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Study on Critical Parachute Opening Speed Based on Air Resistance

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Abstract: *To calculate the maximum safe jump altitude that a skydiver can achieve on a vertically launched rocket, and to calculate the critical opening speed and the landing speed in this condition, this paper deduces the theoretical formula for the critical parachute opening speed based on the inflation volume theory and analyzes the factors affecting the critical parachute opening speed. It is found that the critical opening speed of the parachute is inversely proportional to atmospheric density and fabric permeability. On this basis, the influence of air resistance, gravitational force, Earth rotation, and other factors are analyzed. The movement state and possible fall trajectory of the skydiver are studied, a physical model is established, and the safe landing speed that the human body can bear is combined to analyze and calculate the maximum height at which the skydiver can safely land on the ground. In the end, it is calculated that the maximum jump height at which a skydiver could safely land is approximately H ≈ 5250.63 km.*

Keywords: *Skydiving, Parachute Opening Speed, Air Resistance, Parachute Inflation*

1. Introduction

Skydiving is a highly challenging and risky activity that involves jumping from an aircraft at a high altitude and safely landing using parachute equipment[1, 2]. In the process of skydiving, the design and performance of the parachute equipment are crucial, particularly the parachute opening speed, which is one of the key factors determining the success of the parachute deployment[3, 4]. The opening speed refers to the transition of the skydiver from a free-fall state to a state influenced by the parachute's resistance, eventually achieving a safe descent speed[5, 6].

This study establishes physical models and derives relevant formulas to analyze the effects of air resistance, gravity, atmospheric density, and parachute material properties on the deployment of the parachute and the descent of the skydiver. The findings not only provide a theoretical basis for improving skydiving technology but also offer valuable references for assessing the safety of high-altitude and rocket-launched skydiving.

2. Critical Parachute Opening Speed Analysis

This study begins by analyzing the initial phase of a skydiver's descent from maximum height, during which the skydiver is affected by air resistance. Assuming an initial speed of zero, the skydiver experiences slower acceleration at the early stage of the fall due to lower air density, and the parachute may not be fully deployed. A certain distance of acceleration is required for the parachute to fully open, reaching a speed known as the critical parachute opening speed. This study focuses on calculating this critical opening speed.

To determine the critical opening speed, the parachute is considered to inflate in a bulb-like shape, resembling a combination of a hemispherical shape and a truncated cone, as illustrated in Figure 1. For the parachute to successfully inflate, the volume of gas entering the canopy must exceed the volume of gas exiting it. Assuming no change in gas density, the expression for the change in the volume of gas inside the canopy is derived.

$$\frac{dV}{dt} = k_1 \pi R_1^2 v_1 - (2\pi R_2^2 - \pi R_3^2) v_2 - k_2 \pi R_3^2 v_1 \tag{1}$$

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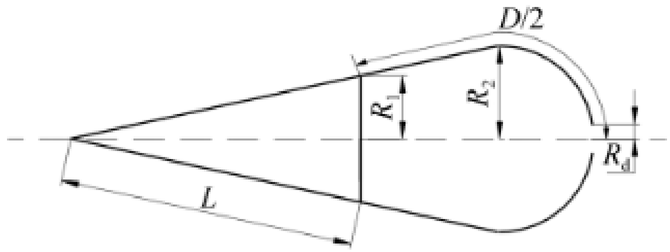

Figure 1: Schematic diagram of parachute inflation configuration

where V represents the volume inside the canopy; v_1 is the velocity of the gas entering the canopy; R_1 is the radius of the bottom edge of the inflated canopy; R_2 is the hemispherical radius at the top of the inflated canopy; R_3 is the radius of the top vent of the canopy, which also indicates the air permeability of the top structure; v_2 is the velocity of the gas exiting the canopy, reflecting the air permeability of the canopy fabric; k_1 and k_2 are correction factors used to adjust the parameters in the equation as necessary. From this equation, when $\frac{dV}{dt} > 0$, the canopy inflates; $\frac{dV}{dt} < 0$, there will be cases that the parachute is closed. Critical parachute opening speed v_k is the value of v_1 when $\frac{dV}{dt} = 0$, as shown in following equation and Table 1:

$$k_1 \pi R_1^2 v_k - (2\pi R_2^2 - \pi R_3^2) v_2 - k_2 \pi R_3^2 v_k = 0 \tag{2}$$

According to the parachute fabric air permeability theory has eq 3:

$$\left.\begin{aligned} \Delta p &= a v_2 + b v_2^2 \\ \Delta p &= \frac{1}{2} C_p \rho v_1^2 \end{aligned}\right\} \tag{3}$$

Table 1: The air permeability of different fabrics under different differential pressure conditions

Type	a	b
518 Anti-scorching white brocade silk	90.9	81.1
K59321 Anti-scorching white brocade silk	9114.3	21260.6
544 Anti-scorching calendered red brocade plaid	5250.2	11694.8
k58326 Anti-scorching calendered red brocade plaid	276.1	129.8
K58326 Anti-scorching red brocade plaid	232.9	109.8
K58326-3 Anti-scorching red brocade plaid	152.4	109.8
K59222 Anti-scorching white brocade lattice	154.1	166.6

Below this study discusses the relationship between the critical parachute opening speed and the density of the atmosphere.

