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Line Planning Model Based on Multibeam Bathymetric Data

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Abstract: *This study is devoted to the line measurement problem under the multibeam measurement system, and through the establishment of the optimization model and the scheme adjustment model, the accurate measurement of the depth of the sea area to be measured and the effective line routing of the survey vessel are realized. For the problems of coverage width and overlap rate between neighboring strips in multibeam bathymetry, this paper establishes a linear model through geometric relations and algebraic operations and calculates the relevant results using MATLAB. For the wiring planning problem, the optimization model of the total length of the survey line is established based on the coverage width and overlap rate model, and the optimal wiring strategy of the survey line is solved by genetic algorithm. Combined with the actual seawater depth data for 3D modeling, the optimization model is used to derive the optimal line routing strategy, which achieves the comprehensive optimization of maximum coverage width, shortest total length of the line, and the smallest missed area. This study will provide important theoretical support and practical guidance for marine topographic surveying.*

Keywords: *Linear Model, Genetic Algorithm, Optimization Model*

1. Introduction

With the advancement of China's ocean development strategy, it has become particularly important to explore and acquire data on the topography and geomorphology of the sea around the islands [1]. In seabed topographic survey, the evolution of bathymetry technology leads to the improvement of measurement accuracy and efficiency, especially the new application of multibeam measurement technology [2]. The purpose of this paper is to study the line measurement problem under the multibeam measurement system, and to establish an optimization model and scheme to achieve the accurate measurement of the depth of the sea area to be surveyed and the effective adjustment of the survey vessel's wiring. By means of geometric theorem, algebraic operation and genetic algorithm, this paper firstly constructs a model of strip coverage width and overlap rate for multibeam bathymetry, then establishes a model of wiring planning, and finally applies the model to actual data. It provides theoretical support and practical guidance for marine topographic surveying and provides technical support and guidance for China's sea area exploration and geological exploration work.

2. Coverage width and overlap rate model

The strip coverage of multibeam bathymetry is closely related to the ocean depth of the current survey line. By analyzing the geometrical relationship, the depth of the ocean, the seafloor slope and the bathymetric strips can be analyzed jointly, and a regular pattern of strip width variation with depth can be obtained [3]. By analyzing the geometrical relationship, the ocean depth and the bathymetric strip are analyzed together to obtain the law of strip width changing with depth YHRYHR, and for the determined ocean slope, the coverage width at different depths and the overlap rate with the previous line can be obtained according to the equation of the strip width and the parameter.

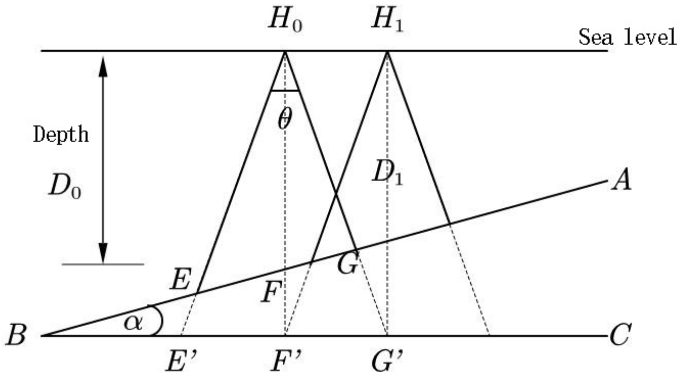

Figure 1: Coverage width and overlap at different distances

As shown in Figure 1, H_0 is the center of the ocean, the depth is D_0, where the center line is located, H_1 is the line with an interval of 200 meters, and the depth is D_1, AB is the seafloor, and BC is the seafloor level, and H_0E and H_0G are the left and right boundaries of the multibeam, and EG is the intersection of the plane of the current multibeam and the seafloor, the length of which is the width of the desired strip W, and the opening transducer position $\theta = 120°$.

In $\triangle BEF$ in which, according to the geometrical relations we have:

$$\angle BFF' = \frac{\pi}{2} - \alpha,\ \ \angle H_oEF = \angle BFF' - \frac{\theta}{2} \tag{1}$$

In $\triangle H_0EF$ in which, by the sine theorem, we have:

$$\frac{D_0}{\sin \angle H_0EF} = \frac{EF}{\sin\frac{\theta}{2}} \tag{2}$$

From the equality of opposite angles, we have:

$$\angle H_0FG = \angle BFF', \angle H_0GF = \pi - \frac{\theta}{2} - \angle H_0FG \tag{3}$$

