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Enhancing Teaching Quadratic Functions: The Benefits, Challenges, and Recommendations of Using GeoGebra

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Abstract: *This paper engages in an extensive literature review to discuss the existing research regarding using GeoGebra in teaching mathematics, specifically quadratic functions. After that, a description of the functionalities and features of GeoGebra and its potential for addressing the challenges of teaching and learning quadratic functions will be introduced. Based on my literature review, I designed an activity sequence designed to teach quadratic functions and explore how the activities align with learning objectives and the arrangement of tasks in line with the TPACK framework. The use of GeoGebra as a teaching tool has the potential to support students' reasoning and visualization skills and to assist teachers' demonstrations and explanations.*

Keywords: *quadratic functions, TPACK, GeoGebra*

1. Introduction

Researchers and educators have recognized quadratic functions as one of the most crucial and challenging areas in algebra education; students found it difficult to understand quadratic functions due to their abstract nature. In particular, applying the arithmetic concept and understanding the coefficients a, b and c on the graphs of quadratic function, $f(x) = ax^2 + bx + c$ [1]. Based on my own teaching experience, without any visual aids, it seems to be hard for students to solve questions related to translating quadratic functions with the vertex form, i.e., $f(x) = a(x-p)^2 + q$. However, every time I used geometric software to demonstrate the translation of quadratic function graphs, students seemed to be excited to see what would happen with the changing of variables. I quite liked using these digital tools to support my mathematical teaching and engage pupils.

During my study of the Digital Technology module, I have gained knowledge about various digital tools that can be valuable in Mathematics Education, such as Desmos and GeoGebra[2], which suggested that incorporating dynamic geometric learning environments in Mathematics classes could improve students' understanding of functions through connecting graphical interpretations to algebraic expressions. The author recommended GeoGebra as a useful geometric software that appears to have a positive impact on teaching mathematics, particularly functions. As someone who advocates for integrating technology in Mathematics lessons, I was prompted to investigate further into enhancing the teaching of quadratic equations with GeoGebra.

However, there appear to be multiple limitations to using GeoGebra in teaching functions[3]. For instance, students might rely on graphic representations without genuinely understanding the algebraic proof [4]. Appropriate teaching approaches are thus crucial in GeoGebra-supported lessons to help students better manipulate digital tools and understand mathematical concepts. Therefore, in line with the framework of Technological Pedagogical Content Knowledge (TPACK), this paper aims to develop a GeoGebra-based activity that can enhance the teaching of quadratic functions, particularly the graph of the "completed square" form. In addition, the advantages and potential challenges of utilizing GeoGebra in this activity are discussed at the end to give further recommendations.

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2. Literature Review

2.1. Difficulties in teaching and learning quadratic functions

Previous studies have identified multiple difficulties in regard to quadratic functions, such as the misinterpretation of symmetrical lines and extreme points[5], sketching a quadratic function by looking at the difference in values of coefficients and solving simultaneous equations graphically. In accordance with my designed activity, this paper focuses on students' common difficulty in building the relationship between the algebraic equation of a quadratic function and a geometric graph [6].

There are three different algebraic forms of quadratic functions: the standard form ($y = ax^2 + bx + c$), the vertex form ($y = a(x-p)^2 + q$) and the factored form ($y = a(x - x_1)(x - x_2)$). Ubah and Bansilal studied students' common misconceptions about understanding the parameters p and q in the vertex form. For instance, some students thought the vertex form was $y = ax^2 + p$ instead of $y = a(x-p)^2 + q$ because they had an inadequate comprehension of the parameters p and q and failed to link them to the turning point on the function graph. Also, he stated that learners seem to easily confound the Y-intercept with the parameter q and struggle with the parameter p to identify the extreme point of a parabola.

Similarly, it has stated various difficulties in teaching and learning quadratic functions and their corresponding graphs, which are summarized as follows:

2.1.1. The conceptual understanding of the definition of a function

Many students struggle to interpret function graphs because they view them as static objects. Teachers also find it challenging to demonstrate the properties of quadratic functions on the board.

