

A Study on Value Assessment of E-Commerce Enterprises Based on the Model of Real Options

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Abstract: With the rapid development of network technology, global e-commerce enterprises are mushrooming, and corresponding economic behaviors such as investment, financing and mergers and acquisitions of e-commerce enterprises are constantly appearing. Therefore, it is very important to construct the value assessment system of e-commerce enterprises. This paper takes the characteristics and value of e-commerce enterprises as a starting point, and explores the value assessment of e-commerce enterprises based on the real option theory of Black-Scholes (B-S), which is a model with good applicability in market transactions. Not only can the model reduce the computational complexity of enterprise value, it can also improve the accuracy of value assessment, which is now an important reference for the value assessment of e-commerce enterprises in the era of big data.

Keywords: e-commerce enterprise; real option; Black-Scholes; value assessment

1. Introduction

In the 1990s, thanks to the advancement of Internet and artificial intelligence, people have strode into the Internet era and smart era. Information economy and network economy have also become one of the hot topics in the economic field. E-commerce was born under such a general trend and rapidly developed, with far-reaching impact on modern society. Since then, capital activities such as investment and acquisition of e-commerce enterprises have become more and more frequent, and it is more and more important to conduct accurate value assessment of such enterprises. However, as a new industry, the intangible assets, the expenses of research and development and goodwill of e-commerce enterprises all account for a relatively large proportion, and the growth is difficult to estimate, and the fluctuation of revenue is also great, which makes e-commerce enterprises valued more difficultly and causes greater investment risks. There are great differences between e-commerce enterprises and traditional industries in terms of asset composition, risk premium, and business model, which makes it hard for traditional methods of enterprise value assessment, such as the income method, cost method, and market comparison method, to effectively assess the value of e-commerce enterprises.

(Higson & Briginshaw, 2000)^[1] argue that although the traditional income approach is accepted by more and more enterprises and has achieved great development results, since most e-commerce enterprises are in the rising stage and have not yet entered the growth stage, and they gradually expand at this time and face large performance fluctuations, many financial indicators data in the financial statements are negative, therefore the method has no way to accurately predict future earnings. (Trueman, Wong, & Zhang, 2000)^[2] found there exist significant valuation differences between e-commerce enterprises, Internet firms, and content community firms when using the traditional market approach. (Lumpkin & Dess, 2004)^[3] used the discounted cash flow model as an example to assess the value of Internet firms and found that the model does not adequately account for the volatility of Internet firms' earnings, resulting in an under-valuation of the firms. (Lin, 2003)^[4] argues that for e-commerce enterprises, the valuation should focus on the asset value of data, such as the number of users of the relevant e-commerce enterprises, the traffic owned by the website, and the analysis capability of big data behind it, etc. The cost of these elements is difficult to be recalculated, and the balance sheet cannot fully reflect these elements, so many challenges appear when enterprises conduct assessment by using the valuation model of cost approach. As a result, a number of new valuation methods, such as real option method, client value method, and DEVA method based on Metcalfe's law, have emerged.

Among them, the real option method has been widely used in company valuation and the related theory has been relatively mature. Regarding real options and their pricing theory, (Alleman, Alleman, & Noam, 1999)^[5] applied this method to the valuation of U.S. telecommunication companies based on a

systematic analysis of real option theory. (Sáenz-Diez, Gimeno, & De Abajo, 2008)^[6] used respectively the real option model and the discounted cash flow model to evaluate the overall value of Internet enterprises, and compared the differences in the results of the two methods, finally concluded that because Internet enterprises have certain option characteristics, the value evaluation results of the real option method are more realistic and reliable, and can reflect the true value of the enterprises. (Nembhard, Shi, & Park, 2000)^[7] used the B-S model to evaluate the value of online marketing projects of Internet enterprises, and found that it was relatively reasonable to use the B-S model to evaluate the potential value of Internet enterprises by examining the characteristics of online marketing projects. (Tsai & Hung, 2009)^[8] proposed a new integrated real option approach combined with analytic hierarchy process, which can investigate the dynamic pricing of Internet retail business under risk volatility conditions. The results show that the integrated real option approach is a model that can provide managers with an effective basis for decision making.

In combing through the literature on e-commerce enterprise value assessment, it is found that although the real option approach has been widely used in enterprise value assessment, most studies have not established a complete enterprise value assessment system based on financial and non-financial indicators. Then, few articles have applied the real option model to the value assessment system of e-commerce enterprises. This paper proposes a complete and efficient value assessment model for e-commerce enterprises based on the characteristics of e-commerce enterprises by deriving a real option value assessment model suitable for e-commerce enterprises and screening the selection of parameters in the model, so as to achieve investment risk control and management in the e-commerce industry. The rest of the paper is organized as follows: the second part analyzes necessity and applicability of the real option method for evaluating e-commerce enterprises, the third part is about the construction of e-commerce enterprise valuation model based on the real options, and the fourth part is the conclusion.

2. Necessity and applicability of the real option method for evaluating e-commerce enterprises

2.1. Necessity of the real option method for valuing e-commerce enterprises

It is very necessary to introduce real options into the valuation of e-commerce enterprises, which is mainly reflected in the following four aspects.

