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Solution of Transient Pressure in Radial Composite Oil Reservoir by Variable Separation Method

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Abstract: *Due to original sedimentary environments and accumulation processes, oil and gas reservoir are usually of great in-homogeneity. After the later-stage reservoir stimulation measures, the formation property of near-well zone and distant-well zone are often quite different. In order to characterize this kind of reservoir more accurately, the reservoir can be radially treated as composite reservoir with different zones. Based on method of variables separation to solve conventional flow problems, the pressure solution is originally obtained for radially dual composite reservoir by using the same method. This case enlarged the application of variables separation method for solving flow problems in oil and gas reservoir, and these work is also of great significance for evaluation of composite reservoirs.*

Keywords: *radial flow, transient flow, method of variables separation, analytical solution, composite reservoir*

1. Introduction

With consistent focus on sustainable development for oil and gas resources in recent years, many unconventional oil and gas resources become the exploitation target for oil companies[1-3]. In order to solve these emerging seepage problems which are becoming more and more complicated in the development of unconventional oil and gas resources, it is necessary to strengthen the study on solution methods for seepage problems. Until now, many scholars have carried out detailed investigation from the perspective of analytical solution, approximate solution and numerical solution for these seepage problems, and significant progress have been made[4-7]. Compared with the practicality of numerical solution, the results obtained form analytical solution can be a good tool for sensitive analysis of each related parameters in the seepage process, and sometimes, the introduction of analytical methods can be helpful for the reduction of computing jobs. In the following, the pressure of unsteady flow in radial composite reservoir is studied, the mathematical model is established based on the characteristics of seepage, and the accurate analytical solution is finally obtained using the variable separation method.

2. Overview of Variable Separation Method

As for the idea, method, and procedure of variable separation method, many references have demonstrated very thoroughly in the view of wave equation[8-10]. Since the equation for seepage problems is usually parabolic equation, the case study is for solution of parabolic equation here first, and this can demonstrate effectiveness of variable separation method in solving simple seepage problems which is the foundation for solution of following seepage problems in composite reservoir. Suppose the pressure p in one dimensional unsteady flow can meet the following mixed problem:

$$\begin{cases} \dfrac{\partial^2 p}{\partial x^2} = \dfrac{1}{\eta}\dfrac{\partial p}{\partial t}, \ 0 < x < l, t > 0 \\ p(x,0) = \varphi(x), \ 0 \le x \le l \\ p(0,t) = 0, \ t \ge 0 \\ \left.\dfrac{\partial p}{\partial x} + hp\right|_{x=l} = 0, \ t \ge 0 \end{cases} \tag{1}$$

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Suppose the solution can be expressed in the format of variable separation as below:

$$p(x,t) = X(x)T(t) \quad (2)$$

Substitute formula(2) into constitutive equation in problem(1):

$$T'(t) + \eta\lambda T(t) = 0 \quad (3)$$

$$X''(x) + \lambda X(x) = 0 \quad (4)$$

Take formula(4) into consideration, in combination with the boundary conditions in problem(1), then function $X(x)$ can meet the following boundary condition:

$$X(0) = 0,\ X'(l) + hX(l) = 0 \quad (5)$$

For eigenvalue problem(4) and (5), it is not hard to find that it has infinitely many eigenvalues λ_k, $k = 1,2,\cdots$. Among them, λ_k is the positive roots of following equation for parameter λ:

$$\sqrt{\lambda}\cos\sqrt{\lambda}l + h\sin\sqrt{\lambda}l = 0 \quad (6)$$

The corresponding eigenfunction is:

$$X_k(x) = B_k \sin\sqrt{\lambda_k}x,\ k = 1,2,\cdots. \quad (7)$$

Substitute λ_k into formula(3), there will be:

$$T_k(t) = A_k e^{-\eta\lambda_k t},\ k = 1,2,\cdots \quad (8)$$

Therefore, suppose $A_k B_k = C_k$, a series of special solutions with variable separation format can be obtained:

$$p_k(x,t) = C_k e^{-\eta\lambda_k t}\sin\sqrt{\lambda_k}x,\ k = 1,2,\cdots \quad (9)$$

Consider the linearity and homogeneity of equation and boundary conditions in problem(1), the solution expressed in series can be obtained based on the theory of superposition:

$$p(x,t) = \sum_{k=1}^{\infty} C_k e^{-\eta\lambda_k t}\sin\sqrt{\lambda_k}x \quad (10)$$

In combination with the initial condition in problem(1), expand function $\varphi(x)$ into sine series, then their coefficients C_k can be determined by the following formula:

$$C_k = \frac{1}{M_k}\int_0^l \varphi(\xi)\sin\sqrt{\lambda_k}\xi d\xi \quad (11)$$

Among them, there is:

$$M_k = \int_0^l \sin^2\sqrt{\lambda_k}\xi d\xi \quad (12)$$

Substitute formula(11)-(12) into formula(10), and the solution of problem 1) in variable separation format will finally be obtained.

