Slope Stability Evaluation Based on Uncertainty Measurement Theory

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Abstract: The notion of unconfirmed measurement was used to develop an evaluation model of slope instability in open pit mines. To establish an evaluation system, construct a linear measurement function, and determine the weights of each evaluation indicator using an objective assignment method, nine indicators from three categories, namely geological structure, hydrogeology, rock structure, slope height, slope angle, angle of internal friction, vibration velocity of blasting mass, seismic intensity, and average annual rainfall, are chosen based on the importance of the causes affecting slope stability. The engineering application demonstrates that the evaluation model has a specific reference value for open pit mine slope stability analysis and mine safety management.

Keywords: slope stability; uncertainty measurement theory; entropy value method; confidence level

1. Introduction

The "2.22" open pit coal mine slope collapse event in Alashan League, Inner Mongolia, in 2023, reminded people that they must always be aware of the dangers created by the instability of open pit mine slopes while pursuing mineral resources. Previous research had indicated that [1-4] slope engineering was a difficult systematic project with numerous aspects influencing the evaluation of slope instability, particularly in open-pit mines, where these evaluation criteria were more complex. The evaluation mechanism for slope stability analysis could not be properly quantified due to complex geological conditions and varied external influences. The association between the assessment elements and the presence of a certain degree of randomness, resulting in the selection of mechanical parameters, has a constraint, which makes the development of mechanical models problematic. These considerations make slope stability evaluation and analysis more difficult and complex.

The theory of uncertainty measurement was developed to handle the problem of determining the state of objects or the extent of the uncertainty of the state when objective knowledge is imperfect under restricted conditions, and it has been widely employed in many domains [4-6]. The uncertainty measure serves as the theoretical foundation for this research, which uses a mine in Liuyang, Hunan Province, as an example to develop a slope stability evaluation model for open pit mines. To determine the weights of each evaluation factor, the entropy weighting method was used, which was a commonly used objective assignment method to determine the weights only by the relationship between the factors or the degree of change, greatly reducing the influence of human subjective factors on the evaluation indexes. Simultaneously, the confidence level was used as the attribute discrimination criterion, resulting in a simple, effective, economical, and practical slope stability evaluation system.

2. Uncertainty measurement evaluation model [7]

Let the research object have n evaluation factors, notation P=P₁,P₂,P₃,...,Pₙ, and the i-th evaluation factor Pi have a total of y evaluation indexes, notation F=F₁,F₂,F₃,...,Fₚ, so that the i-th evaluation factor Pi has measurements for pij on the j-th evaluation indexes Fij. Let there be a total of m evaluation levels, denoted as U={U₁,U₂,U₃,...,Uₘ}, where Uk was the k-th evaluation level, and U₁>U₂>U₃>...>Uₘ, then it was said that {U₁,U₂,U₃,...,Uₘ} as an ordered division of the evaluation level space.
2.1 Establishment of single-indicator measurement function

Let \( \mu \) be the degree to which the measured value \( p_{ij} \) \((p_{ij} \in U_k)\) belongs to the evaluation level of \( U_k \), and \( \mu \) can be said to be an unconfirmed measure when it simultaneously satisfies the following conditions.

\[
\mu (p_{ij} \in U_k) = 1 \quad (1)
\]

\[
0 \leq \mu (p_{ij} \in C_k) \leq 1 \quad (2)
\]

\[
\mu \left( p_{ij} \in \sum_{k=1}^{y} U_k \right) = \sum_{k=1}^{y} \mu (p_{ij} \in U_k) \quad (3)
\]

For single-index measurement functions, the four construction techniques linear, exponential, sinusoidal, and parabolic were more frequently used. In general, the measurement function should be chosen based on the actual engineering, but the linear measurement function was the most popular because it was the simplest and easiest to comprehend. In order to create a linear type measurement function for analysis, the following was the expression used in this paper:

\[
\mu_i(p) = \begin{cases} 
-\frac{p}{a_{i+1}} + \frac{a_i}{a_{i+1} - a_i} & (a_i < p \leq a_{i+1}) \\
0 & (p > a_{i+1})
\end{cases} \quad (4)
\]

\[
\mu_i(p) = \begin{cases} 
\frac{p}{a_{i+1} - a_i} - \frac{a_i}{a_{i+1} - a_i} & (a_i < p \leq a_{i+1})
\end{cases}
\]