In $\triangle H_0FG$ in which, according to the sine theorem, we have:

$$\frac{D}{\sin \angle H_0GF} = \frac{FG}{\sin\frac{\theta}{2}} \tag{4}$$

According to the above equation, the relationship between the width of the strip W and depth D is:

$$W = D \cdot \sin\frac{\theta}{2} \cdot \left[\frac{1}{\cos\left(\frac{\theta}{2}+\alpha\right)} + \frac{1}{\cos\left(\frac{\theta}{2}-\alpha\right)}\right] \tag{5}$$

It is easy to prove the similarity of the triangles formed by the multibeam boundary and the seafloor intersection line, calculate the similarity ratio, and obtain the difference in depth of the sea area where the neighboring lines are located according to the triangulation.

$$\Delta D = d \cdot \tan\alpha \tag{6}$$

Then the similarity ratio is,

$$\mu = \frac{D-\Delta D}{D} \tag{7}$$

According to the property of similar triangles, the sea depths and corresponding strip widths of the lines on both sides of the central sea area can be obtained.

$$D_{i+1} = \mu D_i, W_{i+1} = \mu W_i \tag{8}$$

Then the corresponding overlap rate is,

$$\eta_{i+1} = 1 - \frac{d}{W_i \cos\alpha} \tag{9}$$

D_i is the depth of the sea corresponding to the $i-th$ line, and W_i is the width of the corresponding strip, where $i = 0, \pm 1$, the negative sign means to the left.

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Finally, the calculation of the above model and equations yielded the results shown in Table. 1.

Table 1: Calculation results

Line distance	-800	-600	-400	-200	0	200	400	600	800
Depth of sea	90.95	85.71	80.47	75.24	70.00	64.76	59.53	54.29	49.05
Overlay	315.71	297.53	279.35	261.17	242.99	224.81	206.63	188.45	170.27
Overlap rate	--	0.33	0.28	0.23	0.18	0.11	0.03	-0.06	-0.17

3. Coverage width modeling at different distances

First of all, let the angle between the projection of the survey line on the horizontal plane and the projection on the slope surface is γ , through the trigonometric function and other related geometric formulas, find the relationship between β、 α and γ. Then, by setting the angle between the cover width and the projection of the cover width on the horizontal plane as α_s, and the relationship between β、 α and α_s is found by geometric formulas and algebraic operations [4].

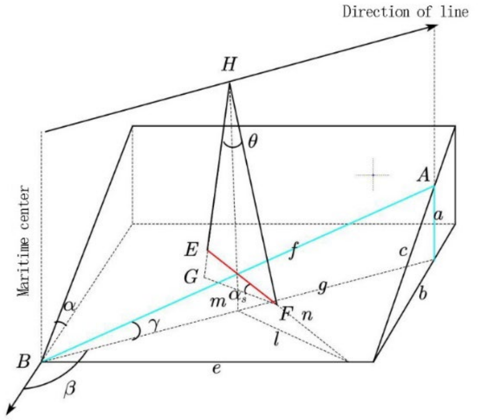

Figure 2: y versus depth at different distances

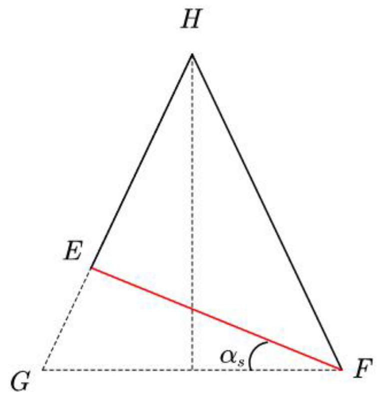

Figure 3: Relationship between depth and α_s

As shown in Figure 2 and Figure 3 above, point B is the center of the sea, α is the angle between the seabed and the horizontal plane, the projection line of the survey line on the seabed surface is AB. The angle between AB and the horizontal plane is γ , the geometric relations is easy to know that γ is a function of the variation of β. The range of which is $0 \sim \alpha$, the plane HEF is the plane perpendicular to the survey line, EF is the intersection line between the beams and the sea floor, let the intersection line EF and the horizontal plane angle is α_s. From the change of the relationship can be seen α_s is a function the change of β. According to the Pythagorean theorem and the trigonometric relationship, the functional equation of γ and α_s with respect to β can be solved.

$$\gamma = \arcsin\sqrt{\frac{\cos^2\beta\cdot\sin^2\alpha}{\cos^2\beta\cdot\sin^2\alpha+\cos^2\alpha}} \quad (10)$$

$$\alpha_s = \arcsin\sqrt{\sin^2\alpha - \sin^2\gamma} \quad (11)$$

The relationship between depth and distance from the center of the sea can be obtained.

$$D(s) = \begin{cases} D_0 - (\tan\alpha_s)s, & 90^\circ \le \beta \le 270^\circ \\ D_o + (\tan\alpha_s)s, & 0 < \beta < 90^\circ, 273^\circ < \beta < 360^\circ \end{cases} \quad (12)$$

S is the distance of points on the survey line from the center of the sea area, D_0 is the sea depth at the center of the sea area to be measured.

