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# Research on location adjustment of UAV based on bearings only passive location

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***Abstract:*** *In recent years, the research on cooperative localization and tracking of moving targets using multiple UAVs under bearings only measurement has attracted extensive attention. In order to locate the UAV and adjust the UAV to the ideal position, this paper determines the distance and polar angle between the UAV and the UAV (FY00) through the received direction information to locate the UAV receiving the information, that is, using the sine cosine theorem and recursive method to locate the position of the UAV receiving the signal.*

***Keywords:*** *UAV positioning adjustment; Sine cosine theorem; Recursive method*

## 1. Introduction

With industrial development and technological progress, the use of UAVs has become increasingly important in all walks of life. There are not only UAVs for flight delivery, but also intelligent UAVs for independent inspection. In this process, UAV flight positioning has become the top priority. Azimuth only UAV positioning refers to adjusting the position of UAV through electromagnetic wave signal. In order to avoid external interference, UAVs generally keep electromagnetic silence and emit a small amount of electromagnetic waves to the outside world when planning to locate. This method is used to maintain the formation of UAVs. Several UAVs in the formation send signals, while other UAVs passively receive signals and extract azimuth information from them to adjust the position of UAVs and keep the relative positions between UAVs unchanged. In this paper, under the premise of determining the signal transmitted by the UAV at the center of the circle (FY00) and two other UAVs with known numbers and no deviation in position, the UAV with slightly deviation in other positions can be located by establishing a mathematical model; On the premise that the signals transmitted by FY00 and FY01 UAVs with no position deviation are known, in order to transmit as few electromagnetic signals as possible, several UAVs are required to transmit signals, which can realize the effective positioning of UAVs with slightly deviated passive receiving signals[1].

## 2. Model assumptions and symbols

### *2.1 Model Assumptions*

1) The aerial adjustment and positioning of UAV is only affected by the direction information sent by the UAV transmitting the signal, and is not affected by wind, gravity, etc.

2) The UAVs transmitting signals are ideally thought to be on the uniformly distributed circumference.

3) The UAV can achieve effective positioning after several adjustments in the air.

4) UAV positioning does not require reference direction information and distance between receiving signal UAV and transmitting signal UAV.

### *2.2 Symbol Description*

The symbols in this document are shown in Table 1.

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*Table 1: Symbol Description Table*

| Symbol | Explain |
|---|---|
| m(FY00-FY09) | Any one UAV with the number of FY00-FY09 as the transmitting signal |
| n(FY00-FY09) | Any one UAV with the number of FY00-FY09 as the transmitting signal |
| r | Uniformly distributed circumferential radius |
| Unmanned aircraft Z | UAV passively receiving signals |
| $\alpha_m$、 $\alpha_n$、 $\alpha_0$ | UAV Z receives the direction information of FY00, m and n |
| $X_{mn}$ | m,n Distance |
| $X_{zm}$ | Z,m Distance |
| $X_{zn}$ | Z,n Distance |
| $\theta_1$ | ∠Zmo |
| $\theta_2$ | ∠Znm |
| $(l,\rho)$ | Polar coordinates of UAV Z |
| $\alpha$ | ∠ BOA changes with adjustment in degrees |
| $\beta$ | ∠ ODA changes with adjustment |
| $k$ | Number of adjustments |
| $x_1$ | Distance between UAV k1 and UAV FY00 |
| $x_2$ | Distance between UAV k2 and UAV FY00 |
| $k_1$ | UAV A |
| $k_2$ | UAV B |
| $\alpha_{k1}$ | Direction information transmitted by UAV $k_1$ to UAV M |
| $\alpha_{k2}$ | Direction information transmitted by UAV $k_2$ to UAV M |
| $a$ | Side length of equilateral triangle |

## 3. Establishment and Solution of Model

### *3.1 Application of sine cosine*

According to the question, the numbers (m, n) of the UAV at the center of the circle (FY00) and the other two signal transmitting UAVs in the formation are known and their positions are all free of deviation. Therefore, the direction information received by the signal receiving UAV can be seen that the connection between the UAV (FY00) and the UAV n can be specified as the polar axis[2]. In combination with the geometric figure, it can be discussed in two cases N is outside the circle with diameter, and the other is that the receiving signal UAV searches for the relationship between edges and angles in the graph within the circle with diameter m and n, and uses the sine theorem and cosine theorem to set equations, so as to find out the distance between the receiving signal aircraft and the UAV (FY00) and the included angle with the specified polar axis, so as to locate the position of the receiving signal UAV.