The measured value \( p_{ij} \) of each evaluation factor was brought into the function \((4)\), and the unconfirmed measured value \( \mu_{ijk} \) of each indicator could be calculated, so as to construct a single-indicator measurement evaluation matrix, which was written as:

\[
\mu_{ijk(nom)} = \begin{bmatrix} 
\mu_{i11} & \mu_{i12} & \cdots & \mu_{i1m} \\
\mu_{i21} & \mu_{i22} & \cdots & \mu_{i2m} \\
\vdots & \vdots & \ddots & \vdots \\
\mu_{im1} & \mu_{im2} & \cdots & \mu_{imm}
\end{bmatrix} \quad (5)
\]

2.2 Determination of indicator weights

Combined with the information entropy theory, the formula for calculating the entropy value of the \( j \)-th evaluation indicator of the \( i \)-th rating factor was:

\[
h_j = 1 + \frac{1}{\ln m} \sum_{k=1}^{m} \mu_{ijk} \ln \mu_{ijk} \quad (6)
\]

Let \( w_j \) be the degree of importance of the evaluation indicator \( F_{ij} \) relative to other indicators in the \( i \)th rating factor, recorded as weight, then:

\[
w_j = \frac{h_j}{\sum_{j=1}^{y} h_j} \quad (7)
\]

And the weight \( w_j \) satisfied: \( 0 \leq w_j \leq 1 \), \( \sum_{j=1}^{y} w_j = 1 \), then \( w=(w_1,w_2,w_3,...,w_y) \) was said to be the weight vector of each evaluation indicator \( F_{ij} \).
2.3 Multi-indicator Uncertainty Measurement

Let $\mu_k$ satisfy: $0 \leq \mu_{ik} \leq 1$, 
$$
\mu_{ik} = \sum_{j=1}^{y} w_j \mu_{jk} \quad (k = 1, 2, \cdots, m),
$$
then the vector $\mu_k$ was said to be the evaluation vector of multi-indicator composite measure of $P$, where $\mu_k = (\mu_{i1}, \mu_{i2}, \mu_{i3}, \cdots, \mu_{im})$.

2.4 Confidence identification criteria

The confidence level was employed for identification for the evaluation space with ordered segmentation, which could more accurately reflect the objectivity of the assessment object. If the confidence level was $\lambda$, for a certain ordered partition of the evaluation space to meet the following equation (8), then the evaluation object was considered to belong to the $k$th evaluation level $C_k$.

$$
k = \min \left\{ k : \sum_{k=1}^{m} \mu_{ik} > \lambda, \ k = 1, 2, \cdots, m \right\}
$$

The confidence level expressed how confident one was that the actual value lies within the range of the measurement results, and the choice of the confidence level was subject to clear limitations due to the subjective nature of the final evaluation level. The relevance of the judgment would be lost if the confidence level was either too high or too low. In order to offer an early warning signal for the management of mine slopes, this article cautiously chose the confidence level of 0.7, which meant that the real risk level was included in the evaluation of the risk level range of the potential risk of 70%.

3. Open pit mine slope stability evaluation system

Open-pit mine slope instability by the coupling of many factors, many domestic experts and scholars have carried out many studies [4-5,8-10] while combining the relevant national technical specifications [11-13] and the results of the field research to determine the following: ① geological structure, hydrogeology, rock structure, slope height, slope angle, internal friction angle (geological conditions), ② blasting vibration velocity of the mass point (engineering conditions), ③ Seismic intensity, average annual rainfall (meteorological environment) three categories of factors, a total of nine indicators, the establishment of open pit mine slope stability evaluation system. The evaluation system is divided into five evaluation levels, recorded as $U = \{U_1, U_2, U_3, \cdots, U_5\}$, corresponding to: very stable, relatively stable, stable, unstable, extremely unstable, forming the main factors affecting the stability of the slope evaluation index system and grading standards, see Table 1.

