

Transmission Optimization Model for Wireless Charging of Electric Vehicles

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Abstract: The purpose of this paper is to study of electric vehicle transmission system of wireless charging, through the analysis of the electromagnetic coupling principle to further understanding of the factors affect the wireless power transmission system, and builds multivariate constrained optimization model for the optimized, among them, mainly concerns the transmitting frequency, the impedance matching, transmission distance and coil deviates four factors, combined with computer simulation, Get more objective quantitative analysis data evidence.

Keywords: Electromagnetic coupling, Transmission efficiency, Mutual inductance, Multivariable constraint optimization, Lagrange multiplier method

1. Restatement of the problem

Electric vehicles are favored by consumers for their advantages of low environmental pollution, low noise, high energy utilization efficiency and convenient maintenance. However, the wired charging method of existing electric vehicles is complex and has safety risks, so the use of wireless charging methods has become the first choice for the electric vehicle industry. However, at present, there are many electric vehicle manufacturers, and wireless charging must meet the principle of "special car use", resulting in a great waste of power resources. Therefore, by optimizing the non-vehicle part of wireless charging, it is of great significance to make the interconnection between wireless charging devices and electric vehicles of different manufacturers.

The wireless charging system is composed of a vehicle part and a non-vehicle part. The on-board part consists of an RLC circuit with the load, and the non-vehicle part consists of a power supply and another RLC circuit. Two inductive elements L_1 (transmit coil inductance, also known as matched impedance) and (receiving coil inductance) transmit electrical energy by coupling to generate a magnetic L_2 field.

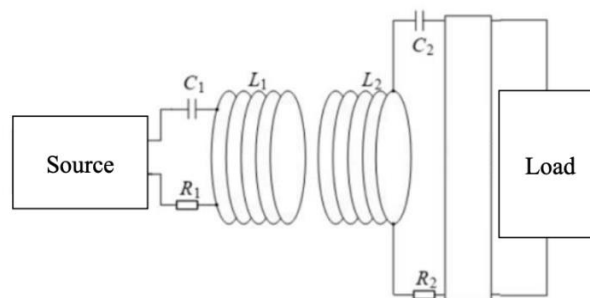


Figure 1: Wireless charging system

Problem 1: Under the complete resonance of the transmission and receiving coils, a mathematical model of transmission frequency, matching impedance and wireless power transmission efficiency is established, and the power transmission efficiency of 10 experimental wireless charges in Annex 1 is calculated by combining the relevant data of Annex 1, Annex 2 and Annex 3.

Problem two: Due to the design of the electric vehicle itself, the distance between the wireless charging part and the ground may be any value within the regulations. The mathematical model of question 1 was reconstructed to establish a mathematical model of transmission frequency, matching impedance, distance between two coils and wireless power transmission efficiency. When the distance between the two coils in the first experiment of Annex 1 was changed to 150mm, 200mm, and 250mm,

the power transmission efficiency of wireless charging was recalculated and compared.

Problem three: Previous studies have shown that transmission efficiency can be improved by changing the emission frequency and changing the matching impedance. Please give the case of the first experiment in Annex 1 (the distance between the two coils), can the transmission efficiency be maximized by adjusting the transmission frequency and matching 100mm the impedance value? What is the maximum value?

Problem four: When the electric vehicle stops for wireless charging, it is difficult to ensure that the transmission coil and the receiving coil are completely vertically coincident, and there will always be more or less deviation, see Figure 1 in the "Related Instructions", where the coil radius is r , and the receiving coil (high h from the transmission coil) deviates from a (mm) in the positive direction of the X axis. Continuing the study in Question 3, calculate the maximum a when the vertical distance from the ground h is 100mm, and the maximum transmission efficiency is greater than 80%.

2. Problem analysis

Electric vehicles conform to the green trend and have been greatly developed. However, the complexity and safety of wired charging methods also restrict their development to a certain extent, so wireless charging methods have become the goal pursued by the electric vehicle industry. Although there are many advantages in the use of wireless charging, technical bottlenecks have also become the main factors restricting its development, so technological breakthroughs in wireless charging have also become the focus of attention of major enterprises.