(1) In the context of economic globalization, the development of e-commerce industry is fickle, with strong product competition and fast update, and the enterprises have the characteristics of high risk, rapid development and technological innovation, which means that the return on investment has high uncertainty. At the same time, investors often face uncertainties such as information asymmetry and incomplete access to information in the process of e-commerce venture investment, which leads to a high risk of investment; moreover, once investors determine to invest, the investment behavior is irreversible. When the uncertain factors in the investment cause that the investment in e-commerce projects cannot be further promoted or cannot produce benefits, it is difficult to recover the large amount of money invested by investors in the early stage until it is transformed into silent costs, which brings great economic losses to investors.

(2) When valuing the enterprise assessment of e-commerce enterprise investment, the traditional evaluation method of enterprise value cannot accurately and reasonably reflect the true value of the invested enterprise, and there will be the situation that the quotation for the proposed e-commerce enterprise is too high or too low. Over-pricing will cause excessive capital pressure on the investment institution and shrink the profit level or even bring about investment losses; however, under-pricing will also influence the decision of the investment institution and lead to the neglect of e-commerce enterprises with good growth potential.

(3) As an effective way for investment institutions to realize capital expansion, the core element of e-commerce venture capital is to make a comprehensive, effective and accurate assessment of the assets, investment risks and investment returns of e-commerce enterprises, and to determine a price acceptable to both investment institutions and enterprises on the basis of a reasonable assessment. Therefore, the evaluation of e-commerce enterprises through a standard and reasonable method is of great importance to both investors, and is a key element to realize the risk investment management of e-commerce enterprises and determine the success of the investment.

(4) Since there are features that meet the criteria of real options in the investment decision process and the subsequent project construction, development and promotion in e-commerce investment, it is very feasible to introduce the mathematical and financial thinking of real options theory into the

investment decision process of e-commerce enterprises, and the enterprise value assessment method of real options can be accurately applied into the investment process of e-commerce projects. The traditional risk management and enterprise value assessment methods of e-commerce enterprises have been improved by applying real option thinking and methods to e-commerce enterprise value assessment, which provides a new perspective and method for investment institutions to conduct risk investment and value assessment.

2.2. Suitability of the real option method for valuing e-commerce enterprises

According to the characteristics of e-commerce enterprises and the analysis on the necessity of using the real option method to evaluate e-commerce enterprises, it is suitable to introduce Real Option when evaluating enterprise assessment, mainly for the following reasons:

(1) E-commerce enterprises are greatly affected by market fluctuations, their own business strategies and other uncertainties in the process of development, and the real option method can well consider these issues and assess the option value of these uncertainties, and if the greater the uncertainties faced by the enterprise in the future, it means that it has a higher value of real options in itself.

(2) For e-commerce enterprises, the potential value such as the intangible assets owned by the enterprise and the technological innovation brought by the enterprise's research team accounts for a large proportion of the enterprise's value composition, so for e-commerce enterprises, the expansion option value brought by this part of the value expansion is very important, which is precisely what the traditional enterprise value analysis method cannot achieve but what the real options method can achieve.

(3) E-commerce enterprises have more obvious stage characteristics in the development process, and the decisions between these stages will affect each other, and the decisions in each stage are dynamic, and the enterprise managers need to adjust the business strategies quickly and accurately according to the unpredictable market environment, and it is difficult for the traditional evaluation methods to evaluate the various options in the process of enterprise development, while the real option method can reflect the value brought by flexible decision-making at each stage.

Therefore, the theoretical value assessment method of real options not only takes into account the value of e-commerce enterprises based on the time value of cash flows, but also takes into account the value of uncertainty, expansion value, and the value of right of choice implicit in the investment. In general, when applying the real option method to assess the value of e-commerce enterprises, the enterprise value includes not only the expected income but also the potential value that the e-commerce enterprises enlarge new markets and expand market share, reduce systematic risk and enhance inherent competitiveness, in order to reckon the risk management and the decision value of flexible choices for e-commerce enterprise investment, and to further evaluate the full value of e-commerce enterprises comprehensively, reasonably and scientifically.

3. Construction of the value assessment model of e-commerce enterprise based on real options

3.1. The value composition model of e-commerce enterprises

(Gupta, Lehmann, & Stuart, 2004)^[9] pioneered the idea that enterprise value assessment can be divided into intrinsic value and potential value to compensate for the shortcomings of traditional enterprise value assessment methods. Specifically, traditional discounted cash flow and other value assessment models cannot accurately estimate the potential value of an enterprise, and this approach provides a more scientific and accurate way to assess the value of an enterprise. E-commerce enterprises are very different from traditional enterprises in terms of business patterns and development forms, so their values cannot be assessed by traditional value measurement methods only.