2.1.2. Multiple representations

Students often believe that a function can only be described by one algebraic expression, which leads to their difficulty in understanding multiple representations of quadratic functions. Thus, teachers need to avoid causing misinterpretation and confusion among learners by using appropriate multi-representational visualizations.

2.1.3. Geometric meaning

Students often have difficulty connecting the geometric meaning of quadratic functions to the algebraic expression, particularly in the vertex form. Teachers are suggested to build the connection between algebra and geometry in teaching quadratic functions.

2.2. Using GeoGebra as a teaching tool in Mathematics Education, focusing on its applications in teaching quadratic functions

In recent decades, the rapid development of Information and Communication Technologies (ICT) has significantly influenced the teaching and learning methods employed by educators and learners[7]. Using digital tools and resources has opened new opportunities for teaching and learning, allowing students to interact with mathematical ideas in dynamic and intuitive ways. GeoGebra, a dynamic geometric software that is both free and open-source (www.geogebra.org), has been extensively employed in 21st-century Mathematics education[8]. GeoGebra was designed to foster students' Mathematical understanding and reasoning, as well as to aid teachers in visualizing Mathematical concepts more effectively and engaging their students.

Yohannes and Chen's article systematically reviews the use of GeoGebra in Mathematics Education. The authors reviewed 66 peer-reviewed journal articles published between 2010 and 2020 and analyzed the trends and findings of using GeoGebra. The study summarized that for algebra, GeoGebra can be used to model and solve equations and graph functions, explore properties of operations, and investigate different types of equations due to various useful features. Corresponding to Yohanne and Chen's study, I summarized three main useful features that I utilized in my designed activity to better demonstrate quadratic functions.

2.2.1. Sliders and dynamic input

Sliders and dynamic input are practical features for investigating how altering a function's coefficients changes its shape and location (Övez, 2018). Students may quickly alter the coefficients of

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a quadratic function and see how these changes impact its graph by using sliders and dynamic input[9]. For instance, students could observe how the parabola transforms from narrow and steep to broad and flat by varying the coefficient of x^2 (Figure 1). By changing the coefficient of x, they could observe how the vertex of the parabola moves. By changing the constant time, pupils could observe how the graph moves up or down.

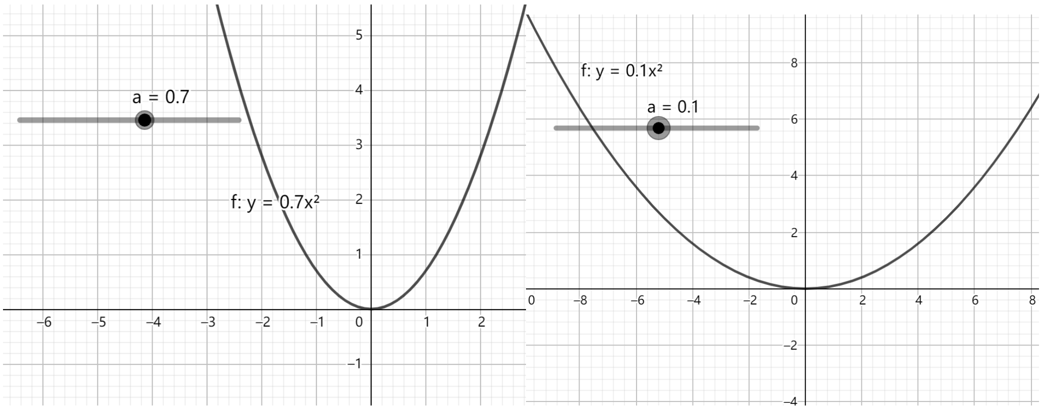

Figure 1: Using a slider

2.2.2. Dragging

The dragging feature allows students to drag points and then see how that might affect other objects. For instance, pupils can use the dragging feature to alter the values of the parameters in a quadratic function and see how these changes affect the graph of the function. By manipulating a point, students might develop a deeper comprehension of how these different parameters affect the shape and position of the graph of a quadratic function (Heid, 2005). They can also explore different types of quadratic functions, such as those in vertex form, and observe how the dragging feature can be used to transform the graph of these functions (Figure 2). According to Heid (2005), the dragging feature in GeoGebra can not only support learners in developing a more intuitive and visual understanding of Mathematical functions but also assist teachers in conducting a more interactive mathematics lesson (Övez, 2018).