On the one hand, including the transmission efficiency problem between the two coils, the premise of wireless charging popularization must be that its efficiency can meet the needs of use. On the other hand, the consideration of position deviation, the transmission efficiency of the coil itself is important, but we must also take into account that it is difficult for customers to always reach the ideal position when parking. In order to achieve higher utilization, a site for customer parking requirements are too high, we can change to a certain range, so that the range of charging is relatively efficient.

2.1. Analysis of problem one

The first problem is to consider the influencing factors of transmission efficiency, mainly to consider the impact of the emission frequency and the matching impedance on it. Where, full resonance simplifies the situation so that the entire circuit is purely resistive. First, we conducted a basic analysis of the entire circuit, sorting out the position of each circuit component in the circuit and the parameters that occur. Then, using Kirchhoff's voltage law as a bridge, the relationship between the primary coil and the secondary coil is established, and the two are regarded as a linear passive two-port network, so as to construct a two-port model of the electromagnetic coupling resonant system. Finalize the wireless power transfer efficiency model for the transmit frequency and matching impedance, and substitute the parameters and data in the annexes for the solution.

2.2. Analysis of question two

Problem two is similar to problem one, mainly on the basis of problem one to consider the impact of the distance between the two coils on the transmission efficiency. The wireless power transfer model mainly realizes energy transfer through electromagnetic conversion, which involves the problem of electromagnetic field. Changing the distance means a change in the strength of the magnetic field, which in turn affects the transmission of energy, which is mainly reflected in the circuit is the influence of the mutual inductance of the two coils, so we can study it from this perspective.

2.3. Analysis of question three

Question three is different from the previous two questions, question three clearly shows that the emission frequency and the matching impedance have an impact on the transmission efficiency, and let's explore the value of the transmission frequency and the matching impedance that make the transmission efficiency the largest. This question is more of a consideration of mathematical ability, and for the most valuable problem, mathematically using the principle of inequality to study it, we can also start from here.

2.4. Analysis of question four

Question Four focuses on another deviation, because the coils are all round, and we consider setting a reasonable range for the customer to be based on the circle. The advantage of a circle is that the center and radius of the circle can establish a circle, so we can reduce the problem to the determination of the radius. This question can be based on the idea of linear programming, and the transmission efficiency requirement of the question is 80%, which is used as a constraint to determine the objective function of the maximum value of the α .

3. Model assumptions

- (1). The circuit reaches a fully resonant state
- (2). The mutual inductance between the two coils is only related to the distance
- (3). the power and transmit coils, receiving coils and loads, and the energy transmission loss between the two RLC circuits are negligible, and their coupling coefficient is considered to be 1
- (4). The change of the distance between the two coils affects the mutual inductance between the coils, which in turn affects the transmission efficiency of wireless power.
- (5). The deviation of the position of the coil will affect the efficiency of energy transmission between the two coils, which mainly affects the mutual inductance between the two.

4. Symbol description

Table 1: Symbol description table

symbol	meaning
M_{12}	Mutual inductance between primary and secondary coils
Z	Equivalent impedance
F	Emission frequency
L_1	Matching impedance
j	Vector representation
η	Transmission efficiency
h	The vertical distance between two horizontal coils
α	The distance at which the center of the circle deviates in the positive direction of the X axis

5. Model building and solving

5.1. Question one

5.1.1. Fundamentals of wireless power transfer systems

The wireless power transfer system completes the exchange of power through the electromagnetic coupling resonance technology between the primary coil and the secondary coil. Among them, the resonant system is the core of the wireless energy transmission system and the core of the wireless charging of electric vehicles. Resonant circuits are common circuit models, that is, in ac circuits with resistor R, inductor L and capacitor C elements, the voltage at both ends of the circuit is generally different from the current phase in it. If the parameters or power supply frequencies of circuit elements (L or C) are adjusted, they can be phased together and the entire circuit appears to be purely resistive. The circuit reaches this state called resonance. In full resonance, the total impedance of the circuit reaches an extreme value.