According to the evaluation theory and method of real options, a complete assessment of e-commerce enterprises should be carried out from two aspects. On the one hand, it is the intrinsic value of the e-commerce enterprise itself, i.e., the existing value, called static net present value value, which is mostly calculated by traditional enterprise value assessment methods, and is also calculated by the discounted cash flow method and net present value and other models apart from the free cash flow discounting mentioned in the theoretical part. On the other hand, it is the value of real options of e-commerce enterprises, which is actually an option held by investors on the assets of e-commerce enterprises, and this part can be calculated through the method of real options. Therefore, the value of investment in e-commerce enterprises is the sum of the intrinsic asset value assessed by traditional methods and the value

of real options, which can be expressed by the formula:

$$V = V_1 + V_2 \quad (1)$$

Where, V denotes the total value of e-commerce enterprise, V_1 denotes the static net present value value of e-commerce enterprise, and V_2 denotes the value of real option of e-commerce enterprise.

For the existing value of e-commerce enterprises V_1 , it can be calculated by the traditional asset value assessment model, and most scholars calculate it by the discounted free cash flow model, which is a more mature and generally accepted model among the evaluation models of assets of e-commerce enterprises at present and an important basis for conducting the feasibility assessment of e-commerce enterprise venture investment (Kalyebara & Islam, 2014; Tan, 2017).^[10-11]

Based on this, this paper evaluates the intrinsic value of e-commerce enterprises by selecting the discounted free cash flow model. In order to forecast the future cash flows of the investee company, the investor needs to analyze the financial statements and related indicators stored in the investee e-commerce company in the past years, so as to understand the historical results of the company. Next, the future operating income of the enterprise is forecasted by combining the development prospect of the enterprise and the market prospect of the corresponding industry, and a suitable discount rate is chosen to calculate the present value of the possible future free cash flow of the enterprise, and the existing value of the e-commerce enterprise is evaluated by this method.

3.2. Free cash flow discounting model (V_1)

This paper specifically adopts a two-stage free cash flow discounting model, which is divided into two stages according to the development of e-commerce enterprises, the first stage being the rapid growth stage and the second stage being the stable growth stage. In the first stage, the e-commerce enterprise achieves rapid and sustained growth in revenue and profit, and the annual free cash flow of the enterprise is discounted and summed up by predicting the future revenue growth rate of the investee enterprise. In the second stage of e-commerce enterprise development, the efficiency of the invested enterprise has entered the stage of stable growth, and at this time, after prophase continuous precipitation, the enterprise development is more mature and the growth rate gradually tends to be stable. In summary, the value of e-commerce enterprises is the sum of the value of these two stages.

$$V_1 = \sum_{t=1}^n \frac{FCFF_t}{(1+WACC)^t} + \frac{FCFF_{n+1}}{(WACC-g)(1+WACC)^n} \quad (2)$$

where $FCFF_t$ denotes the cash flow generated by the firm in year t , g denotes the stable growth rate of the firm after n years, and $WACC$ denotes the firm's weighted average cost of capital, or discount rate.

3.2.1. Determination of discount rate

With the widespread use of discounted free cash flow models, how to determine the discount rate accurately has become a difficult part of the valuation process. The average cost of capital, also known as the discount rate, refers to that the projected cash flows are converted into some undetermined ratio of present value. The determination of the discount rate is not a complex and redundant task. Due to the estimation of cash flows is full of uncertainties and the investor will face certain risks as long as the investment is made, the investor is bound to ask for partial compensation for the risks. Because of the uncertainty of the investment and the risk avoidance of the investor, it requires that the discount rate of the free cash flow discounting model must be higher than the rate of return of the risk-free investment. Hence, determining a reasonable discount rate is essentially determining a reliable rate of return on investment. The methods for determining the discount rate are further divided into the risk accrual method, the average industry return on assets method, and the weighted average cost of capital method. Among them, the risk accrual method is an estimation method that relies on professional appraisers, and the accuracy of the appraisal results is very dependent on the professional level of appraisal experts; The average industry return on assets method is a method that uses the average return on assets of the industry in which the target enterprise is located as the discount rate of the appraised enterprise, and because of the great variability among global e-commerce platforms and the different risk factors faced by each enterprise, measuring the return on investment by this method does not meet the actual situation; The last method is the weighted average cost of capital method, which is a method that uses the weighted average cost of capital of all assets of the appraised enterprise as the discount rate, and its calculating formula is:

$$WACC = R_d \times (1 - T) \times \frac{D}{D + E} + R_e \times \frac{E}{D + E} \quad (3)$$

Where $WACC$ is the discount rate, D is the debt capital, E is the equity capital, T is the income tax rate, R_d is the cost of debt capital, and R_e is the cost of equity capital.

Consistent with most researches, the paper also selects the interest rate of treasury bonds used for the forecast period at the same maturity to be the cost of debt capital, and adopts the Capital Asset Pricing Model (CAPM) to determine the cost of equity capital, calculated as follows:

$$R_e = R_f + \beta(R_m - R_f) \quad (4)$$

where R_f is the risk-free rate of return; R_m is the expected rate of return; $R_m - R_f$ is the risk premium; and β is the systematic risk factor. The parameters are determined in the following manner.