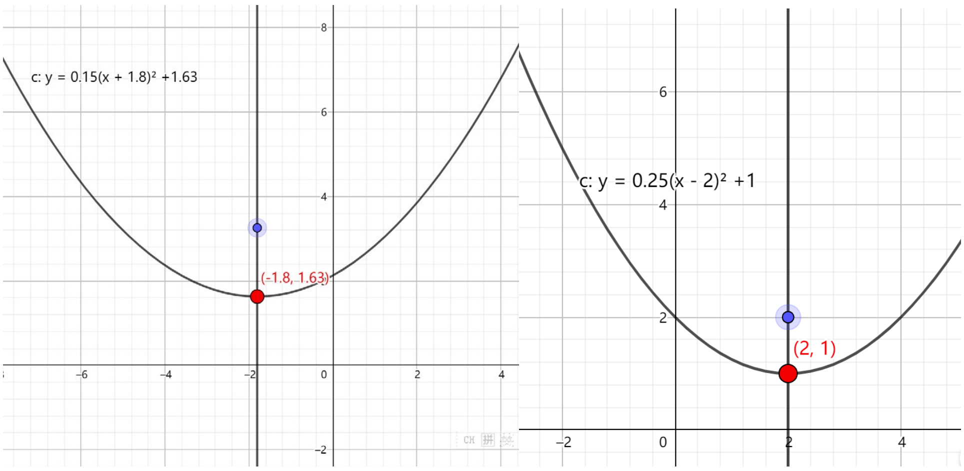

Figure 2: Dragging the vertex point

2.3. The TPACK framework

The TPACK framework involves the relationships among the pedagogical content knowledge[10], technological pedagogical knowledge (TPK) and technological content knowledge (TCK), as shown in Figure 3.

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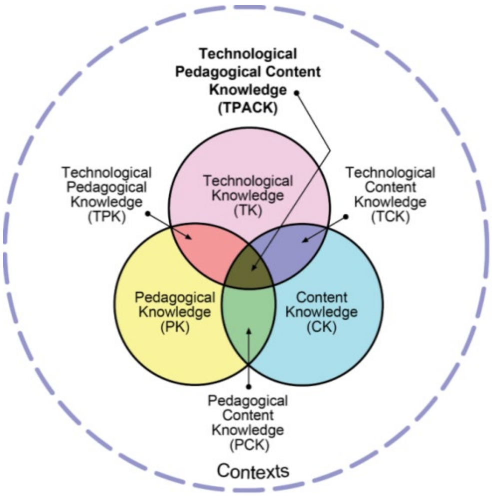

Figure 3: A Venn diagram of TPACK

As shown in the Venn diagram, the framework includes three big parts of knowledge: pedagogical knowledge (PK), content knowledge (CK), and technological knowledge (TK). The intersections of these three knowledge categories are four other parts: pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK) and technological pedagogical content knowledge (TPACK). As indicated in the diagram, I believe that TPACK appears to be the core among those knowledge categories.

There have been multiple definitions of the TPACK concepts in the last decades [11]. This paper would like to choose the definitions from Cox (2008). CK stands for content knowledge, which is the understanding of important facts and concepts within a particular subject area. PK refers to pedagogical knowledge, which is knowledge of general teaching strategies that teachers might employ. TK involves knowledge of computer hardware. PCK refers to the knowledge of how subject-specific representations can be used in classroom activities to enhance students' understanding. TPK is the knowledge of how computer hardware and software tools can be used in general teaching strategies. TCK is the knowledge of how digital technology can be used to represent content. Lastly, TPACK involves understanding the interactive relationship between technology, pedagogy, and content and how this relationship impacts student learning in a classroom setting (Cox, 2008).

The term "TPACK" refers to the knowledge and abilities required to successfully incorporate technology into the classroom[12]. Dynamic visualization seems to be one of the innovative strategies that teachers might apply to conduct TPACK. Specifically, GeoGebra's utilization of sliders and dynamic info permits understudies to investigate what changing the coefficients of quadratic capabilities means for their shape and area. Students' ability to connect algebraic and geometric representations and their spatial reasoning skills may benefit from this feature [13].