Scenario 1:

As shown in Figure 1, except for the circle with the diameter of m and n of the two UAVs transmitting the signal, the angle between the 9 UAVs receiving the signal is $360^{\circ}/9=40^{\circ}$, if the 9 UAVs except FY00 are finally uniformly distributed on the circle. Where the polar coordinate of Z is (l, ρ), The polar coordinates of n are $\left(r,0^{\circ}\right)$, the polar coordinates of m are $(r,(m-n)*40^{\circ}$ and the direction information of FY00, m, n received by UAV Z is $\alpha_m$, $\alpha_n$、, $\alpha_0$, and m, n distance is $x_{mn}$, Z, m distance is $x_{zm}$, and angle Zmo is $\theta_1$. The angle Znm is $\theta_1$.

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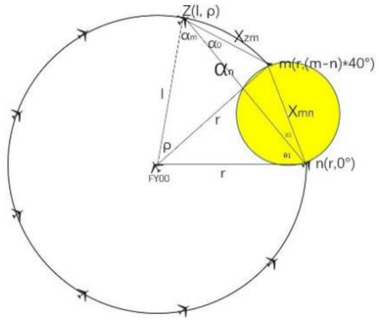


*Figure 1: UAV Location Map (1)*

In △omn

$$2\times \mathrm{r}^2 - x_{mn}^2 = 2\times r^2 \times \cos\left[(m-n)\times 40^{\circ}\right] \quad (1)$$

In △ozm

$$l^2 + \leftarrow x_{zm}^2 - r^2 = 2\times l \times x_{zm} \times \cos\alpha_n \quad (2)$$

In △ozn

$$\frac{\mathrm{r}}{\sin\alpha_m} = \frac{x_{zn}}{\rho} \quad (3)$$

In △zmn

$$\frac{\mathrm{x}_{mn}}{\sin\alpha_0} = \frac{x_{z\mathrm{m}}}{\sin\theta_2} \quad (4)$$

In △omn

$$2\times(\theta_1+\theta_2)+(m-n)\times 40^{\circ} = 180^{\circ} \quad (5)$$

In △ozn

$$\rho + \theta_1 + \alpha_{\mathrm{m}} = 180^{\circ} \quad (6)$$

In the above six equations, there are $x_{mn}$, $x_{zm}$, ρ、, l, $\theta_1$, $\theta_2$ Six unknowns, so that the polar coordinate of Z can be obtained, that is, the UAV receiving the information can be located[3].

Scenario 2:

As shown in Figure 2, the drone Z receiving the signal is in the circle with the diameter m and n of the two drones transmitting the signal, where the polar coordinate of Z is (l, ρ), The polar coordinate of n is $\left(\mathrm{r}, 0^{\circ}\right)$, and the polar coordinate of m is $(r,(m-n)*\ 40^{\circ}$, the direction information of FY00, m, n received by UAV Z is $\alpha_m$, $\alpha_n$, $\alpha_0$.The distance between m and n is $x_{mn}$, and the distance between Z and m is $x_{zm}$.

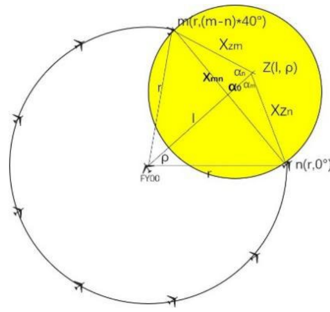


*Figure 2: UAV Location Map (2)*

In △omn

$$2\times \mathrm{r}^2 - x_{mn}^2 = 2\times r^2 \times \cos\left[(m-n)\times 40^\circ\right] \tag{7}$$

In

$$x_{zm}^2 + x_{zn}^2 - x_{mn}^2 = 2x_{zm}\times x_{zn}\times \cos\alpha_0 \tag{8}$$

In

$$\frac{\mathrm{r}}{\sin\alpha_m} = \frac{x_{zn}}{\rho} \tag{9}$$

In

$$\frac{\mathrm{x}_{zm}}{\sin\left[(m-n)\times 40^\circ - \rho\right]} = \frac{\mathrm{r}}{\sin\alpha_n} \tag{10}$$

In the above four equations, there are $X_{mn}$, $X_{zm}$ ρ, $X_{zn}$ four unknowns, so that the polar coordinate of Z can be obtained, that is, the UAV receiving the information can be located[4].