(1) Determination of risk-free rate of return (R_f)

The risk-free rate of return to be determined in the paper is the market rate at which the investor can make loans and lend funds at will in the market. The risk-free rate is generally determined in three ways: the first is to consider the interest rate of short-term treasury bonds as the risk-free rate, the second is to consider short-term government debt as the risk-free rate, and the third is to consider the interest rate of long-term treasury bonds as the risk-free rate. However, in the real evaluation practice, the interest rate of long-term treasury bonds is preferred as the risk-free rate because of its high safety factor, low volatility and high stability.

(2) Determination of expected rate of return (R_m)

Expected rate of return refers to the rate of return on investment that the investor hopes to receive in the end. Since stock market yields are subject to a variety of factors and often fluctuate, it is almost impossible to predict the average level of yields if only the more short-term stock fluctuations are used. Therefore, the expected rate of return is generally obtained by calculating the average of the long-term market returns, and usually the average of the long-term stock market returns for the last ten years is used as the expected rate of return.

(3) Determination of risk factor (β)

The indicator used in the capital asset pricing model to measure the magnitude of systematic risk is β . If β is greater than 1, it means that the systematic risk in the calculation period is greater than the average market risk, and if β is less than 1, it means that the average market risk in the calculation period is greater than the systematic risk. Usually, the estimation method of β is to fit the yield of the enterprise to be valued with the yield of the whole market for the analysis. For the listed enterprise with valuation, the factor β of the enterprise can be directly queried through the software.

(4) Determination of discount factor (R_i)

After discount rate is calculated, the discount factor for each year will also be calculated by the number of various discount periods, which is calculated as follows

$$R_i = \frac{1}{(1 + i)^n} \quad (5)$$

Where, R_i denotes the discount factor, i denotes the discount rate, and n denotes the number of discount periods.

3.2.2. The forecast of future cash flow

Before evaluating the value of e-commerce enterprises, it is necessary to make basic assumptions for evaluation. It should be first analyzed from two aspects: the future development prospect of the enterprise to be evaluated and the future development of the industry to which the enterprise to be evaluated belongs. Through the analysis of the historical data and future development strategy of the invested enterprise, the future operating income of the enterprise is estimated in conjunction with the future development trend of the industry in which the enterprise is located, and the future free cash flow of the enterprise is further calculated, and finally, the cash flow value of the stable growth period of the enterprise is calculated based on the free cash flow value obtained from the previous estimation.

To forecast the cash flow of the assessed enterprise for the forecast period, it is necessary to collect the financial history data of the enterprise for at least the past five years and calculate the growth rate of sales revenue, and then the sales revenue of the future forecast period is forecasted by the growth trend.

Subsequently, the average values of operating income, depreciation and amortization, proportion of increase in capital expenditures, various expenses and increase in working capital for the previous five years are calculated separately, which is multiplied with the value of sales revenue previously predicted, then the predicted value of each index in the future years is obtained. The calculating formula is:

$$\text{free cash flow} = \text{profit before interest and tax} \times (1 - \text{income tax rate}) + \text{depreciation and amortization} - \text{working capital changes} - \text{capital expenditures}$$

Through the formula, the free cash flow for each year of the expected life is calculated, and the free cash flow is multiplied with the previously calculated discount factor for each year to obtain the net present value for each year, and the net present value for each year is added up to obtain the enterprise value for the forecast period of the enterprise to be evaluated.

3.2.3. Estimation of the subsequent value of the investee enterprise

After reaching the forecast period, the investee enterprise starts to enter the stable development stage, and the main work in this stage is to determine a reasonable perpetual growth rate, that is, we assume that the investee e-commerce enterprise's development has entered a stable period and will grow at a stable growth rate in the future, and the growth rate is the perpetual growth rate. The setting of the perpetual growth rate usually takes into account two factors. One is to take the growth rate of the last year of the forecast period as the perpetual growth rate, because it has been assumed that in the last year of the forecast period, the development of the investee enterprise has stabilized, so it is reasonable to use this method. Another is to consider the GDP growth rate of each country, in general, the perpetual growth rate of the investee enterprise will not exceed the GDP growth rate. The formula is:

$$\text{cash flow in stable period} = \frac{FCFF_{n+1}}{(WACC - g)(1 + WACC)^n} \quad (6)$$

where $FCFF_{n+1}$ denotes the cash flow generated by the enterprise in year n+1 of the forecast period, g denotes the perpetual growth rate of the enterprise after n years, and $WACC$ denotes the weighted average cost of capital of the enterprise, i.e., the discount rate.

The final existing value of the investee enterprise is equal to the expected value plus the subsequent value.

3.3. Study on real option valuation model for e-commerce enterprises (V_2)

3.3.1. B-S option pricing model

The B-S option pricing model was proposed by Black and Scholes in 1973, which is the first more complete option pricing model that can reasonably and accurately estimate the European call options, laying a theoretical foundation for the pricing of financial derivatives within stocks and bond funds, and becoming the most widely used option pricing model at present.