However, the need for teachers to effectively incorporate GeoGebra into their teaching methods is another obstacle (Övez, 2018). Teachers' ought to receive appropriate training to gain TK and opportunities for professional development to enhance their TPACK and successfully incorporate GeoGebra into their teaching methods. Further, GeoGebra should be used in conjunction with other teaching methods, such as inquiry-based activities, to help students learn how to solve problems, think critically, and communicate effectively[14].

3. Activity Sequence

As discussed above, teaching quadratic functions can be challenging since students often struggle with abstract and symbolic representations of functions and may find it difficult to visualize and make sense of the geometric properties of parabolas. My designed activity sequence aims to provide students with a dynamic and interactive learning experience using GeoGebra as a teaching tool. Those questions in the sequence involved students' common difficulties with definitions, multiple representations and geometric meanings of quadratic functions, as summarized in section 2.1. This is in line with the TPACK framework (Coz, 2008). I took innovative and interactive activities with the use of various features in GeoGebra (section 2.2) to visualize quadratic functions, demonstrate the properties and dynamically show the connection between the geometric meaning and algebraic forms (Övez, 2018).

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3.1. Activity

The objective of Activity 1 is to assist students in addressing difficulties 1 and 3, which were discussed in section 2.1. To compare the differences between a non-technology teaching method and a GeoGebra-based teaching method, I began by creating two questions without any visual aids. After that, I invited participants to manipulate the dynamic function graphs using GeoGebra's sliders and dynamic input capabilities[15].

Participants were encouraged to observe the translation of the quadratic function, $y = a(x-p)^2+q$, and how the coordinates of the vertex change as the parameters a, b, and c are altered. The dynamic manipulation of the graphs in GeoGebra aims to engage participants and help them develop a concrete understanding of the abstract variables. As demonstrated in Figure 4, participants were able to visualize the changes in the graph and observe how they correlated with the values of the parameters. During the process, I gave guidance and explanations of the GeoGebra screen. According to Koehler and Mishra (2009), dynamic visualisation seems to be a helpful and effective strategy to conduct TPACK in mathematics lessons.

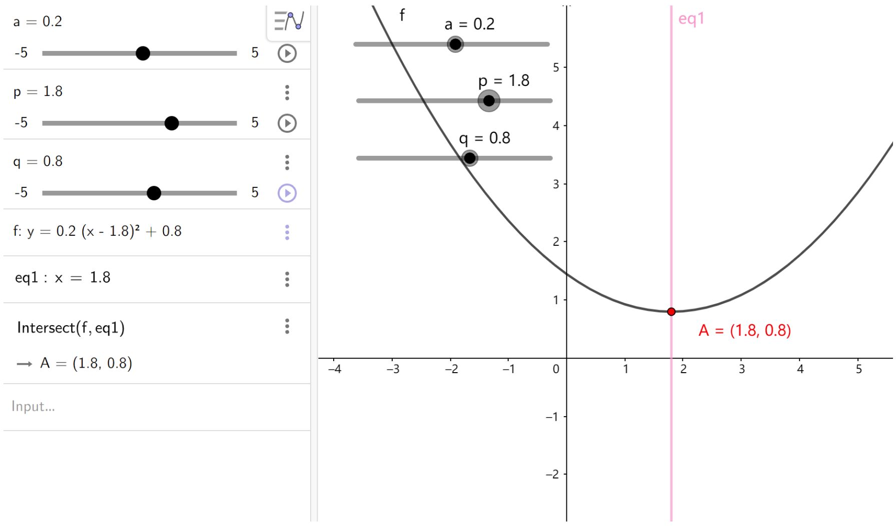

Figure 4: Designed GeoGebra activity

Then, participants were given another 2 questions about translation to check if they could have a better understanding of the graphs after observing and manipulating GeoGebra.

How do the parameters a, p and q affect the graph of the quadratic function $y = a(x-p)^2+q$.