The American Society of Automotive Engineers (SAE) regulates the wireless charging frequency of electric vehicles at around 85kHz, Ordinary coils are difficult to achieve 85kHz if they use self-resonance, Generally in the MHz level. Compensation is required to increase the transmission efficiency, generally by adding capacitance. This article adopts a two-sided compensation method, that is, adding compensation capacitors to both the primary and secondary coils, and the simplified circuit diagram is shown in Figure 2.

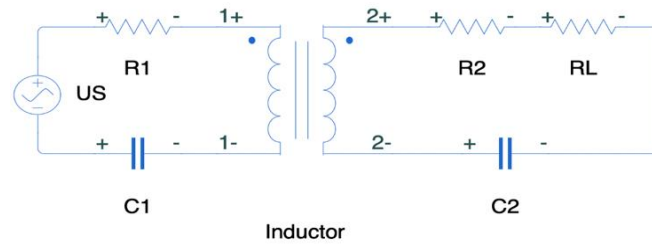


Figure 2: Simplifies the circuit diagram

According to the title, this article is a compensation structure for primary series-secondary series (SS). Wherein, the supply voltage is U , the primary coil resistance and inductance are R_1, L_1 , and the secondary coil resistance inductances are R_2, L_2, R_L, R_L is the load impedance, and M_{12} is the mutual inductance between the coils, C_1, C_2 Compensation capacitance of the primary coil and secondary coil, I_1, I_2 Current flowing through the primary coil and secondary coil, respectively, Q_1, Q_2

$$Q = \frac{\omega L}{R} \tag{1}$$

Where $\omega = 2\pi F$, F is the emission frequency

Two inductive elements, L_1 and L_2 transmit electrical energy by coupling to generate a magnetic field, and their mutual inductance is only related to distance, the value of which is, M_{12} , and the coupling coefficient is expressed as in Equation 2

$$k_{12} = \frac{M_{12}}{\sqrt{L_1 L_2}} \tag{2}$$

Considering the influence of the coil, we use the equivalent impedance (equation 3) to represent,

$$\begin{aligned} Z_1 &= R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \\ Z_2 &= R_L + R_2 + j\omega L_2 + \frac{1}{j\omega C_2} \end{aligned} \tag{3}$$

Thereinto,

$$\begin{aligned} j\omega L_1 + \frac{1}{j\omega C_1} &= 0 \\ j\omega L_2 + \frac{1}{j\omega C_2} &= 0 \end{aligned}$$

so:

$$Z_1 = R_1, Z_2 = R_2 + R_L$$

5.1.2. Two-port model of an electromagnetically coupled resonant system

According to Kirchhoff's voltage law, the algebraic sum of all electromotive forces along the closed loop is equal to all voltage drops. Combined with the circuit structure of this problem, the following KVL equation (Equation 4) can be obtained.

$$\begin{cases} I_1 Z_1 - j\omega M I_2 = U \\ I_2 Z_2 - j\omega M I_1 = 0 \end{cases} \tag{4}$$

For the study of transmission problems. Both can be thought of as a linear passive two-port network. When the reference direction of the current is selected as the incoming two-port network, the relationship between the output signal and the input signal can be established by parameter A, and the transmission equation can be expressed as Equation 5

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{5}$$

To simplify the problem, the two-port network is considered to have a symmetrical structure. Defines

the normalized angular frequency coefficient $\omega_{12} = \omega/\omega_0$, That is, the ratio of the actual working angle frequency to the natural harmonic frequency, and the natural harmonic frequency $\omega_0 = 1/\sqrt{L_1 C_1}$ is taken.

