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Pure azimuth passive positioning in attempted UAV formation flight

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Abstract: *In order to maintain the formation of UAVs during flight, pure azimuthal passive positioning is generally used to adjust the position of UAVs. In order to accurately perform UAV positioning, this paper establishes a single-objective optimization model of UAV with passively received signals based on the polar coordinates of UAVs, combined with geometric knowledge such as the sine theorem and triangle properties. And then the model is optimized and improved, i.e., when the position information of two UAVs is known, how many additional UAVs are needed to perform UAV localization. Therefore, it was found that two additional UAVs with unknown numbers were needed to determine the UAV position by establishing a new set of equations to achieve the positioning of the UAVs with slightly deviated positions. The model is important to achieve effective positioning of UAVs and adjust the solution.*

Keywords: *pure azimuthal passive localization of UAVs, sine theorem, single objective optimization model, readjustment plan*

1. Introduction

As technologies for UAV flight progress, UAVs are being used in various applications and the method of pure orientation passive positioning is important for adjusting the position of UAVs and maintaining the formation [1]. Suppose there are 10 UAVs in a circular formation, one of which is at the center of the circle and the other 9 are evenly distributed on a certain circumference. The UAVs fly at the same altitude based on their perceived altitude and receive signals as directional information[2-3].

When a UAV with a slightly deviated position receives signals from the UAV at the center of the circle and another UAV (with no deviation in position) on the circumference, this paper optimizes the model based on the above-mentioned model, i.e., adding several additional UAVs with unknown numbers to transmit signals to obtain a new set of equations about the angle, and solving for the number of UAVs needed to achieve its effective positioning.

2. Method

2.1 Model building based on UAV polar coordinates

From the fact that the UAVs all remain flying at the same altitude, it is clear that the problem is a two-dimensional planar problem. The UAV located at the center of the circle and the other two UAVs in the formation can be known to transmit signals, of which the radius of the circular formation and the value of the polar angle of the UAV transmitting signals can be determined since the position of the UAV transmitting signals is not deviated and the number is known. The remaining UAVs with deviated positions passively receive signals, and the model can be derived by combining the sine theorem and the geometric properties of the triangle part.

By observing the circular formation formed by the UAVs we can know the distribution of the UAVs that passively receive signals in four ways.

In the first case, the image of the position relationship of the UAV is (see Figure 1).