The B-S option pricing model usually constructs a portfolio of risk-free securities when deriving the value of an option. The movement of the underlying asset price needs to be consistent with the Geometric Brownian motion, frictionless market and other basic assumptions to construct a specific portfolio of risk-free securities, such as holding a certain stock while selling a certain percentage of call options, such method can insulate the return of the portfolio from stock price fluctuations, reducing the investment risk brought to investors due to changes in stock prices. If the capital market is in perfect equilibrium, according to the CAPM model, the return on this portfolio is equal to the risk-free rate of return, so the return on the stock option can be replicated by a particular combination of the underlying asset and the risky asset. If in the case of no-arbitrage equilibrium, the option price is the price at which the portfolio is purchased. Therefore, the option price in the B-S option pricing model is independent of the expected return on the stock and various market risk factors, but is affected by five factors: the price of the underlying asset, the risk-free interest rate, price volatility, option exercise price and exercise time. Based on the above, the final mathematical expression of the model is:

$$R_t = P_t N(d_1) - Ke^{-rt} N(d_2) \quad (7)$$

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(\gamma + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad (8)$$

$$d_2 = d_1 - \sigma\sqrt{t} \quad (9)$$

Where R_t represents the European call option price; P_t represents the asset price of the underlying asset on pricing date t ; K represents the option exercise price and r represents the risk-free interest rate when calculated according to continuous compound interest; σ represents the price volatility of the underlying asset, t represents the option term, and $N(X)$ represents the cumulative probability function of standard normally distributed variables.

3.3.2. The establishment of real option value assessment model for e-commerce enterprises

In this paper, based on the characteristics of e-commerce enterprises and the B-S option pricing model, a tradable security is constructed, which has the same risk characteristics with e-commerce enterprises. Meanwhile the paper derives the real option valuation model of the e-commerce enterprise by replicating the return characteristics of the real option of the invested e-commerce enterprise. Based on the theoretical basis of the B-S option pricing model, the following assumptions are made for the real options of e-commerce enterprises.

Assumption 1: The project asset of an e-commerce enterprise is a kind of asset by risk gain or loss, and the relevant trading market of the e-commerce enterprise is open, and investors are free to buy and sell and the transaction is not restricted to costs and taxes.

Hypothesis 2: The cost of the project of the invested e-commerce enterprise when it is opened and closed is not considered, and the permanent abandonment of the project is not considered as well.

Assumption 3: The corresponding real option type of the investee e-commerce enterprise is a European option and can only be executed on the contract expiration date.

Assumption 4: Given the risk-free interest rate r of the investee e-commerce enterprise, the rate is a constant that does not vary with time.

Assumption 5: There will be no risk-free arbitrage opportunity for the investment in the e-commerce enterprise.

Assumption 6: The investor can borrow or lend funds at the same risk-free rate.

Assumption 7: The product price of the e-commerce firm fluctuates randomly and the variation needs to conform to the Geometric Browning motion, in other words, the product price P of the e-commerce firm should obey a logarithmic distribution and the variance of returns is a fixed value that does not change (Zhou et al. 2009)^[12]. It is expressed in the formula as:

$$dP = uPdt + vPdz \quad (10)$$

Where, u is a constant, representing the expected return per unit time of the e-commerce enterprise product, v is also a constant, representing the volatility of the e-commerce enterprise product price, and dz represents a Wiener process, $dz = \varepsilon\sqrt{dt}$, obeying (0,1) normal distribution.

Suppose that the value of the real option R of the invested e-commerce enterprise at moment t is a function of the price of the e-commerce enterprise product at moment t , which is denoted as $R(P, t)$; and also according to the core differential tool ITO theorem (Cox & Ross, 1976)^[13], $Y = Y(x, t)$ for any function of x and t , the following equation always holds:

$$dY = \left(\frac{\partial Y}{\partial x} \mu + \frac{\partial Y}{\partial t} + \frac{1}{2} \frac{\partial^2 Y}{\partial x^2} \sigma^2 \right) dt + \frac{\partial Y}{\partial x} \sigma dz \quad (11)$$

For the above equation, taking $x = P$; $u = u(P, t) = uP$; $\sigma = \sigma(x, t) = \sigma P$, the following is obtained:

$$dR = \left(\frac{\partial R}{\partial P} \mu + \frac{\partial R}{\partial t} + \frac{1}{2} \frac{\partial^2 R}{\partial P^2} \sigma^2 \right) dt + \frac{\partial R}{\partial P} \sigma P \sqrt{dt} \xi \quad (12)$$

The discrete form of Eq.(10) and Eq.(12):

$$\Delta P = \mu P \Delta t + \sigma P \sqrt{\Delta t} \xi \quad (13)$$

$$\Delta R = \left(\frac{\partial R}{\partial P} \mu + \frac{\partial R}{\partial t} + \frac{1}{2} \frac{\partial^2 R}{\partial P^2} \sigma^2 \right) dt + \frac{\partial R}{\partial P} \sigma P \sqrt{dt} \xi \quad (14)$$