A parameter (Equation 6) can be expressed as:

$$\begin{cases} A_{11} = \frac{V_1}{V_2} \Big|_{I_2=0} = \frac{1}{k_{12}} \left(1 - \frac{j}{\omega_n^2 Q_1} \right) = A_{22} \\ A_{12} = \frac{V_1}{V_2} \Big|_{I_2=0} = \frac{-j\omega L_1}{k_{12}} \left[k_{12}^2 - \left(1 - \frac{j}{\omega_n^2 Q_1} \right) \right] \\ A_{21} = \frac{V_1}{V_2} \Big|_{I_2=0} = \frac{1}{j\omega L_1 k_{12}} \end{cases} \quad (6)$$

When the system is fully resonant, the two-port network exhibits pure resistance characteristics, so the admitting parameter can be transferred by the A_{12} (Equation 7) Derive the resonant frequency to meet the requirements of the formula.

$$k_{12}^2 - \left(1 - \frac{j}{\omega_n^2 Q_1} \right)^2 = 0 \quad (7)$$

At the same time, for electromagnetic coupling resonance systems, in order to obtain better energy transmission effects, the Q value is generally at the 10^3 level, and the final characteristic frequency can be reduced to Equation 8

$$\omega_{12} = \frac{\omega_0}{\sqrt{1 - jQ_1^{-1} \pm k_{12}}} \approx \frac{\omega_0}{\sqrt{1 \pm k_{12}}} \quad (8)$$

The gain characteristics of the output response to the input excitation signal can be expressed by the voltage transfer function KV and the current transfer function KI, i.e. Equation 9

$$\begin{aligned} k_v &= \frac{V_2}{V_1} = \frac{V_2}{A_{11}V_2 + A_{12}(-I_2)} \\ k_I &= \frac{I_2}{I_1} = \frac{I_2}{A_{21}V_2 + A_{22}(-I_2)} \end{aligned} \quad (9)$$

Introducing the concept of power factor in a resonant circuit, the power factor of a resonant circuit (Equation 10) can be expressed as reflecting the ratio of active power to real power in the circuit, so it can also reflect the transmission efficiency of the system through this parameter.

$$\eta = \frac{|V_2 I_2|}{|V_1 I_1|} = \frac{|V_2| |I_2|}{|V_1| |I_1|} = |k_v| |k_I| = \left(1 + \frac{2}{k_{12}^2 Q_1^2} + \frac{Q_2}{k_{12}^2 Q_1} + \frac{k_{12}^2 + Q_1^{-2}}{k_{12}^2 Q_1 Q_2} \right)^{-1} \quad (10)$$

By the above relationship, it is possible to obtain a function relationship image of the transmission efficiency with respect to the coupling coefficient as shown in figure3.

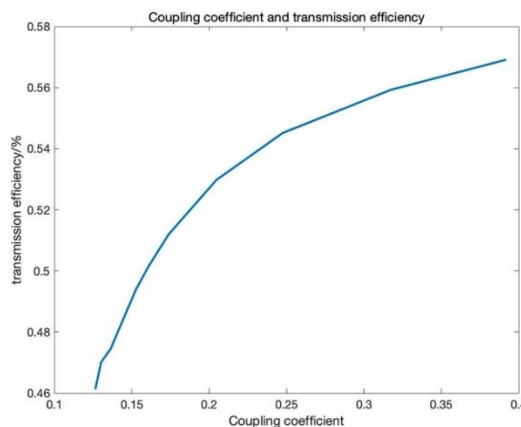


Figure 3: Coupling coefficient and transmission efficiency

5.1.3. Solve the model

According to the above model, we link the transmission frequency, the matching impedance and the wireless power transfer efficiency, add the parameters in Annex 2 ($R_1=1.55 \Omega$, $R_L=432.24 \Omega$, transmit frequency $F=30\text{kHz}$), and calculate the transmission efficiency of the 10 sets of data in Annex 1 as shown below:

Table 2: Inductance and mutual inductance data at different distances

Experimental serial number	Distance between two coils	Emission coil inductance	Receive coil inductance	Coil mutual inductance	Transmission efficiency
	$h(\text{mm})$	$L_1(\mu\text{H})$	$L_2(\mu\text{H})$	$M_{12}(\mu\text{H})$	%
1	100	162.21	163.6	63.83	56.91%
2	125	161.46	163.04	51.49	55.93%
3	150	161.36	163.15	40.17	54.52%
4	175	161.75	162.87	33.24	52.98%
5	200	161.79	162.96	28.24	51.20%
6	225	161.59	163.15	26.17	50.18%
7	250	161.53	163.25	24.78	49.38%
8	275	161.35	162.72	22.12	47.46%
9	300	162.61	163.74	21.25	47.01%
10	325	161.39	163.26	20.53	46.13%

5.2. Question two

5.2.1. Mutual inductance model for coil transmission

The change in the distance between the two coils will affect the electromagnetic field strength, which in turn will affect the mutual inductance between the coils, and the change of mutual inductance will affect the transmission efficiency of wireless power. Because both coils are circular, to simplify the model, it can be assumed that both circles are circles with a radius r . Ideally, the center of the two coils should be on a vertical line and the two coils should be parallel so that the distance between the two circles is h .

For ease of calculation, we put two coils into the coordinate axis so that both circles are located on the vertical z -axis, one coil center is at point A, the coordinate of point A is $(0,0,z_1)$, the other coil is at point B, and the coordinate of point B is $(0,0,z_2)$, where, $z_1 + z_2 = h$. Take two points C and D on each of the two coils, and make $C(r, \theta, z_1)$, $D(r, \varphi, z_2)$. The distance d between the two points of C and D can be expressed as Equation 11

$$d = |r_2 - r_1| = \sqrt{[2r^2 - 2r^2 \cos(\varphi - \theta) + h^2]} \tag{11}$$

According to Newman's formula, the mutual inductance between the two coils can be obtained as shown in Equation 12

$$M = \frac{\mu_0}{4\pi} \iint \frac{dI_1 dI_2}{|r_2 - r_1|} \tag{12}$$

The distance d between C and D is substituted into the mutual inductance calculation formula (Equation 13) under the mutual inductance model that can be obtained by coil transmission

$$M = \frac{\mu_0}{4\pi} \int_0^{2\pi} d\varphi \int_0^{2\pi} \frac{r^2 \cos(\varphi - \theta) d\theta}{\sqrt{2r^2 - 2r^2 \cos(\varphi - \theta) + h^2}} \tag{13}$$

Through the above relationship, we can obtain a function relationship image of M and transmission distance as shown in Figure 4.

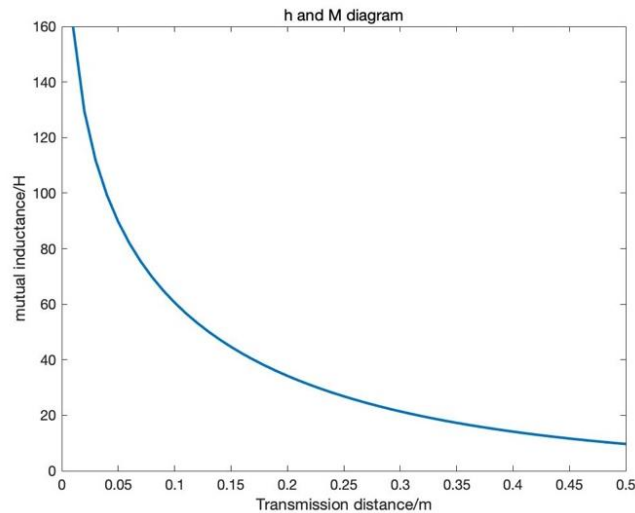


Figure 4: *h and M diagram*

5.2.2. Solve the model

By Equation 2, the relationship between the mutual inductance of the two coils and the coupling coefficient can be established, and then the size of the transmission efficiency is determined under the two-port model of the electromagnetic coupling resonance system in the first problem, and the relationship can be obtained from the function relationship image of the transmission efficiency on the distance of the two coils as shown in Figure 5. In addition, with the above relationship and the parameters given in the question, the distance between the two coils can be calculated to be 100mm, 150mm, 200mm, and 250mm respectively, and the power transmission efficiency of wireless charging is calculated as shown in Table 3.