3. Mathematical Model Establishment for Seepage Problem

Consider to enlarge the application of above variable separation method, suppose the radial two-egion composite reservoir is as that shown in figure 1: the formation is horizontal and it consists of two regions with permeability of K_1 and K_2 respectively. The two regions are distributed radially and the interval ranges are $0 \le r \le a$ and $a \le r \le b$ respectively, and the pressure transmission coefficients

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are η_1 and η_2 respectively. The pressure at the outer boundary remains constant. After production for some time, the pressure change from that(the initial formation pressure is p_i) at the initial time $t=0$ are $F_1(r)$ and $F_2(r)$ respectively. Suppose there are good connectivity between two regions, and the purpose is to find out the pressure distribution in each region when $t>0$.

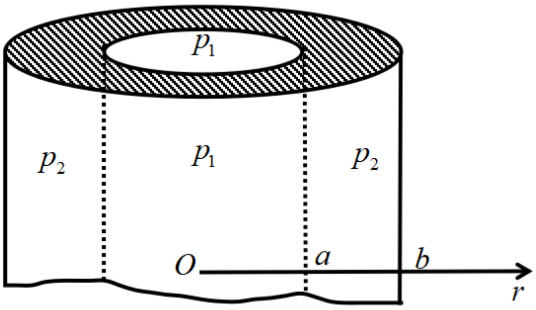

Figure 1: Sketch map of seepage area for radial two-region composite reservoir.

In order to make outer boundary conditions homogeneous, suppose $p_1(r,t)$ and $p_2(r,t)$ are the pressure change compared with that of initial pressure in two regions respectively, refer to the model establishment methods for seepage problem[11-12], the mathematical expression of this problem can be established as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_1}{\partial r}\right)=\frac{1}{\eta_1}\frac{\partial p_1}{\partial t},\quad 0<r<a \tag{13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_2}{\partial r}\right)=\frac{1}{\eta_2}\frac{\partial p_2}{\partial t},\quad a<r<b \tag{14}$$

$$p_1=p_2,\quad r=a \tag{15}$$

$$K_1\frac{\partial p_1}{\partial r}=K_2\frac{\partial p_2}{\partial r},\quad r=a \tag{16}$$

$$p_2=0,\quad r=b \tag{17}$$

$$p_1(r,t)=F_1(r),\quad 0\le r\le a,\quad t=0 \tag{18}$$

$$p_2(r,t)=F_2(r),\quad a\le r\le b,\quad t=0 \tag{19}$$

4. Solution of Seepage Problem by Variable Separation Method

Suppose $p_1(r,t)=R_1(r)T_1(t), p_2(r,t)=R_2(r)T_2(t)$, substitute into the formulas(13)-(19), introduce the separation constant $\lambda\equiv\beta^2$, therefore, the separation results will be:

$$T'+\beta^2T=0 \tag{20}$$

and $R_1(\beta,r), R_2(\beta,r)$ can meet the following eigenvalue problems:

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$$\begin{cases} \frac{1}{r}\frac{d}{dr}(r\frac{dR_1}{dr})+\frac{\beta^2}{\eta_1}R_1=0, 0<r<a \\ \frac{1}{r}\frac{d}{dr}(r\frac{dR_2}{dr})+\frac{\beta^2}{\eta_2}R_2=0, a<r<b \\ R_1=R_2, r=a \\ K_1\frac{dR_1}{dr}=K_2\frac{dR_2}{dr}, r=a \\ R_2=0, r=b \end{cases} \quad (21)$$