Consider such a portfolio of assets like selling an option on a project asset, i.e., losing value R , while buying $\frac{\partial R}{\partial P}$ project asset, i.e., gaining $\frac{\partial R}{\partial P}$ asset, then the value of the portfolio at time t is:

$$Q = -R + \frac{\partial R}{\partial P} \Delta P \tag{15}$$

After the corresponding period of time Δt , the change in the value of the portfolio is:

$$\Delta Q = -R + \frac{\partial R}{\partial P} \Delta P \tag{16}$$

Substituting Eq. (13) and Eq. (14) into Eq. (16) and solving for them, It can be gained:

$$\Delta Q = \left(\frac{\partial R}{\partial t} + \frac{1}{2} \frac{\partial^2 R}{\partial P^2} \right) \Delta t \tag{17}$$

Eq. (17) has eliminated the stochastic term $\sqrt{\Delta t} \xi$, indicating that the change ΔQ in the value of this portfolio has the same value as the portfolio whose return is the risk-free rate.

$$\Delta Q = Q \times \gamma \times \Delta t \tag{18}$$

Where γ represents the instantaneous risk-free rate. Substituting Eq. (15) and Eq. (17) into Eq. (18), the simplified equation is obtained:

$$\frac{1}{2} \sigma^2 P^2 \frac{\partial^2 R}{\partial P^2} + \gamma P \frac{\partial R}{\partial P} - \gamma R + \frac{\partial R}{\partial t} = 0 \tag{19}$$

For the European option, the boundary condition for the real option value in the period $t = T$ is: $P[P(T), T] = \text{Max}[P(T) - K, 0]$, so a stochastic differential equation can be obtained as:

$$\begin{cases} \frac{1}{2} \sigma^2 P^2 \frac{\partial^2 R}{\partial P^2} + \gamma P \frac{\partial R}{\partial P} - \gamma R + \frac{\partial R}{\partial t} \\ P[P(T), T] = \text{Max}[P(T) - K, 0] \end{cases} \tag{20}$$

3.3.3 The solution of the real option valuation model for e-commerce enterprises

A stochastic differential equation for a real option valuation model of e-commerce firms has been established in the previous section, and now the model is to be solved. Let $P = Ke^y$, $t = T - 2 \frac{\alpha}{\sigma^2}$, $R = K * V(y, t)$, and it is obtained:

$$\begin{cases} \frac{\partial V}{\partial a} + \frac{2\gamma}{\sigma^2} V = \frac{\partial^2}{\partial y^2} + \left(\frac{2\gamma}{\sigma^2} - 1 \right) \frac{\partial V}{\partial y} \\ V(y, 0) = \text{Max}(e^y - 1, 0) \end{cases} \tag{21}$$

Let:

$$V(y, a) = \exp \left[\left(\frac{1}{2} - \frac{\gamma}{\sigma^2} \right) y - \left(\frac{1}{2} + \frac{\gamma}{\sigma^2} \right) a \right] U(y, t) \tag{22}$$

It is obtained that:

$$\begin{cases} \frac{\partial U}{\partial a} = \frac{\partial^2 U}{\partial y^2} \\ U(y, 0) = \text{Max} \left\{ \exp \left[\left(\frac{1}{2} + \frac{\gamma}{\sigma^2} \right) y - \left(\frac{1}{2} - \frac{\gamma}{\sigma^2} \right) y \right], 0 \right\} \end{cases} \tag{23}$$

Where $\frac{\partial U}{\partial a} = \frac{\partial^2 U}{\partial y^2}$ is a heat conduction equation, whose solution is:

$$U(y, v) = \frac{1}{2\sqrt{\pi a}} \int_{-\infty}^{+\infty} U(\mu, 0) \exp \left(-\frac{(y - \mu)^2}{4a} \right) d\mu \tag{24}$$

Substituting $U_{y,0}$ into Eq. (24), It can be obtained:

$$U(y, a) = \frac{1}{2\sqrt{\pi a}} \int_0^{+\infty} \exp \left[\left(\frac{1}{2} + \frac{\gamma}{\sigma^2} \right) \mu - \frac{(y - \mu)^2}{4a} \right] d\mu - \frac{1}{2\sqrt{\pi a}} \int_0^{+\infty} \exp \left[\left(\frac{1}{2} - \frac{\gamma}{\sigma^2} \right) \mu - \frac{(y - \mu)^2}{4a} \right] d\mu \tag{25}$$

Let:

$$B_1 = \frac{1}{2\sqrt{\pi a}} \int_0^{+\infty} \exp\left[\left(\frac{1}{2} + \frac{\gamma}{\sigma^2}\right)\mu - \frac{(y - \mu)^2}{4a}\right] d\mu \quad (26)$$

$$B_2 = -\frac{1}{2\sqrt{\pi a}} \int_0^{+\infty} \exp\left[\left(\frac{1}{2} - \frac{\gamma}{\sigma^2}\right)\mu - \frac{(y - \mu)^2}{4a}\right] d\mu \quad (27)$$