<table>
<thead>
<tr>
<th>Indicator Grade</th>
<th>Very stable</th>
<th>Relatively stable</th>
<th>Stable</th>
<th>Unstable</th>
<th>Very unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geological structure</strong></td>
<td>No or minor faults that do not affect the project, slightly developed fissures</td>
<td>Minors faults</td>
<td>relatively well developed fissures, relatively intact rock mass</td>
<td>relatively well developed fissures, broken rock mass</td>
<td>well-developed joints and fissures, loose rock mass</td>
</tr>
<tr>
<td><strong>Hydrogeology</strong></td>
<td>Very low permeability</td>
<td>Low permeability, weak surfaces that do not affect the project</td>
<td>Weak permeability</td>
<td>weak surfaces that affect the project</td>
<td>interbedded weak surfaces Strong permeability</td>
</tr>
<tr>
<td><strong>Rock structure</strong></td>
<td>Integral structure</td>
<td>Blocky structure</td>
<td>Layered structure</td>
<td>Fractured structure</td>
<td>Scattered structure</td>
</tr>
<tr>
<td><strong>Slope height/m</strong></td>
<td>&lt; 50</td>
<td>50~100</td>
<td>100~200</td>
<td>200~500</td>
<td>&gt; 500</td>
</tr>
<tr>
<td><strong>Slope angle°</strong></td>
<td>&lt; 48</td>
<td>48~60</td>
<td>60~85</td>
<td>65~70</td>
<td>70~75</td>
</tr>
<tr>
<td><strong>Internal friction angle°</strong></td>
<td>&gt; 42</td>
<td>33~42</td>
<td>25~33</td>
<td>16~25</td>
<td>&lt; 16</td>
</tr>
<tr>
<td><strong>Blasting mass vibration velocity/cm/s</strong></td>
<td>&lt; 2</td>
<td>2~3</td>
<td>3~4</td>
<td>4~5</td>
<td>&gt; 5</td>
</tr>
<tr>
<td><strong>Seismic intensity</strong></td>
<td>&lt; 3</td>
<td>3~5</td>
<td>5~7</td>
<td>7~8</td>
<td>8~12</td>
</tr>
<tr>
<td><strong>Average annual rainfall/mm</strong></td>
<td>&lt; 300</td>
<td>300~700</td>
<td>700~1000</td>
<td>1000~1600</td>
<td>&gt; 1600</td>
</tr>
</tbody>
</table>

The nine evaluation indexes were divided into three qualitative indexes and six quantitative indexes,
of which the qualitative indexes were assigned according to the evaluation level in Table 1 to achieve quantitative transformation. According to the formula (4), the linear measurement function of each evaluation index of open pit mine slope was established, in which the linear measurement function of single index for 3 qualitative indexes was shown in Fig. 1, and the linear measurement function of single index for 6 quantitative indexes was shown in Figs. 2 to 7.

**Figure 1** Qualitative indicators measurement function  

**Figure 2** Slope height measurement function
Figure 3 Slope angle measurement function

Figure 4 Internal friction angle measurement function

Figure 5 Blasting mass vibration velocity measurement function
4. Engineering Application

This paper took a mine in Liuyang as the research object, and evaluates and analyses its slope stability by using the theory of unconfirmed measurement. This mine is a limestone mine, the mineral is endowed in the strata of She Tianqiao Formation (D₃s) of Devonian system and Ciziqiao Formation (D₂q) of Middle Devonian system, which belongs to carbonate rock depositional type of the Basin phase of the shallow open sea confined inter-table basin of the Devonian system of the Paleoproterozoic world, and is controlled by the stratigraphic level, and the rock stratum is the ore stratum. The geological structure of the ore layer is simple, with a gentle and broad back-slope output, slightly undulating folds, and no soft and weak interlayers. The surrounding rock is medium-thick limestone, with high hardness and good stability, and the karst development is mainly controlled by joints and fissures. Hydrogeological conditions were simple, the ore layer is a water-rich weak rock layer, containing a small amount of dissolution fissure water. Mining area is subtropical monsoon climate, according to Liuyang city meteorological observation data over the years shows that the average annual rainfall is 1578mm, rainfall is concentrated in March to June. The seismic intensity of the mine area is basically less than 6 degrees, with good stability. The maximum slope height of the mine is 101m, and the slope angle is 60°-65°.