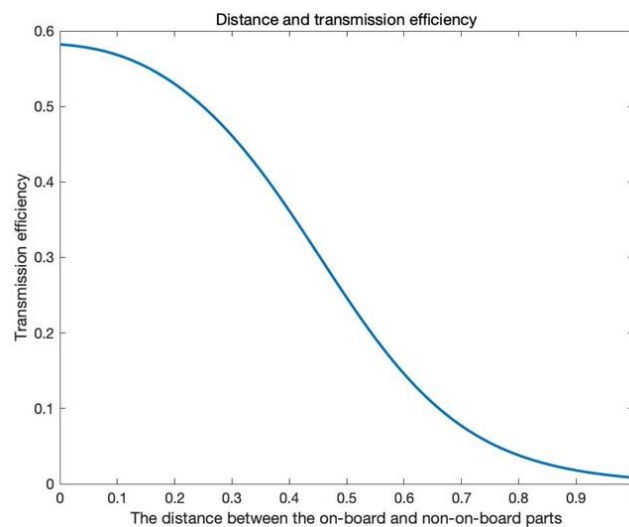


Figure 5: *Distance and transmission efficiency*

Table 3: *Distance and transmission efficiency*

<i>h</i>	Transmission efficiency
100	56.82%
150	55.26%
200	52.98%
250	49.95%

5.2.3. Comparison of models

Comparing the transmission efficiency of wireless energy under question one and problem two, it can be seen that from the model itself, the second problem is more comprehensive, which focuses on the consideration of the distance between the two coils; from the calculation results, the transmission

efficiency of problem two is higher, although the transmission efficiency under both models has a downward trend with the increase of distance h , but it is clear that the efficiency of model two is higher, and the minimum value of model two (49.95%) is still higher than the highest value of model one (46.13%). It shows that the distance between the two coils has a significant impact on the efficiency of wireless power transmission.

5.3. Question Three

5.3.1. Lagrange optimal

In mathematical optimality problems, the Lagrange multiplier method is a method of finding the extremum of a multivariate function whose variable is constrained by one or more conditions. Question 3 has made it clear that transmission efficiency can be improved by changing the emission frequency and changing the matching impedance. That is, in the case of adjusting the transmission frequency and matching the impedance value, the transmission efficiency is maximized.

Based on Equation 1 and Equation 2, obtain the variables ω about the emission frequency and the variables about the matching impedance L_1 , substitute for Equation 10, to obtain Equation 14 as shown below

$$\eta(\omega, L_1) = \frac{M^2 \omega^4 L_1^2 L_2 R_L}{M^2 \omega^4 L_1^2 L_2 R_L + 2 \omega^3 L_1 L_2^2 R_L^2 + M^2 \omega^2 L_1 L_2 R_L^2 + L_2 R_L^3 R_L^2} \quad (14)$$

Using the Lagrange multiplier method, the above variable is used to obtain partial derivatives ω, L_1

$$\frac{\partial \eta(\omega, L_1)}{\partial \omega} = \frac{4M^2 \omega^3 L_1^2 L_2 R_L}{M^2 \omega^4 L_1^2 L_2 R_L + 2 \omega^3 L_1 L_2^2 R_L^2 + M^2 \omega^2 L_1 L_2 R_L^2 + L_2 R_L^3 R_L^2} = 0 \quad (15)$$

$$\frac{\partial \eta(\omega, L_1)}{\partial L_1} = \frac{2M^2 \omega^4 L_1 L_2 R_L}{M^2 \omega^4 L_1^2 L_2 R_L + 2 \omega^3 L_1 L_2^2 R_L^2 + M^2 \omega^2 L_1 L_2 R_L^2 + L_2 R_L^3 R_L^2} = 0 \quad (16)$$

5.3.2. Solving the Model

This approach introduces a new scalar unknown, the Lagrange multiplier: the coefficient of each vector in the variable about the emission frequency ω and the variable about the matching impedance L_1 . The proof of this method involves partial differentiation, full differentiation, and thus finding a value that allows the differential division of the set hidden function to be zero. Join these two equations and solve them:

$$\omega = 114\pi, L_1 = 160\mu H$$

That is, in this case the wireless power transfer system reaches its maximum value. By reversing these two values back to Equation 10, the maximum value that can be solved η is **87.2%**.