Let $\mu = y + \left(\frac{1}{2} + \frac{\gamma}{\sigma^2}\right) \times 2a - \sqrt{2a} \times g$, substitute it into B_1 and simplify the equation, it can be obtained:

$$B_1 = \exp\left[\left(\frac{1}{2} + \frac{\gamma}{\sigma^2}\right)y + \left(\frac{1}{2} + \frac{\gamma}{\sigma^2}\right)^2 v \times \sqrt{2a} \times \int_{-\infty}^{d_1} e^{-\frac{1}{2}g^2} dg\right] \quad (28)$$

In the formula, $d_1 = \frac{y}{\sqrt{2a}} + \left(\frac{1}{2} + \frac{\gamma}{\sigma^2}\right)\sqrt{2a}$

Let: $\mu = y + \left(\frac{\gamma}{\sigma^2} - \frac{1}{2}\right) \times 2a - \sqrt{2a} \times g$, and substitute it into B_2 , it can be obtained:

$$B_2 = \exp\left[\left(\frac{\gamma}{\sigma^2} - \frac{1}{2}\right)y + \left(\frac{\gamma}{\sigma^2} - \frac{1}{2}\right)^2 v \times \sqrt{2a} \times \int_{-\infty}^{d_1} e^{-\frac{1}{2}g^2} dg\right] \quad (29)$$

In the formula, $d_2 = \frac{y}{\sqrt{2a}} + \left(\frac{\gamma}{\sigma^2} - \frac{1}{2}\right)\sqrt{2a}$

Substituting Eq. (28) and Eq. (29) into Eq. (27) and simplifying it, we can obtain the equation of the asset value assessment model for e-commerce enterprises.

$$R = R(P, t) = P_t N(d_1) - Ke^{-rt} N(d_2) \quad (30)$$

Among them, $d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + (\gamma + \sigma^2/2)t}{\sigma\sqrt{t}}$, $d_2 = d_1 - \sigma\sqrt{t}$

When $t=0$, the initial value of the real option value of the e-commerce enterprise can be obtained.

Where, R_t represents the option value of the invested e-commerce enterprise; P_t represents the asset price of the underlying asset at pricing date t ; K represents the option exercise price, r represents the risk-free interest rate when calculated according to continuous compounding; σ represents the price volatility of the underlying asset, t represents the option term, and $N(X)$ represents the cumulative probability function of standard normally distributed variables.

3.3.4. Determination of parameters of the real option valuation model for e-commerce enterprises

(1) Determination of the intrinsic value (P_t) of the assets of the invested e-commerce enterprise

In financial options, the underlying assets are the financial products such as stocks, bonds, and funds purchased by the enterprise, then the current value of the underlying assets is the current price of these financial products, and the final execution price is the price of the financial products executed on the exercise date. However, in real options, it is difficult to determine the underlying asset, i.e., the assets of the e-commerce enterprise, and the choice of the exercise price within the exercise period. The real option method is currently used more often for a specific project or projects invested in the enterprise, so these invested projects can be identified as the underlying assets. The intrinsic value of the underlying asset is the discounted value of the future cash flows that the investment project or portfolio of projects can bring to the enterprise, and further the exercise price of the underlying asset is the cost (investment amount) of the investment project or portfolio.

There are various methods to determine the current value of an enterprise, such as the traditional market valuation method and the discounted cash flow model, both of which can calculate the intrinsic value of an investee enterprise by forecasting the future cash flows of the target enterprise. However, both methods have certain shortcomings. The traditional market approach requires sufficient comparable enterprises for reference by the target enterprise and is not applicable to the valuation of e-commerce enterprises. The disadvantages of the discounted cash flow model have been described in the theoretical section. The main disadvantage of the discounted cash flow model is that the forecast of future cash flows is too subjective, and if the present value of the future cash flows of the investee e-commerce enterprise is taken as the intrinsic value of the subject assets, it greatly intensifies the difficulty and workload of the valuation, because the method is based on the forecast period, it is also necessary to obtain the intrinsic value of the e-commerce enterprise on the base date of the valuation by discounting the cash flows. When the value of real options of an enterprise is evaluated in a large number of literature, the total assets

visible on the books of the enterprise are directly considered as the intrinsic value of the underlying assets (Coopersmith & Cunningham, 2002)^[14]. Based on this, the paper also chooses to use all the assets on the books of the investee e-commerce enterprise as the intrinsic value of the underlying assets and obtain the total assets on the books of the enterprise on the base date of valuation by looking up the financial statements of the investee e-commerce enterprise.

(2) Determination of the option exercise price (K) of the investee e-commerce enterprise

Generally speaking, shareholders buy a business stock because the business has strong development prospects or the development of the business meets the needs of the market at that time and can bring great returns to the business in the future, so the shareholders' investment in the target business can be regarded as a call option. The option cost of that call option is what the shareholder pays when purchasing the stock, which is the transaction price of the stock. The shareholders themselves enjoy the option on the residual value, specifically when the enterprise's total assets are less than its debt portfolio, the shareholders will not receive any income at this time, the value of the option is zero, and all the assets of the enterprise will be used for debt repayment; When the enterprise's liabilities are less than the capital, the shareholders enjoy the option on the residual value of the capital after deducting the debt, and the value of the call option in financial options is calculated in a similar way, which is equal to the current price minus the exercise price.

