) Evaluation: The fitness of each individual in the population is assessed by calculating its quality. Fitness is commonly associated with the objective function of the problem being addressed.

3) Selection: Superior individuals are chosen for the next generation based on their fitness levels. Individuals with higher fitness have a greater likelihood of being selected. Common selection techniques include roulette wheel selection and tournament selection.

4) Crossover: Selected individuals are paired, and random crossover points are chosen to exchange genetic information, resulting in the creation of new individuals. Crossover mimics the biological process of reproduction.

5) Mutation: Within the population of generated individuals, some are randomly chosen to undergo changes in one or more genes within their genetic sequence, known as mutation. Mutation introduces new genetic material to the population, enhancing its diversity.

6) Termination Condition Check: The algorithm verifies if the termination condition has been met, which could involve reaching a specified number of iterations, fulfilling solution quality criteria, or no longer improving fitness. If the termination condition is satisfied, the algorithm concludes; otherwise, it returns to step 2 to proceed with the subsequent iteration.

## 2.2 Lake Simulation Model

### 2.2.1 Model Indicator Construction

Lake water-level fluctuation is a complex and dynamic process, characterized by high stochasticity and nonlinearity, and is difficult to model and forecast [9]. Maintaining the optimal water level of the Great Lakes requires the consideration of both climate and human activities. To visualize the inflow and outflow of water in the Great Lakes, inspired by the Navier-Stokes equations [10], we have chosen to establish a system dynamics model that can more comprehensively show the dynamic changes of the Great Lakes.

Reflow Principle: When the level of the river is higher than the level of the lake, the direction of movement of the river is: river a lake. When the lake level is higher than the river level, the direction of movement of the river is lake to river. This is the relationship between rivers and lakes have a mutual

recharge relationship.

The following is the realization formula:

$$\begin{cases} \frac{dx_1}{dt} = Inflow_t - \alpha_1(x_1 - x_{1-rl(1)}) * N \\ \dots \\ \frac{dx_5}{dt} = Inflow_t + \sum_{k=i=1}^4 \alpha_k(x_i - x_{i-rl(k)}) * N - \alpha_5(x_5 - x_{5-rl(5)}) * N \end{cases} \quad (8)$$

where the  $Inflow_t$  is the inflow of rivers,  $Outflow_t$  is the outflow of rivers,  $\alpha_i$  is Flow rate of river.

$$N = \begin{cases} 1 & x_i \geq x_{i-rl(k)} \\ -1 & x_i \leq x_{i-rl(k)} \end{cases} \quad (9)$$

According to the question, excessive water level fluctuations can have a significant impact on relevant stakeholders, so we limit the water level variation of the lake to within 2 feet; At the same time, in order to maintain the balance of the lake water level, we limit the lake's water level to fluctuate around 5% of the mean.

The constraint conditions are as follows:

$$\begin{cases} \left| \frac{dx_i}{dt} \right| \leq 2 \\ \left| \frac{x_i - x_{average(i)}}{x_{average(i)}} \right| \leq 5\% \\ \min Inflow_t \leq Inflow_t \leq \max Inflow_t \end{cases} \quad (10)$$

where  $x_{average(i)}$  is Average water level of the lake,  $x_i$  is water level of the lake.

### 3. Results

#### 3.1 Data Preprocessing

First, we divided the annual data into monthly data by removing missing values as well as outliers from the Great Lakes water level data with multiple-factor data. Then we normalize and regularize the multiple factors to provide a strong foundation for model building.

$$X_{new} = \frac{X - X_{min}}{X_{max} - X_{min}} \quad (11)$$

Turning quantitative expressions into dimensionless expressions facilitates the ability to compare and weigh indicators of different units or magnitudes. Normalization can also simplify calculations by turning dimensioned data sets into pure quantities.

#### 3.2 Multi-objective Programming Result

To fully consider the interests of all parties, it is necessary to build bridges between water levels and benefit costs and to start from the maximum benefit and the minimum cost to obtain the optimal water level.

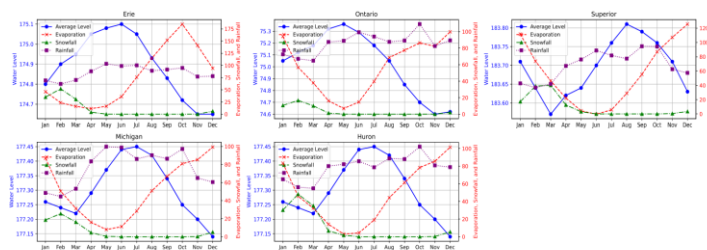


Figure 2: Great Lakes Data

From Figure 2, we can see that the average water level of the Great Lakes peaks in June and July and then gradually declines. Evaporation from the lakes drops to a minimum in April and May and then keeps rising, with a generally parabolic shape. Snowfall is generally higher in the winter months of January and February and almost non-existent in other months. Monthly rainfall for each lake generally showed a steady trend with small fluctuations in rainfall.

Table 1: Monthly water level predictions for the Great Lakes

Month	Erie	Ontario	Superior	Michigan	Huron
Jan	174.82	75.13	183.75	177.29	177.25
Feb	174.85	75.08	183.6	177.3	177.28
⋮	⋮	⋮	⋮	⋮	⋮
Dec	174.65	74.67	183.73	177.24	177.18

Table 1 above shows the optimal water level of the Great Lakes for each of the twelve months and the maximum water level was examined to be no more than 2 meters from the actual water level, with good modeling results.