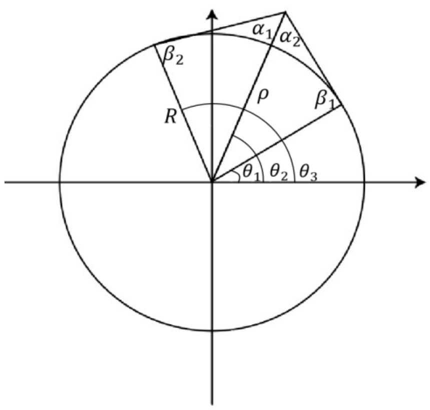

Figure 1: First case image

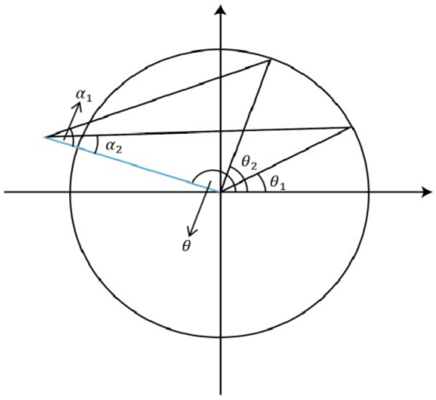

Figure 2: Second case image

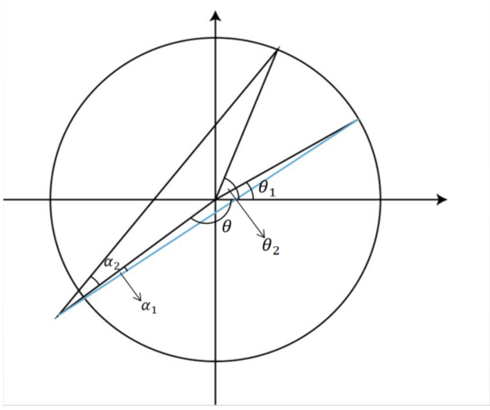

Figure 3: Third case image

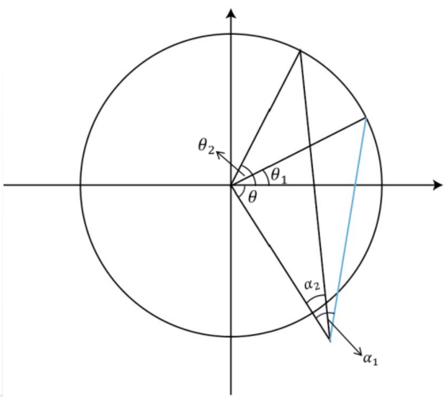

Figure 4: Fourth case image

Where, ρ is the polar diameter of the receiving signal UAV, θ is the polar angle of the receiving signal UAV, R is the radius of the circular formation i.e. the polar diameter of the transmitting signal UAV, θ_1 and θ_2 are the polar angles of the transmitting signal UAV, and the rest of the angles are shown in Figure 1.

The positioning model of the UAV can be obtained by combining the sine theorem and the triangle interior angle relationship.

$$\begin{cases} \dfrac{R}{\sin \alpha_1} = \dfrac{\rho}{\sin \beta_1} \\ \dfrac{R}{\sin \alpha_2} = \dfrac{\rho}{\sin \beta_2} \\ \beta_1 = \pi - \alpha_1 - (\theta - \theta_1) \\ \beta_2 = \pi - \alpha_2 - (\theta_2 - \theta) \end{cases} \tag{1}$$

The image of the position relationship of the UAV in the second case is shown in Fig. 2. This model building process is similar to the first case, so the positioning model built for the second case is (see Eq. 2)

$$\begin{cases} \dfrac{R}{\sin(\alpha_1 + \alpha_2)} = \dfrac{\rho}{\sin(\pi - (\theta - \theta_2) + \alpha_1 + \alpha_2)} \\ \dfrac{R}{\sin \alpha_1} = \dfrac{\rho}{\sin(\pi - (\theta - \theta_1) + \alpha_1)} \end{cases} \tag{2}$$

The image of the position relationship of the UAV in the third case is shown in Fig. 3. This model is

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also built similarly to the first case, so the positioning model built for the third case is (see Eq. 3)

$$\begin{cases} \dfrac{R}{\sin\alpha_1} = \dfrac{\rho}{\sin(\alpha_1 - \theta)} \\ \dfrac{R}{\sin\alpha_2} = \dfrac{\rho}{\sin(\theta - \theta_1 + \alpha_2)} \end{cases} \tag{3}$$

The image of the position relationship of the UAV in the fourth case is shown in Fig. 4. This model is also built similarly to the first case, so the positioning model built for the fourth case is (see Eq. 4)

$$\begin{cases} \dfrac{R}{\sin\alpha_1} = \dfrac{\rho}{\sin(\alpha_1 - \theta)} \\ \dfrac{R}{\sin\alpha_2} = \dfrac{\rho}{\sin(\alpha_2 - \theta + \theta_2)} \end{cases} \tag{4}$$

2.2 Optimization based on the above model

When a UAV with a slightly deviated position receives the signal from the UAV at the center of the circle and another UAV (with no deviation in position) on the circumference, the image of the additional two unknown numbered UAVs, built with the UAVs numbered FY00 and FY01 as horizontal axes, is (see Figure 5).