Thus, if the invested e-commerce business is considered a real option, the final exercise price of the underlying asset is the total liability of the business. A review of the relevant literature shows that it is feasible to use the total liabilities on the books of the target enterprise as the exercise price of the underlying asset (Harun & Abdullah, 2019)^[15]. The method is: the total liabilities on the books of the enterprise are obtained as of the base date of valuation by looking up the most recent financial statements of the investee enterprise.

(3) Determination of the expiration period (t) of the real options of the investee e-commerce enterprise

The exercise period refers to the period from the purchase of the option by the investee to the final exercise of the option. In the paper, the total liabilities of the investee e-commerce enterprise are regarded as the execution price of the option, but this part of the data involves the internal data of the enterprise, so there is no way to know the time of repayment of liabilities at maturity, and the period of option execution needs to be determined from other perspectives instead.

For financial options, the option's expiration date is agreed upon with a very clear time span, but real options usually do not have a precise exercise period, and the exercise period can be estimated by the expected best execution time. The intrinsic value of the invested e-commerce enterprise can be regarded as an option, and then the holder of the option, i.e., the management of the enterprise, will choose a reasonable exercise date to achieve the goal of maximizing the profit of the enterprise according to the current market environment and the current development status of the enterprise. Therefore, from the definition of the exercise date, the investor usually chooses a period that does not change the value of the enterprise too much. The paper follows the most common method used in the valuation of enterprise real options, and regards the number of years used in forecasting the future cash flows of the enterprise as the expiration date of the real options of the investee e-commerce enterprise.

(4) Determination of the volatility (σ) of the asset value of the investee e-commerce enterprise

The volatility of the underlying asset refers to the volatility of the value of evaluated enterprise, then the volatility is the standard deviation of the price return calculated from previous years' data of the price of the underlying asset, and the financial option usually chooses the price of the stock as the volatility. The selection of volatility for real options is slightly different from that of financial options. Real options theory holds that the potential value of an enterprise is composed of both debt value and equity value, and usually the debt option of an enterprise is relatively stable, whose volatility can be ignored, so the volatility of the final enterprise value is the volatility of the equity value. The volatility can be estimated from the data of the evaluated enterprise in previous years, or from the stock price volatility of companies with similar development scale. There are a small number of comparable companies due to the special nature of e-commerce enterprises, then the paper suggests that the standard deviation can be calculated by counting the stock price data of previous years and it can be regarded as the volatility of the underlying asset. The specific calculating method is as follows:

$$\gamma_t = P_t - P_{t-1} \quad (31)$$

$$\bar{\gamma} = \frac{1}{n} \sum_{t=1}^n \ln \gamma_t \quad (32)$$

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (\ln \gamma_t - \bar{\gamma})^2} \quad (33)$$

Where, P_t represents the daily closing price, γ_t represents percentage rate of return of stock price, $\bar{\gamma}$ represents the mean of the continuous compounded return; σ represents the standard deviation which is also the volatility to be calculated in the paper.

In addition, the daily volatility is converted to annual volatility in the calculation, which is calculated according to 252 trading days a year, and the formula is:

$$\sigma_{year} = \sigma_{Day} \times \sqrt{252} \quad (34)$$

(5) Determination of the risk-free interest rate (r) for valid period of the real options of the investee e-commerce enterprise

The risk-free volatility is the market rate at which investors can loan funds at will in the market. At present, there are about three views on the choice of the risk-free interest rate in academia, the first view is to take the interest rate of short-term treasury bonds as the risk-free interest rate, the second view is to take the spot short-term government bonds as the risk-free interest rate, and the third view is to take the interest rate of long-term treasury bonds as the risk-free interest rate. In the practice of real option value assessment, researchers usually prefer to estimate the long-term treasury bond interest rate as the risk-free rate because of its advantages of low volatility and high safety factor. And the paper adopts the long-term treasury rate as the risk-free rate.

The values of the corresponding parameters are calculated by applying the evaluation model of the real option value of the invested e-commerce enterprise which is constructed in the previous paper, and the value of the real option of the invested e-commerce enterprise can be estimated, that is, the evaluation of the risk value of the e-commerce enterprise can be realized.

4. Conclusion

There are characteristics similar to those of real options in the process of making venture investments in e-commerce enterprises and there are various kinds of real options in e-commerce enterprises, which makes it possible to assess the value of e-commerce enterprises with the evaluation system of real options theory. It is an improvement of the traditional e-commerce enterprise value assessment method to assess the value of e-commerce enterprises with the real option method, which provides a new method and perspective for investment institutions to conduct investment risk assessment and value assessment of e-commerce enterprises. Based on the analysis of the applicability and necessity of the real option valuation method for e-commerce enterprises and the research on theoretical methods of real option valuation, the paper constructs and derives a real option valuation model for e-commerce enterprises by analyzing the factors influencing the value of real options of e-commerce enterprises and combining it with the B-S option pricing method. The model is simple to calculate and easy to operate. All that is needed is the determined parameters in the model, and the value of real options of e-commerce enterprises can be evaluated.

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