Table 4: Impedance and frequency corresponding to maximum efficiency

Matching impedance (μH) L_1	Emission frequency F (kHz)	Maximum Efficiency (%)
160	57	87.2%

5.4. Question four

5.4.1. Mutual inductance model for coil deviation

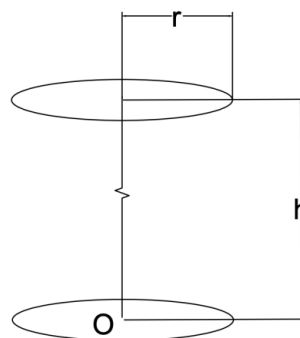


Figure 6: Coil vertical diagram

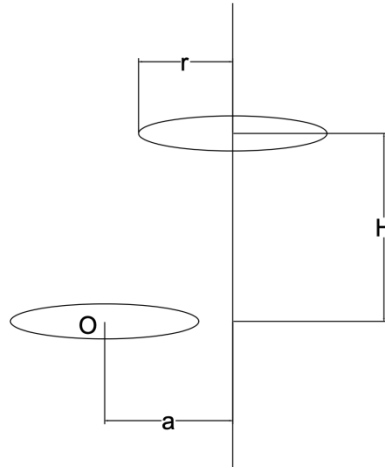


Figure 7: Horizontal coil offset plot

The ideal state of the level of the primary coil and the secondary coil and the projection completely coinciding is difficult to reach in real life, deviation is the norm, and deviation will affect the energy conversion efficiency between the two coils, which is mainly to change the transmission efficiency by affecting the mutual inductance between the two, so we mainly consider the change of self-perception.

Assuming that the horizontal direction of the two coils does not change, but relative movement occurs in the vertical direction, because the coil is circular, the situation is consistent in all directions, so we assume that the center of the circle deviates from the α in the positive direction of the X axis, based on the three-dimensional space of problem two, the two coil centers are $(\mathbf{0}, \mathbf{0}, z_1)$ $(\mathbf{0}, \mathbf{0}, z_2)$ take the dots on the circle separately:

$$E(r \cos \beta, r \sin \beta, z_1)$$

$$F(r \cos \gamma, r \sin \gamma + \alpha, z_2)$$

Equation 17 can be calculated

$$\begin{cases} d I_1 = r(\cos \beta - \sin \beta) d \beta \\ d I_2 = r(\cos \gamma - \sin \gamma) d \gamma \\ d I_1 d I_2 = r^2 \cos(\beta - \gamma) d \beta d \gamma \end{cases} \quad (17)$$

According to Newman's formula, it can be obtained that the mutual inductance of the two coils in this case is Equation 18

$$M = \frac{\mu_0}{4\pi} \int_0^{2\pi} d\beta \int_0^{2\pi} d\gamma \frac{r^2 \cos(\beta - \gamma)}{r^2(\cos \beta - \cos \gamma)^2 + (\alpha + \gamma \sin \beta - \gamma \sin \gamma)^2 + h^2} \quad (18)$$