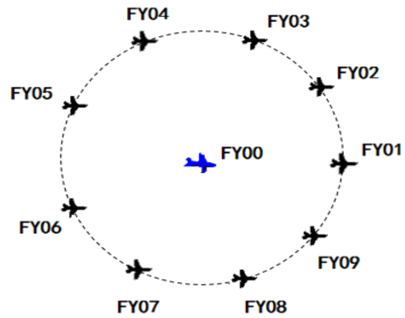

Figure 5: Schematic of circular UAV formation

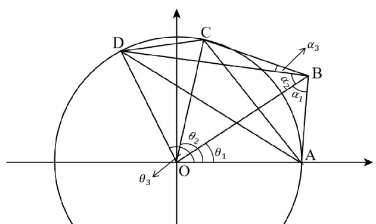

Figure 6: Image of adding two unknown numbered UAVs

Where $\alpha_1 = \angle ABO$, $\alpha_2 = \angle DBO$, $\alpha_3 = \angle CBD$, $\theta_1 = \angle BOA$, $\theta_2 = \angle COA$, $\theta_3 = \angle AOD$ (see Figure 6). *A* is the drone numbered FY01, *O is the drone* numbered FY00, *B is the drone with* slightly deviated position, *C* and *D are* two additional drones with unknown numbers. α_1 are the orientation angles of UAVs *A* and *O* to *UAV B*, α_2 are the orientation angles of *UAVs O* and *D* to UAV *B,* and α_3 are the orientation angles of UAVs *D* and *C* to UAV *B.*

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According to the triangle interior angle relationship and the sine theorem it can be concluded that

$$\begin{cases} \dfrac{R}{\sin\alpha_1} = \dfrac{\rho}{\sin\angle BAO} \\ \dfrac{R}{\sin\alpha_2} = \dfrac{\rho}{\sin\angle BDO} \\ \dfrac{BD}{\sin\angle BAD} = \dfrac{AD}{\sin(\alpha_1+\alpha_2)} \\ \dfrac{CD}{\sin\alpha_3} = \dfrac{BD}{\sin\angle BCD} \end{cases} \tag{5}$$

3. Results

3.1 Solving the positioning model

In the first case, the polar coordinates of the receiving signal drone are found by solving equation (1), i.e., by deformation of Eq.

First of all, you may want to make

$$K_1 = \frac{R}{\sin\alpha_1} = \frac{\rho}{\sin\beta_1} \tag{6}$$

$$K_1 = \frac{R}{\sin\alpha_2} = \frac{\rho}{\sin\beta_2} \tag{7}$$

That is, we can obtain.

$$\frac{K_1}{K_2} = \frac{\sin\beta_2}{\sin\beta_1} = \frac{\sin(\pi-\alpha_2-\theta_2+\theta)}{\sin(\pi-\alpha_1-\theta+\theta_1)} \tag{8}$$

It can be assumed that

$$\gamma_1 = \pi - \alpha_1 + \theta_1 \tag{9}$$

$$\gamma_2 = \pi - \alpha_2 - \theta_2 \tag{10}$$

Bringing equation (9), with equation (10) to equation (8) gives.

$$(K_1\sin\gamma_1 - K_2\sin\gamma_2)\cos\theta = (K_2\cos\gamma_2 + K_1\cos\gamma_1)\sin\theta \tag{11}$$

Jointly (6), (7), (9), (10) yields the polar coordinates of the passively received signal drone.

$$\begin{cases} \theta = \arctan(\dfrac{K_1\sin\gamma_1 - K_2\sin\gamma_2}{K_2\cos\gamma_2 + K_1\cos\gamma_1}) \\ \rho = K_1\sin\gamma_1\cos\theta - K_1\sin\theta\cos\gamma_1 \end{cases} \tag{12}$$

Similarly, the polar coordinates of the passively receiving signal drone in the second case can be obtained as

$$\begin{cases} \theta = \arctan(\dfrac{K_2\sin\gamma_1 - K_1\sin\gamma_2}{K_1\cos\gamma_2 + K_2\cos\gamma_1}) \\ \rho = \dfrac{R\sin(\pi-(\theta-\theta_1)+\alpha_1)}{\sin\alpha_1} \end{cases} \tag{13}$$

Polar coordinates of the passively received signal UAV in the third case.

$$\begin{cases} \theta = \arctan\left(\dfrac{K_1 \sin\alpha_1 - K_2 \sin(\alpha_2 - \theta_2)}{K_1 \cos\alpha_1 + K_2 \cos(\alpha_2 - \theta_2)} \right) \\ \rho = \dfrac{R\sin(\alpha_1 - \theta)}{\sin\alpha_1} \end{cases} \tag{14}$$

Polar coordinates of the passive reception signal UAV in the fourth case.