For fabrics with different air permeability, the critical opening speed decreases with the increase of atmospheric density. The reason for this phenomenon is that the density of the atmosphere affects the effective air permeability of the fabric. Effective air permeability.

$$W_y = \frac{v_2}{v_1} \tag{4}$$

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Combined with this formula, the effective air permeability of the fabric is obtained:

$$W_y = \frac{r^2}{8\mu H}\sqrt{\frac{\rho \Delta p}{2}} \tag{5}$$

Where is the radius of the small hole in the fabric; is the air viscosity coefficient; for the thickness of the fabric?

The inflated state of the parachute is illustrated in Figure 2. Initially, the parachute lines are straightened, as shown in Fig.2 (a). Then, the canopy expands from a folded state to an approximately cylindrical shape, as depicted in Fig. 2 (b). This expansion continues outward until the descent speed reaches the critical opening speed, as shown in Fig.2 (d), at which point the canopy stops inflating.

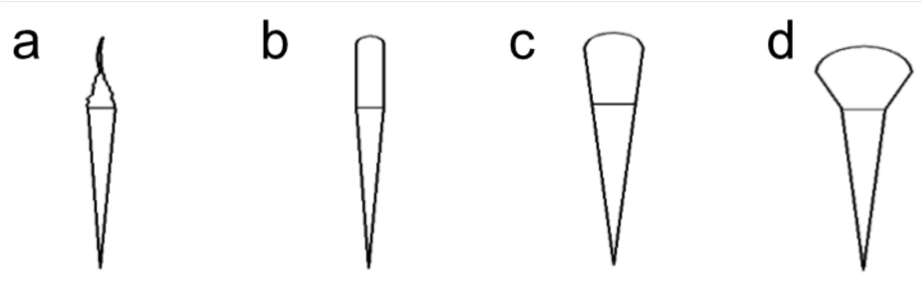

Figure 2: Typical inflation process of a closed parachute; (a) The air mass extends to the top of the parachute; (b) Cylindrical inflatable state; (c) The canopy extends outward; (d) The canopy stops inflating.

Considering that the density of the atmosphere is small at the beginning of acceleration, and the windward area before the parachute is fully opened is constantly changing and much smaller than the windward area after it is fully opened, therefore, we ignore the effect of air resistance during this phase and regard it as a free-fall motion, and the direction of movement of the parachute is directed towards the center of the earth, that is the following Eq 6:

$$F_G = \frac{GMm}{(R+h)^2} \tag{6}$$

The relationship between gravitational force and height above the ground is shown in the figure 3:

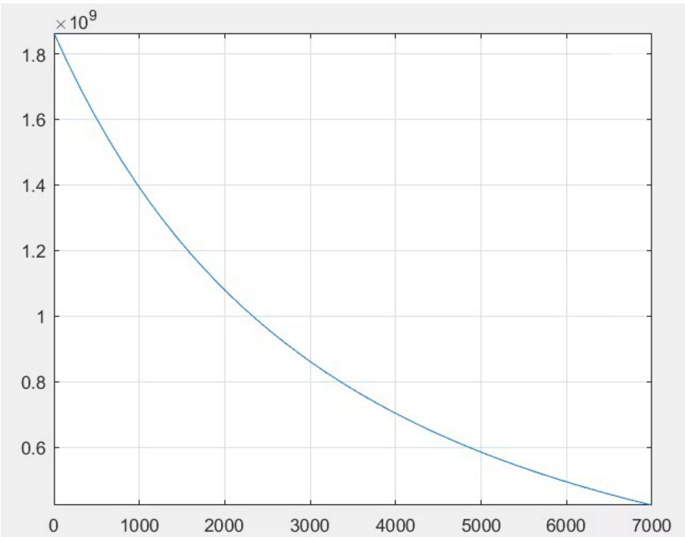

Figure 3: Gravitational Force vs. Height

When the human and parachute velocity reaches, the total kinetic energy is related to the height above the earth.