The graph of g(x) is a translation 4 units left and 1 unit down of the graph of $f(x) = x^2$, write g(x) in the form $y = a(x-p)^2+q$.

Finally, to strengthen learners' understanding and encourage peer feedback, participants were given the opportunity to work together to verify their answers using GeoGebra. This approach is in parallel with Coz's TPACK framework (2008). Teachers need to promote active participation and allow students to engage in an interactive learning experience. By collaborating with each other, participants were able to exchange their Mathematics insights. In addition, by interacting with GeoGebra, participants were able to visually see and explore the concepts by themselves [16].

3.2. Analysis and evaluations

Before engaging with GeoGebra, two participants displayed a lack of awareness regarding how the parameters affect the function graph. Participant A, for example, answered the first question by stating, "I believe that the 'a' in $y = a(x-p)^2+q$ can change the coordinate of the vertex since it is a variable." This response indicated a deficiency in the definitions of variable and vertex. Similarly, participant B mistakenly thought that a translation 7 units to the left of x^2 resulted in the expression $(x-7)^2$ instead of $(x+7)^2$, highlighting a misconception about the direction of the translation.

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After interacting with GeoGebra, utilizing Sliders, dynamic input and the tracing tool, the participants appeared to gain a basic understanding of the geometric meaning of each parameter in the vertex form, $y = a(x-p)^2 + q$. By dragging the slider and observing the coordinate, both participants were able to describe how “a”, “p”, and “q” affect the graph of the quadratic function $y = a(x-p)^2 + q$. As shown in Figure 5, participants realized that the modification of the parameter "a" only influences the shape of the parabola and does not impact the position of its vertex.

Participant A noted, "In fact, I discovered that changing the value of "a" does not affect the position of the parabola but rather alters its width, making it either narrower or wider."

Additionally, they mentioned that the parameter "p" determines the x-coordinate of the vertex, while "q" controls the vertical movement of the graph, shifting it up or down.

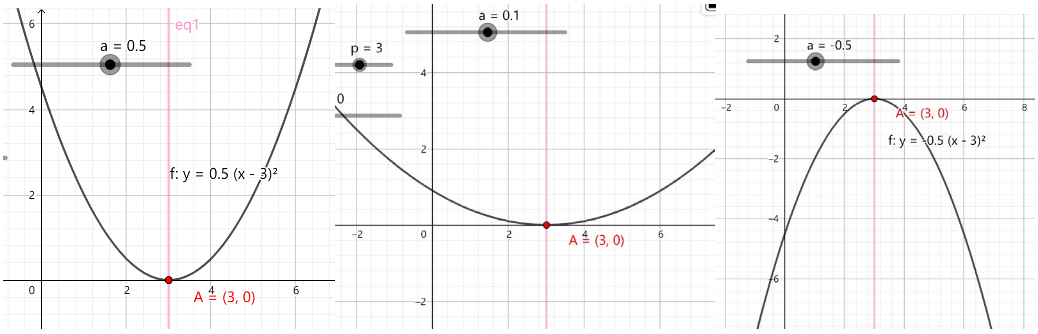

Figure 5: Participants’ work of observing the changing “a”

After this activity, both participants gave the correct expression of $g(x) = (x-4)^2 + 1$, which translated 4 units right and 1 unit up from the graph of $f(x) = x^2$. Also, participate B checked his answer by dragging and tracing the vertex of $f(x)$ (Figure 6), and he stated: “After the movement, the vertex became (4, 1) and I have learnt that the vertex of a quadratic function $y = a(x-p)^2 + q$ is (p, q), which means my $g(x)$ is correct.” His justification of his answer indicates that he developed his geometric reasoning by connecting the graph of a quadratic function to the vertex form expression[17].