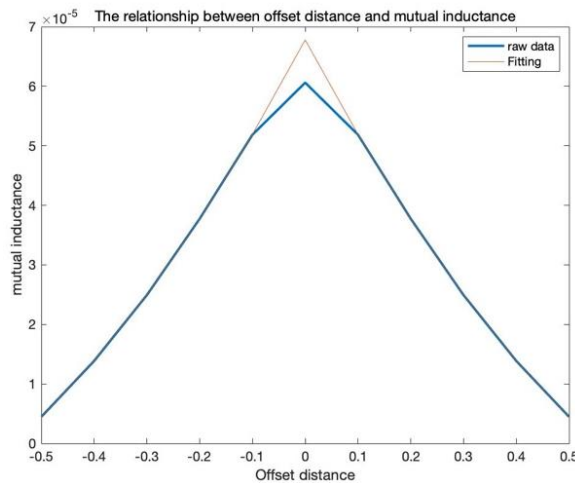


Figure 8: Offset distance

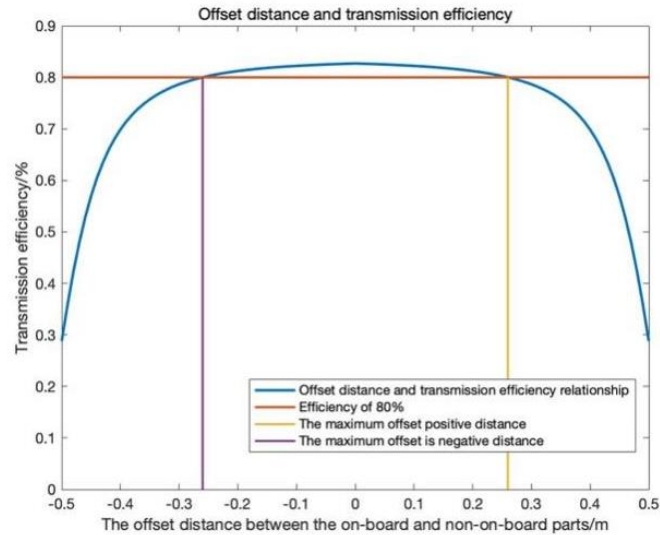


Figure 9: Offset distance and transmission efficiency

5.4.2. Solve the model

Equation 2 establishes the relationship between the mutual inductance and coupling coefficient of the two coils in the off-the-coil state, and then determines the magnitude of the transmission efficiency under the two-port model of the electromagnetic coupling resonant system in problem 1. The maximum value of the emission frequency, the matching impedance, and the distance between the two coils obtained by the above relationship and the first two questions. Substitution can obtain the mathematical relationship between α and wireless power transmission efficiency. According to the requirements of the topic, the transmission efficiency is 80%, that is, the maximum value of the α can be obtained is 260mm, That is, in the range of the primary coil center as the center of the circle and the α as the radius, even if the coil deviation occurs, the wireless charging transmission system can still maintain good transmission efficiency.

Table 5: Offset distances at good transmission efficiency

Offset distance (mm)
260

6. Evaluation of the model

6.1. Advantages of the model

(1). The electromagnetic coupling resonance model established by this problem simplifies the complex electrical problems and extracts the mathematical equations, thus realizing the discussion of transforming the microscopic electromagnetic change process into a mathematical equation, and advantageously establishing a transformation bridge between mathematical physics, and controlling the physical process more deeply through the control of mathematical variables.

(2). Through model optimization and analysis, some unnecessary processes and steps are simplified and optimized to make the model more clear and intuitive, and some data with relatively large deviations are re-adjusted and modified, which greatly improves the solution efficiency and is conducive to the discussion of other optimization problems on such problems.

6.2. Disadvantages of the model

(1). The electromagnetic coupling resonance model established by this problem does not fully take into account the actual situation that the power supply has internal resistance, heat consumption during transmission, equipment aging and other problems that cannot be overcome.

(2). does not reflect a more variable and flexible circuit situation is only applicable to the series to series of two-port conversion problem, circuit components are relatively simple, can control and change the variables can not meet the actual situation of multi-scenario change problems.

7. Generalization of the model

This paper in consideration can change the emission frequency, matching impedance, two coil distance in the case of the control of wireless electromagnetic transmission efficiency, through the study of the given variables can theoretically deduce the maximum efficiency in the electromagnetic transmission process, in the use of simulation software to verify and correct the optimal value so that there is a good service for practical applications and energy saving effect, this model is suitable for general electromagnetic propagation resonance coupling problems, the general wireless charging, magnetic levitation, electromagnetic detection and other fields have a good application value.

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