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$$E_K = \frac{1}{2} m v_k^2 = \frac{GMm}{R+H_1} - \frac{GMm}{R+H} \tag{7}$$

3. Dynamic Analysis of Fully Opened Parachute

After the parachute is fully opened, the effect of air resistance is not negligible, and its magnitude is:

$$f = \frac{1}{2} C_d \rho S v^2 \tag{8}$$

An expression that is expressed by the change in atmospheric density with altitude.

$$\rho = \rho_0 e^{-1.256 \times 10^{-4} \times h} \tag{9}$$

$$\rho_0 = 1.29 kg/m^3 \tag{10}$$

when a skydiver in a vertically launched rocket jumps from a high altitude, the relationship between the air resistance and atmospheric density is shown in figure 4:

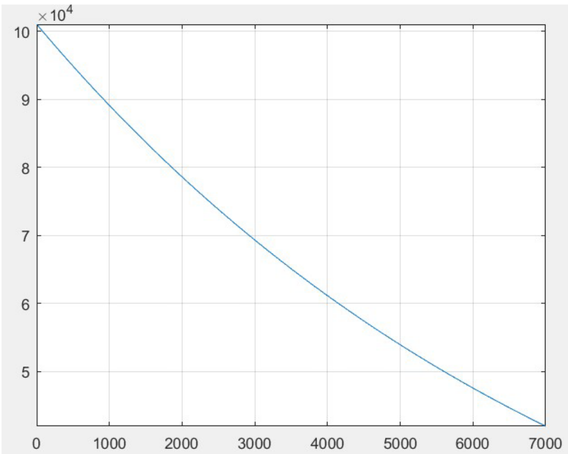

Figure 4: Air Resistance vs. Atmospheric Density at High Altitude

Performing a dynamic analysis of the parachute, taking a spherical model of the Earth and taking into account the rotation of the Earth, approximating the windward area to the area of a circle, is $S=\pi R^2$, then the equation of motion for a single flying body can be written as follows:

$$\begin{cases} \frac{dh}{dt} = v \sin\gamma \\ \frac{d\theta}{dt} = \frac{v \cos\gamma \cos\psi}{(R+h)\cos\phi} \\ \frac{d\phi}{dt} = \frac{v \cos\gamma \sin\psi}{R+h} \\ \frac{d\gamma}{dt} = -\frac{GM}{(R+h)^2}\cos\gamma + \frac{v^2}{R+h}\cos\gamma \\ +2\omega v \cos\psi \cos\phi \\ +\omega^2 (R+h)\cos\phi(\cos\gamma\cos\phi + \sin\gamma\sin\psi\sin\phi) \\ \frac{d^2\psi}{dt^2} = -\frac{v^2}{(R+h)}\cos\gamma\cos\psi\tan\phi \\ +2\omega v(\tan\gamma\sin\psi\cos\phi - \sin\phi) \\ -\frac{\omega^2(R+h)}{\cos\gamma}\cos\psi\sin\phi\cos\phi \end{cases} \tag{11}$$

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Simplifying the above equation yields a differential equation for the change in the velocity of a person and a parachute.

$$\frac{dv}{dt} = -\frac{C_d}{2m}\rho v^2 S - \frac{GM}{(R+h)^2}\sin\gamma + \omega^2 (R+h)\cos\phi(\cos\gamma\cos\phi - \cos\gamma\sin\psi\sin\phi) \tag{12}$$

Integrals for the above formula:

$$v(h,t) = -\frac{C_d}{2m}\rho v^2 St - \frac{GM}{(R+h)^2}\sin\gamma t + \omega^2 (R+h)\cos\phi(\cos\gamma\cos\phi - \cos\gamma\sin\psi\sin\phi)t \tag{13}$$

The speed is v_k when the parachute is fully opened, and the height is H_1, so

$$v_k = v(H_1, 0) \tag{14}$$

From the review of the data, it can be seen that the safe landing speed of people is about $6m/s$, so when $h=0$.

$$v(0, t_{arr}) \leqslant 6 \tag{15}$$

By solving the above formula, it can be obtained H_1

Bring H_1 to Eq7, we can know that:

$$\frac{1}{2}mv_k^2(H_1) = \frac{GMm}{R+H_1} - \frac{GMm}{R+H} \tag{16}$$

And then we can find $H \approx 5250.63km$.

4. Conclusions

This study analyzes the skydiving process from maximum altitude, focusing on the critical parachute opening speed, which is essential for successful deployment and skydiver safety. During the initial fall, where the skydiver is slower and air density is lower, the parachute requires a certain distance of acceleration to fully inflate. The volume change of gas inside the canopy was modeled, determining that inflation occurs when the inflow volume exceeds the outflow. The relationship between critical opening speed and atmospheric density was examined, revealing that the critical speed decreases with increasing atmospheric density due to its effect on fabric air permeability.

After full parachute deployment, air resistance becomes significant, and the equations describing the skydiver's motion were derived, considering atmospheric density changes with altitude and Earth's rotation. The study established that the safe landing speed is approximately 6 m/s, concluding that the critical parachute opening speed and the height at which it opens are crucial for a safe descent. This research provides a concise understanding of parachute dynamics, offering insights for enhancing skydiving safety protocols.

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