It has been determined that FY00 and FY01 UAVs can transmit signals. Since we need to transmit as few electromagnetic signals as possible, we need to determine at least several UAVs to determine the position of the UAV receiving the information. It is better to determine whether the UAV receiving the information can be located by adding another UAV[5]. If it can, just add another aircraft to send signals; If not, continue to discuss adding two more aircrafts, discuss the same and add them in turn, and finally determine that at least several more UAVs are needed to send signals.

(1) When adding a UAV, under the model established in the first question, determine that n=1, m is the unknown number with the value of 2-9, and record it as i, then there is

$$2\times \mathbf{r}^2 - x_{i1}^2 = 2\times r^2 \times \cos\left[(i-1)\times 40^\circ\right] \tag{11}$$

Obviously, $X_{21}=X_{91}$, $X_{31}=X_{81}$, $X_{41}=X_{71}$, $X_{51}=X_{61}$, that is, for any x, the above equation corresponds to two i, there is no unique solution, so it is impossible to accurately locate the received signal UAV. Therefore, it is not feasible to add one UAV.

(2) When two UAVs are added, with the help of the geometric relationship in Figure 3, the UAV FY00 in the figure is the point O, the UAV FYO1 is the point 1, the UAV $k_1$ is the point A, the UAV $k_2$ is the point B, and r is the uniform circumference radius. Wherein, the UAV FY00, FYO1, $k_1$, and $k_2$ are all UAVs transmitting signals, the UAV $k_1$ and $k_2$ are two newly added UAVs, and the UAV M is the UAV receiving signals. It is necessary to locate the position of the UAV M, The polar coordinates of M are defined as (l, ρ), The polar coordinate of point A is $\left(r,\quad (k_1-1)\times 40^\circ\right)$, the polar coordinate of point B is $\left(r,\quad (k_2-9)\times 40^\circ\right)$, the polar coordinate of point 1 is (r, 0), and the direction information transmitted by UAV FY00, FYO1 and $k_1$ to UAV M is $\alpha_{k1}$, $\alpha_1$, $\alpha_0$, UAV FY00, FYO1. The direction information transmitted by $k_2$ to UAV M is is $\alpha_{k2}$ , $\alpha_1$, $\alpha_0$, $\theta_1$=∠O1M, $\theta_2$=∠ A1M, the length from point A to point 1 is $X_1$, and the length from point M to point 1 is $X_2$, so there is

$$\alpha_0 + \alpha_1 = \alpha_{k1} \tag{12}$$

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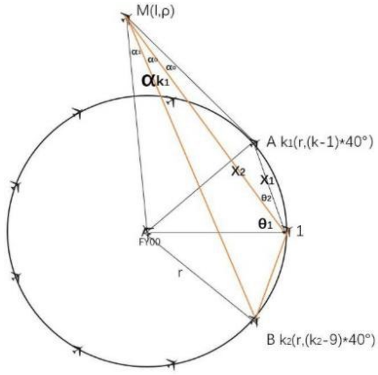


*Figure 3: UAV Location Map (3)*

In △OA1

$$\cos\left[\left(k_1-1\right)\times 400^{\circ}\right]=\frac{2\times r^2-x_1{}^2}{2\times r^2} \tag{13}$$

In MO1

$$\frac{\mathrm{r}}{\sin\alpha_{k1}}=\frac{l}{\sin\left(\pi-\alpha_{k1}-\rho\right)} \tag{14}$$