Based on formula(20), the following results can be obtained:

$$T(t)=e^{-\beta^2 t} \quad (22)$$

Problem(21) is the eigenvalue problem of this composite reservoir, the eigenvalue β_m and the eigenfunction can be solved and expressed in two intervals:

$$R(\beta_m,r)=\begin{cases} R_1(\beta_m,r), 0\le r\le a \\ R_2(\beta_m,r), a\le r\le b \end{cases} \quad (23)$$

It can be proved that the eigenfunction has following orthogonality:

$$\frac{K_1}{\eta_1}\int_0^a rR_1(\beta_m,r)R_1(\beta_n,r)dr+\frac{K_2}{\eta_2}\int_a^b rR_2(\beta_m,r)R_2(\beta_n,r)dr=\begin{cases} 0, \quad m\ne n \\ N(\beta_m), \quad m=n \end{cases} \quad (24)$$

Among them, the norm is:

$$N(\beta_m)=\frac{K_1}{\eta_1}\int_0^a rR_1^{\,2}(\beta_m,r)dr+\frac{K_2}{\eta_2}\int_a^b rR_2^{\,2}(\beta_m,r)dr \quad (25)$$

So the solution of problem(13)-(19) can be expressed in two intervals:

$$p(r,t)=\begin{cases} p_1(r,t), 0\le r\le a \\ p_2(r,t), a\le r\le b \end{cases} \quad (26)$$

Among them,

$$p_1(r,t)=\sum_{m=1}^{\infty}c_m e^{-\beta_m{}^2 t}R_1(\beta_m,r), 0\le r\le a \quad (27)$$

$$p_2(r,t)=\sum_{m=1}^{\infty}c_m e^{-\beta_m{}^2 t}R_2(\beta_m,r), a\le r\le b \quad (28)$$

Take initial condition into consideration, the superposition coefficient c_m can be determined:

$$F_1(r)=\sum_{m=1}^{\infty}c_m R_1(\beta_m,r), 0\le r\le a \quad (29)$$

$$F_2(r)=\sum_{m=1}^{\infty}c_m R_2(\beta_m,r), a\le r\le b \quad (30)$$

Make use of orthogonality formula(24), do operation on each side of formulas (29) and (30) with

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operator $\frac{K_1}{\eta_1}\int_0^a rR_1(\beta_m, r)dr$ and $\frac{K_2}{\eta_2}\int_a^b rR_2(\beta_m, r)dr$, and then add results together, there will be:

$$c_m = \frac{1}{N(\beta_m)}[\frac{K_1}{\eta_1}\int_0^a rR_1(\beta_m, r)F_1(r)dr + \frac{K_2}{\eta_2}\int_a^b rR_2(\beta_m, r)F_2(r)dr] \quad (31)$$

Substitute formula(31) into formula(27) and (28), there will be:

$$p_1(r,t) = \sum_{m=1}^{\infty} e^{-\beta_m{}^2 t} R_1(\beta_m, r)\frac{1}{N(\beta_m)}\cdot [\frac{K_1}{\eta_1}\int_0^a rR_1(\beta_m, r)F_1(r)dr + \frac{K_2}{\eta_2}\int_a^b rR_2(\beta_m, r)F_2(r)dr] \quad (32)$$

$$p_2(r,t) = \sum_{m=1}^{\infty} e^{-\beta_m{}^2 t} R_2(\beta_m, r)\frac{1}{N(\beta_m)}\cdot [\frac{K_1}{\eta_1}\int_0^a rR_1(\beta_m, r)F_1(r)dr + \frac{K_2}{\eta_2}\int_a^b rR_2(\beta_m, r)F_2(r)dr] \quad (33)$$

By now, the formal solution has been obtained, and the remaining task is to solve the eigenvalue problem 21):

First, the general solution of equations 21) can be solved as:

$$R_1(\beta, r) = A_1 J_0(\frac{\beta}{\sqrt{\eta_1}}r) + B_1 N_0(\frac{\beta}{\sqrt{\eta_1}}r),\ 0 \le r \le a \quad (34)$$

$$R_2(\beta, r) = A_2 J_0(\frac{\beta}{\sqrt{\eta_2}}r) + B_2 N_0(\frac{\beta}{\sqrt{\eta_2}}r),\ a \le r \le b \quad (35)$$

The unknown coefficients A_1 , B_1 , A_2 , B_2 and the eigenvalue β has to be determined by boundary conditions and connecting conditions.