According to the current characteristics of the slopes of the mine, the stability evaluation indexes of open pit mine slopes were established, such as Table 2, to evaluate the stability of open pit slopes.
The value of each evaluation indicator in Table 2 was brought into function (4) to calculate the value of single-indicator unconfirmed measurement, so as to construct the single-indicator measurement evaluation matrix:

$$\mu_{jk}(9,5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.3467 & 0.6533 & 0 \\ 0 & 0 & 0.5000 & 0.5000 & 0 \\ 0 & 0 & 0 & 0.4444 & 0.5556 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ \end{pmatrix} \begin{pmatrix} 0.6667 \\ 0.3333 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

According to the entropy weight method, the value of the unconfirmed measure of each indicator was bring into the formula (6) to calculate the entropy value of each evaluation indicator:

$$h_1 = h_2 = h_3 = h_7 = h_8 = 1 + \frac{1}{\ln 5} (1 \times \ln 1) = 1$$

$$h_4 = 1 + \frac{1}{\ln 5} (0.3467 \times \ln 0.3467 + 0.6533 \times \ln 0.6533) = 0.5990$$

$$h_5 = 1 + \frac{1}{\ln 5} (2 \times 0.5 \times \ln 0.5) = 0.5693$$

$$h_6 = 1 + \frac{1}{\ln 5} (0.4444 \times \ln 0.4444 + 0.5556 \times \ln 0.5556) = 0.5732$$

$$h_9 = 1 + \frac{1}{\ln 5} (0.6667 \times \ln 0.6667 + 0.3333 \times \ln 0.3333) = 0.6045$$

The entropy value of each evaluation indicator was bringing into the formula (7) to calculate the weight of each evaluation indicator. And the weight vector of evaluation indicators was obtained as w=(0.1361,0.1361,0.1631,0.0815,0.0775,0.0780,0.1361,0.1361,0.0823). Then the vector of composite indicator measures $\mu=(0.0549,0.0274,0.2031,0.1267,0.5879)$ was calculated by multiplying the weight vector of each evaluation indicator and the single indicator measure matrix.

Taked the confidence level $\lambda=0.7$, which was obtained from the confidence level identification criteria described above, the value of the composite measure was arranged from smallest to largest:

$$k = 0.0274+0.0549+0.1267+0.2031+0.5879=1 > 0.7$$, the stability of the open pit slope of the mine was very stable;

Arranged from large to small, it geted: $k = 0.5879+0.2031=0.791 > 0.7$, the stability of mine open slope was more stable. The results of the two discriminations were inconsistent, but it was conservatively judged that the open pit slope stability of the mine was better and belongs to the second level of more stable.

The mine's local slope angle was close to 90 degrees, the mining slope angle ranged from 70 to 80 degrees, the slope height was close to 100 meters (historical legacy), and no collapse phenomenon had occurred for many years, indicating that its slope stability was better. This result was also in line with
the expert group's assessment of the mine's safety status quo in December 2022, which stated that "the slope stability was good, there were no geological disasters in the area, and the current status of all kinds of geological disasters was of small danger." No unusual changes in the slopes were discovered during the inspection and monitoring of the slopes.

5. Conclusion

(1) The open pit mine slope stability evaluation model was established by using the theory of unconfirmed measurement. The weight of each evaluation factor was determined by the entropy value method. The slope stability was analyzed by combining the confidence level identification criterion and multi-indicator comprehensive measurement evaluation. The results demonstrated that the open pit slope stability evaluation method was economical and practical.

(2) The model was used to evaluate the stability of the open pit slopes of a mine in Liuyang, Hunan Province. The evaluation results were consistent with the actual scenario, demonstrating the applicability of the evaluation approach. The evaluation's findings were consistent with the situation as it is, demonstrating the method's applicability. For the mine's subsequent safety management and the early detection of slope instability, it will have specific reference value.

(3) So that it can be used more frequently in engineering practice, it is necessary to further study the evaluation system, optimize the evaluation indexes and grading, and implement the "one mine one policy" with the support of a large number of observation data.

References