In △OMA

$$\frac{\mathrm{r}}{\sin\alpha_1}=\frac{l}{\sin\left[\pi-\rho+\left(k_1-1\right)\times 40^{\circ}-\alpha_0-\alpha_{k1}\right]} \tag{15}$$

In △MOB

$$\frac{\mathrm{r}}{\sin\alpha_1}=\frac{l}{\sin\left[\pi-\alpha_1-\rho-\left(9-k_2\right)\times 40^{\circ}\right]} \tag{16}$$

In △OM1

$$\frac{x^2}{\sin\rho}=\frac{r}{\sin\alpha_{k1}} \tag{17}$$

In △OM1

$$\frac{x_2}{\sin\rho}=\frac{l}{\sin\theta} \tag{18}$$

In △MA1

$$\frac{x_1}{\sin\alpha_0}=\frac{x_2}{\sin\left(\pi-\alpha_0-\theta_2\right)} \tag{19}$$

$$\theta_1+\theta_2=\frac{\pi-\left(k_1-1\right)\times 40^{\circ}}{2} \tag{20}$$

In the above nine equations, there are $k_1$, $k_2$, $x_1$, $x_2$, l, ρ, $\alpha_1$, $\alpha_0$, $\alpha_{k1}$ nine unknowns, so that the polar coordinates of M (l, ρ) So as to locate the UAV receiving the signal. Therefore, it can be concluded that when two UAVs are added, the UAV can be effectively located[6].

### *3.2 Application of Recursive Method*

According to the initial position of the UAV, it can be found that the UAV FY00 is on the center of

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a circle with a radius of 100 m, and the UAV FY01 is exactly on the circumference. Because when the 9 UAVs except FY00 are evenly distributed on the circumference, the angle between the 9 aircraft is $360^{\circ}/9=40^{\circ}$. Make the position of UAV FY00 as point O, the position of UAV FY01 as point A, the position of UAV FY02 as point B, and the point adjacent to point A on the uniform circumference as point C, then $\angle$ COA=$40^{\circ}$, $\angle$ OCA=$40^{\circ}$. First, compare the relationship between the polar angle of B and $40^{\circ}$. If it is greater than $40^{\circ}$, draw an arc with point O as the center and OB as the radius, and adjust it clockwise along the arc; If it is equal to 40 °, no adjustment is required; If it is less than $40^{\circ}$, it shall be adjusted counterclockwise along the arc, and the final point is D. After adjusting the polar angle, adjust the polar diameter. By comparing the size relationship between $\angle$ODA and $\angle$OCA, if $\angle$ODA is greater than $\angle$OCA, adjust along the direction of the polar diameter deviating from the O point; If $\angle$ODA is equal to $\angle$OCA, no adjustment is required; If $\angle$ODA is less than $\angle$OCA, it shall be adjusted along the polar diameter near the O point until $\angle$ODA is equal to $\angle$OCA, and then it returns to the uniform circumference[7].

In the above, for the adjustment of the polar angle, it is necessary to take a step t first, rather than t=0.01, then each time according to α= $\angle$OBA±$kt_1$, k=1,2,3,..., k are the adjustment times until α=$40^{\circ}$; For the adjustment of the polar diameter, another step size $t_2$ is taken in the same way. If $t_2$=0.01 is taken, then every time β= OCA ± $kt_2$, k=1, 2, 3, ..., k are the adjustment times.

Then, using the recursive method, after adjusting the UAV FY02 to point B, let the UAV FY00 and the UAV FY02 send information and adjust the position of the UAV FY03 until the UAV FY09 is also adjusted to the uniform circumference, so that one UAV is located at the center of the circle, and the other nine UAVs are uniformly distributed on the circumference with a radius of 100 m.

First adjust the polar angle

If $\angle$BOA>$40^{\circ}$ (as shown in Figure 4 and Figure 5), point B draws an arc with point O as the center and OB as the radius, clockwise along the arc, and in steps of $t_1$=0.01, $\alpha=\angle \mathrm{OBA}-k_{t1}$, k=1,2,3,..., k is the number of adjustments until $\alpha=40°$.