$$\begin{cases} \theta = \arctan\left(\dfrac{K_2 \sin(\alpha_1 + \theta_2) - K_1 \sin\alpha_1}{K_2 \cos(\alpha_2 + \theta_2) - K_1 \cos\alpha_1} \right) \\ \rho = \dfrac{R\sin(\alpha_1 - \theta)}{\sin\alpha_1} \end{cases} \tag{15}$$

3.2 Optimization of positioning model

First, in *△OAB* and *△OBD*, the relationship between the angles of the triangles and the sine theorem are known, such that

$$k_1 = \frac{R}{\sin\alpha_1} = \frac{\rho}{\sin\angle BAO} \tag{16}$$

$$k_2 = \frac{R}{\sin\alpha_2} = \frac{\rho}{\sin\angle BDO} \tag{17}$$

Combining equations (4) and (5) while making $\beta = \alpha_2 + \theta_3$ we can obtain

$$k_1 \sin(\theta_1 + \alpha_1) = k_2 \sin(\beta - \theta_1) \tag{18}$$

Expanding equation (18) and combining like terms yields

$$\theta_1 = \arctan\left(\frac{k_2 \sin(\alpha_2 + \theta_3) - k_1 \sin\alpha_1}{k_1 \cos\alpha_1 + k_2 \cos(\alpha_2 + \theta_3)} \right) \tag{19}$$

The lengths of *BD* and *AD and CD* can be known from *△AOD* and *△BOD* and *△COD* as

$$BD = k_2 \sin(\theta_3 - \theta_1) \tag{20}$$

$$AD = 2R\sin\frac{\theta_3}{2} \tag{21}$$

$$CD = 2R\sin\frac{\theta_3 - \theta_2}{2} \tag{22}$$

From the relationship between the angles of a triangle, we know that

$$\angle ACB = \pi - (\alpha_1 + \alpha_2 + \alpha_3) - \frac{\pi - 2\alpha_1 - 2\theta_1 + \theta_2}{2} \tag{23}$$

Thus obtained.

$$\angle BCD = \frac{3\pi - \theta_3 - 2\alpha_1 - 2\theta_1 + \theta_2}{2} \tag{24}$$

Combining equation (3) with the above equation yields the simplified model.

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$$\begin{cases} \theta_1 = \arctan\left(\dfrac{k_2\sin(\alpha_2+\theta_3)-k_1\sin\alpha_1}{k_1\cos\alpha_1+k_2\cos(\alpha_2+\theta_3)}\right) \\ \dfrac{k_2\sin(\theta_3-\theta_1)}{\sin(\pi-\theta_1-\alpha_1-\frac{\theta_3}{2})} = \dfrac{2R\sin\frac{\theta_3}{2}}{\sin(\alpha_1+\alpha_2)} \\ \dfrac{2R\sin\left(\frac{\theta_3-\theta_2}{2}\right)}{\sin\alpha_3} = \dfrac{k_2\sin(\theta_3-\theta_1)}{\sin\left(\frac{3\pi-\theta_3-2\alpha_1-2\theta_1+\theta_2}{2}\right)} \\ \dfrac{R}{\sin\alpha_1} = \dfrac{\rho}{\sin(\pi-\theta_1-\alpha_1)} \end{cases} \tag{25}$$

4. Discussions

The drone flying in the air is a three-dimensional space problem, and the drones in this paper are all guaranteed to be at the same altitude, which transforms the three-dimensional space problem into a two-dimensional plane problem, making the process of solving the model easier and reducing the possibility of special cases[4].

In the process of building the model, some properties of triangles such as the sine theorem of triangles are used reasonably, and by the mathematical model finally obtained is made more rigorous by the constant transformation between the angles[5]. In the process of solving the problem, the model is considered in a variety of situations to make the problem more comprehensive.

5. Conclusions

Passive positioning system has the advantages of no electromagnetic radiation and good concealment. This paper is based on a model established in a two-dimensional plane and uses triangular properties such as the sine theorem in its process to establish polar coordinates to simplify the calculation and achieve pure azimuthal passive positioning of UAVs. The determination of the position between ships at sea and the position between cars on land is also carried out on the two-dimensional plane, so the model established in the problem can be extended to the problem of positioning between ships at sea, and can also be applied to the positioning between cars on the road.

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