Take the engineering reality into consideration, p_1 is certain to have limitation at $r = 0$, so R_1 also has limitation correspondingly. Without loss of generality, suppose $A_1 = 1$, formula (34) and (35) above can be simply expressed as:

$$R_1(\beta, r) = J_0(\frac{\beta}{\sqrt{\eta_1}}r),\ 0 \le r \le a \quad (36)$$

$$R_2(\beta, r) = A_2 J_0(\frac{\beta}{\sqrt{\eta_2}}r) + B_2 N_0(\frac{\beta}{\sqrt{\eta_2}}r),\ a \le r \le b \quad (37)$$

Make use of the boundary condition at $r = b$ and the connecting condition at $r = a$, the equations concerning A_2, B_2, and β can be obtained as follows:

$$J_0(\frac{\beta}{\sqrt{\eta_1}}a) = A_2 J_0(\frac{\beta}{\sqrt{\eta_2}}a) + B_2 N_0(\frac{\beta}{\sqrt{\eta_2}}a) \quad (38)$$

$$\frac{K_1}{K_2}\sqrt{\frac{\eta_2}{\eta_1}}J_1(\frac{\beta}{\sqrt{\eta_1}}a) = A_2 J_1(\frac{\beta}{\sqrt{\eta_2}}a) + B_2 N_1(\frac{\beta}{\sqrt{\eta_2}}a) \quad (39)$$

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$$0 = A_2 J_0(\frac{\beta}{\sqrt{\eta_2}} b) + B_2 N_0(\frac{\beta}{\sqrt{\eta_2}} b) \quad (40)$$

Or formulas(38)-(40) can be expressed in following equation of matrix:

$$\begin{bmatrix} J_0(\xi_1) & -J_0(\frac{a}{b}\xi_2) & -N_0(\frac{a}{b}\xi_2) \\ KJ_1(\xi_1) & -J_1(\frac{a}{b}\xi_2) & -N_1(\frac{a}{b}\xi_2) \\ 0 & J_0(\xi_2) & N_0(\xi_2) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Among them,

$$\xi_1 = \frac{\beta}{\sqrt{\eta_1}} a, \ \xi_2 = \frac{\beta}{\sqrt{\eta_2}} b, \ K = \frac{K_1}{K_2}\sqrt{\frac{\eta_2}{\eta_1}}$$

Since the equation above has non-zero roots, the determinant of its coefficients must be 0, there is:

$$\begin{vmatrix} J_0(\xi_1) & -J_0(\frac{a}{b}\xi_2) & -N_0(\frac{a}{b}\xi_2) \\ KJ_1(\xi_1) & -J_1(\frac{a}{b}\xi_2) & -N_1(\frac{a}{b}\xi_2) \\ 0 & J_0(\xi_2) & N_0(\xi_2) \end{vmatrix} = 0 \quad (41)$$

Eigenvalue β is just the root of this equation. There are infinitely many roots: $\beta = \beta_m, m = 1,2,\cdots$.

Due to the validity of formula(41), formula(38)-(40) are not totally independent, therefore, any two of the three formulas can be chosen for determination of coefficients. Coefficients A_2 and B_2 can be determined by combination of formula(38) and (40):

$$A_2 = \frac{J_0(\xi_1) N_0(\xi_2)}{M} \quad (42)$$

$$B_2 = -\frac{J_0(\xi_1) J_0(\xi_2)}{M} \quad (43)$$

Among them,

$$\mathrm{M} = \begin{vmatrix} J_0(\frac{a}{b}\xi_2) & N_0(\frac{a}{b}\xi_2) \\ J_0(\xi_2) & N_0(\xi_2) \end{vmatrix} \quad (44)$$

Substitute the results of formula(42)-(44) into formula(37), the eigenfunctions can be completely determined. Actually, the coefficients A_2, B_2 are actually related to eigenvalue, for each eigenvalue β_m, there are corresponding coefficients A_{2m}, B_{2m}.

5. Conclusions

The solutions directly obtained is actually the pressure change compared with that of original formation pressure. To get the pressure distribution in each region, just add the initial pressure p_i with $p_1(r,t)$ or $p_2(r,t)$ respectively.

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Choose the seepage problem in radial composite reservoir as an example, the joining condition between two regions are considered, on the basis of solution of seepage problems by traditional variable separation method, the solution of pressure in composite reservoir is obtained. This study enlarged the application of variable separation method, and which significantly improve the methodology for the evaluation of oil and gas reservoirs.

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