**3.3 Lake Simulation Model Result**

Visualization parameters: Set parameters to visually represent water levels by node size-larger nodes represent higher water levels, and smaller nodes represent lower water levels. Likewise, adjust edge thickness to represent flow magnitude thicker edge represents higher flow, and a thinner edge represents lower flow.

Analysis tools: Based on system dynamics, combine the analysis tool Python to build a river simulation system.

After constructing the analysis graph, the following is a partial display of the results of our analysis process. The lake simulation flow charts are as follows: Among the 12 months, June is selected as the representative month for analysis. In June, the size of the network lines on the lake simulation network map indicates influence weights; the thicker the line, the more it can influence water level changes. This is shown in Figure 3, the final optimal water level results were obtained as 182.69 for Lake Superior, 176.63 for Lake Michigan, 176.64 for Lake Huron, 174.25 for Lake Erie, 72.55 for Lake Ontario, and 1.15 for the Ottawa River.

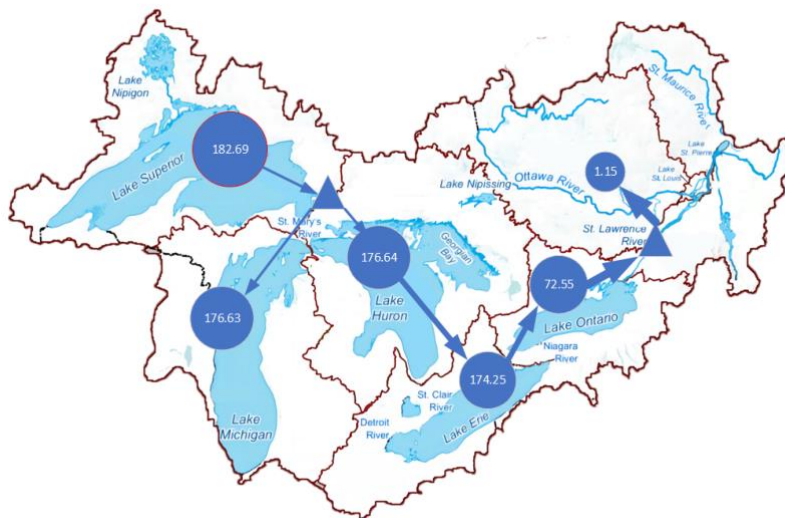


Figure 3: Lake Simulation Network Diagram June

**4. Conclusion and Outlook**

Using multi-objective programming combined with a genetic algorithm, the lake system model was constructed, and the effective solution to the lake water level regulation problem was realized through

data preprocessing and an index system to maximize benefits and minimize costs. A system dynamics model was established, which could simulate the fluctuation of lake water level, and showed the dynamic process of water level change through visual analysis, which provided a decision-making basis for water level control. The model considers the needs of different stakeholders, balances the interests of all parties through constraints, and uses a genetic algorithm to search for the optimal solution globally, which improves the reliability of the model.

There is still room for improvement in the construction of the indicator system, and more influencing factors can be considered in the future to make the model more comprehensive. At the same time, the sensitivity analysis of system dynamics model parameters and model calibration are also the future improvement directions. This model provides important support for the decision-making of lake water level regulation and has the value of popularization and application for similar lake systems. Combined with advanced algorithms such as machine learning, model performance can be further improved and a more reliable basis for relevant decisions.

## References

- [1] Woolway R I, Kraemer B M, Lenters J D, et al. *Global lake responses to climate change*[J]. *Nature Reviews Earth & Environment*, 2020, 1(8): 388-403.
- [2] Zhu S, Hrnjica B, Ptak M, et al. *Forecasting of water level in multiple temperate lakes using machine learning models*[J]. *Journal of Hydrology*, 2020, 585: 124819.
- [3] Chaudhari S, Felfelani F, Shin S, et al. *Climate and anthropogenic contributions to the desiccation of the second largest saline lake in the twentieth century*[J]. *Journal of Hydrology*, 2018, 560: 342-353.
- [4] Nguyen X H. *Combining statistical machine learning models with ARIMA for water level forecasting: The case of the Red river*[J]. *Advances in Water Resources*, 2020, 142: 103656.
- [5] Mohammadi B, Guan Y, Aghelpour P, et al. *Simulation of Titicaca lake water level fluctuations using hybrid machine learning technique integrated with grey wolf optimizer algorithm*[J]. *Water*, 2020, 12(11): 3015.
- [6] Xu J, Fan H, Luo M, et al. *Transformer based water level prediction in Poyang Lake, China*[J]. *Water*, 2023, 15(3): 576.
- [7] Alhijawi B, Awajan A. *Genetic algorithms: Theory, genetic operators, solutions, and applications*[J]. *Evolutionary Intelligence*, 2023: 1-12.
- [8] Jike G, Yuhui Q, Chunming W, et al. *Review of genetic algorithm research*[J]. *Application research of computers*, 2008, 25(10): 2912-2913.
- [9] Zhu S, Lu H, Ptak M, et al. *Lake water-level fluctuation forecasting using machine learning models: a systematic review*[J]. *Environmental Science and Pollution Research*, 2020, 27(36): 44807-44819.
- [10] Ershkov S V, Prosviryakov E Y, Burmasheva N V, et al. *Towards understanding the algorithms for solving the Navier–Stokes equations*[J]. *Fluid Dynamics Research*, 2021, 53(4): 044501.