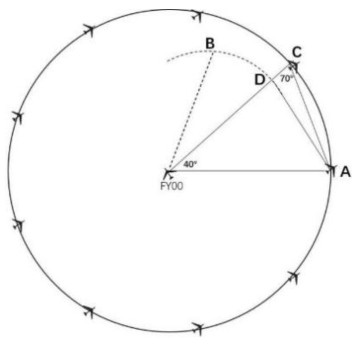


*Figure 4: UAV Location Map (4)*

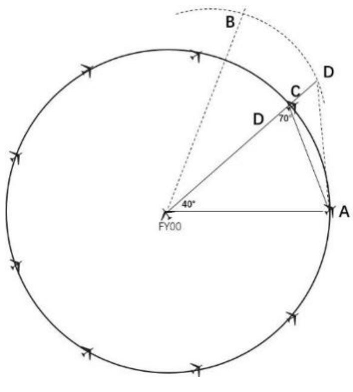


*Figure 5: UAV Location Map (5)*

If $\angle$BOA=$40^{\circ}$, no adjustment.

If $\angle$BOA< $40^{\circ}$ (as shown in Figure 6 and Figure 7), then point B draws an arc with point O as the center and OB as the radius, along the clockwise direction of the arc, in the step of $t_1$=0.01, $\alpha = \angle \mathrm{OBA} + k_{t1}$, k=1, 2, 3 ,..., k is the number of adjustments until $\alpha = 40°$.

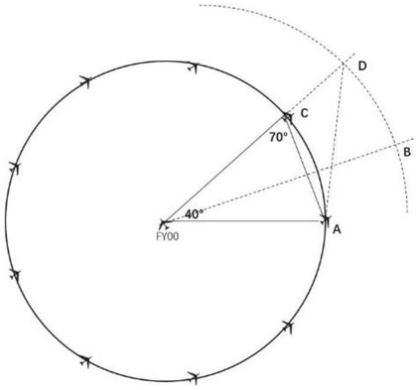


*Figure 6: UAV Location Map (6)*

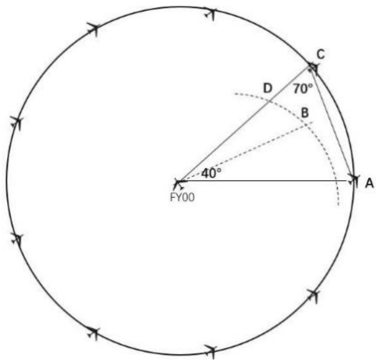


*Figure 7: UAV Location Map (7)*

Secondly, adjust the polar diameter.

If $\angle$ODA> $70^{\circ}$ (as shown in Figures 4 and 7), point D deviates from point O along the polar radial direction, according to the step size $t_2$=0.01, $\beta = \angle \mathrm{OCA} - k_{t2}$, k=1,2, 3 ,..., k is the number of adjustments until $\beta = 70°$.

If $\angle$ODA= $70^{\circ}$ , it will not be adjusted;

If $\angle$ODA< $70^{\circ}$ (as shown in Figures 5 and 6), point D is close to point O along the extreme radial direction, and the step size t2=0.01, $\beta = \angle \mathrm{OCA} + k_{t2}$, k=1,2,3,..., k is the number of adjustments until $\beta = 70°$.

Finally, using the recursive method, after adjusting the UAV FY02 to point B, let the UAV FY00 and the UAV FY02 send information, adjust the position of the UAV FY03,until the UAV FY09 is also adjusted to the uniform circumference, so that finally one UAV is located at the center of the circle, and the other nine UAVs are evenly distributed on the circumference with a radius of 100 m[8].

## 4. Conclusion

In this paper, mathematical ideas are flexibly used to analyze the UAV positioning model, which is easy to realize and understand and can be understood by most people. Through the combination of numbers and shapes, the relationship between angles and sides is summarized into rules. The combination of numbers and shapes is more readable. The model provides an indirect and practical method for decision-making and sequencing of various factors related and restricted to this paper, and avoids the difficulties of the problem itself, making the problem solving process more convenient and efficient. The model can not only analyze the precise positioning of UAV, but also take certain measures with high accuracy. And this paper selects a more appropriate step size, which not only makes the error small, but also makes the UAV positioning adjustment more efficient and rapid.

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