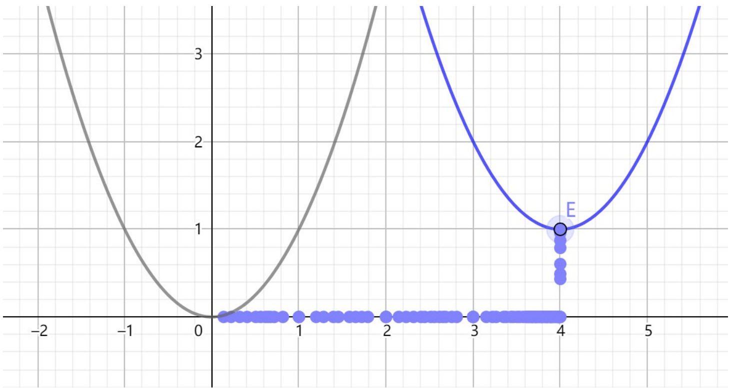

Figure 6: Traces the vertex E

3.3. Summarized benefits and challenges

As evident from my above analysis, this activity appeared to contribute to the enhancement of teaching quadratic functions to some extent. The utilization of GeoGebra enables participants to visualize and manipulate dynamic function graphs, facilitating the exploration and verification of various problems. Notably, both participants were able to generate the correct algebraic expressions from the geometric function graphs. This aspect is beneficial in developing a conceptual understanding of the parameters in different forms and promoting geometric reasoning when solving problems (Caglayan, 2014).

Moreover, by incorporating the TPACK framework, the activity integrates technology, pedagogy, and content knowledge to enhance learning and encourage the development of technological skills

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(Cox, 2008). Participants interacted with different features on GeoGebra to explore the Mathematical objects, such as dragging the function, tracing the vertex and using animation sliders. Particularly, by effectively controlling the sliders and dynamics, participants were eager to see the relationship between the parameters and the graphs and noticed the prompt visual criticism. This approach encourages a more profound comprehension of quadratic functions and the capability to upgrade critical thinking abilities [18]. Additionally, as suggested by Cox (2008), I encouraged self-exploration and peer feedback during the activities, fostering an environment conducive to active learning and promoting collective learning experiences for students. An illustrative example of this is participant A's critical thinking, as demonstrated by their ability to critique participant B's explanation and return to examining the algebraic parameters.

4. Conclusion

This paper designed this activity to support the teaching of quadratic functions regarding students' three common difficulties: the conceptual understanding of the definition of a function, multiple representations and geometric meaning. My designed activity sequence provides an innovative and engaging approach to teaching quadratic functions using GeoGebra, a powerful tool for visualization, exploration, and problem-solving (Dockendorff & Solar, 2018). As suggested by the TPACK framework (Coz, 2008), my designed activities incorporated self-exploration, visualisation, interaction with digital tools and collaboration to facilitate learners' understanding and appreciation of the geometric properties of quadratic functions. With the use of various features on GeoGebra, I was able to demonstrate three different forms of quadratic functions and their properties, such as the vertex, the roots and the direction. The use of GeoGebra as a teaching tool has the potential to support students' reasoning and visualization skills and to assist teachers' demonstrations and explanations. Overall, this activity sequence highlights the value of integrating technology into mathematics education and the importance of promoting innovation, critical thinking, and collaboration in teaching and learning practices.

However, this paper also acknowledged a few challenges associated with using GeoGebra in teaching quadratic functions, such as the combination of technological knowledge (TK) and pedagogical knowledge (PK), students' potential overreliance on graphical representations and a lack of rigor algebraic proof for the graphic properties. In light of my analytical findings, it becomes evident that students may be influenced by the visual nature of graphs presented in GeoGebra when attempting to convert between different forms of quadratic functions.

To address this issue, this paper has proposed some recommendations. Teachers might need to establish a strong connection between the GeoGebra screen and traditional algebraic proofs. As suggested by Coz (2008), teachers need to develop their Technological Pedagogical Content Knowledge (TPACK) skills to effectively explain the dynamic processes depicted in GeoGebra, moving beyond mere demonstrations. For instance, in future lessons, I will incorporate the cross method for factorizing quadratic functions when explaining the roots displayed on the graph. By integrating algebraic reasoning with graphical representations in technology-oriented lessons, learners might develop a deeper conceptual understanding of quadratic functions.

Although this paper has identified several limitations and benefits of using GeoGebra in teaching quadratic functions, it is essential to recognize that the designed activities focused solely on manipulating and observing three different forms of quadratic functions. Further research might still be needed to explore the most effective ways of integrating GeoGebra into quadratic function instruction.

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