Given a β given, each determined D corresponds to a different coverage width W, so the width W is a function of β. Replace α in Equation (5) with α_s, we get W as a function of $D(s)$.

$$W = D(s)\cdot\sin\frac{\theta}{2}\cdot\left[\frac{1}{\cos\left(\frac{\theta}{2}+\alpha_s\right)}+\frac{1}{\cos\left(\frac{\theta}{2}-\alpha_s\right)}\right] \quad (13)$$

Finally, the results are derived from the above relational expressions, as shown in Table. 2.

Table 2: Calculation results

Coverage width		Distance/nautical mile from the center of the sea by the survey vessel							
		0	0.3	0.6	0.9	1.2	1.5	1. 8	2.1
Line direction	0	415.69	466. 09	516.49	566.89	617.29	667.69	718.09	768. 48
	45	416. 19	451.87	487.55	523.23	558.91	594.59	630.27	665.95
	90	416.69	416.69	416.69	416.69	416.69	416.69	416.69	416.69
	135	416. 19	380.51	344.83	309.15	273. 47	237.79	202.11	166. 43
	180	415.69	365.29	314.89	264.50	214. 10	163. 70	113.30	62.90
	225	416. 19	380.51	344.83	309. 15	273. 47	237.79	202.11	166.43
	270	416.69	416.69	416.69	416.69	416.69	416.69	416.69	416.69
	315	416.19	451.87	487.55	523.23	558.91	594.59	630.27	665.95

4. Measuring line wiring strategy

4.1 Cabling planning model

As shown in Figure 4 and Figure 5, assuming that the starting point of a survey line is the shallowest point of the sea in the northeast corner, the survey line is L_0, the direction of the survey line is southwest, and the point is set to be point O, and a right-angled coordinate system is established with the x-positive direction to the west and the y-positive direction to the south. The angle between L_0 and the x-positive direction is β, ranging from $0 < \beta < \pi/2$. According to the literature of relevant marine surveys, the survey line should be planned parallel to the isobath, according to the above data estimation [5]. When $0 < \beta < \pi/4$, the overlap rate in the deeper part is too large, the overlap rate in the shallower part is too small, in order to simplify the calculation, it can be reasonably rounded off, and take $\pi/4 < \beta < \pi/2$.

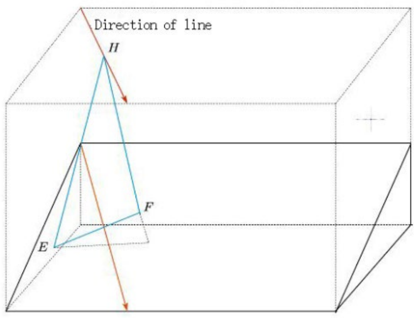

Figure 4: Coverage width in relation to β, line distance

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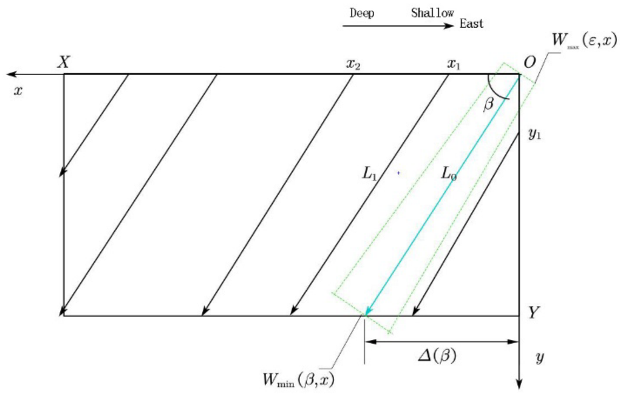

Figure 5: Wiring under different cabling scenarios

The pattern of change in depth in the east-west direction:

$$D(x) = D_0 + x(\tan\alpha) \tag{14}$$

L_0 The distance between the end point and the y axis is Δ , the starting depth is the shallowest depth $D_{\min}(x)$, the end point is the deepest depth $D_{\max}(x)$.

$$D_{\min}(x) = D(x), D_{\max}(x) = D(x+\Delta) \tag{15}$$

Rewriting the function relationship between W and $D(x)$:

$$W(D) = D(x) \cdot \sin\frac{\theta}{2} \cdot \left[\frac{1}{\cos\left(\frac{\theta}{2}+\alpha_s\right)} + \frac{1}{\cos\left(\frac{\theta}{2}-\alpha_s\right)}\right] \tag{16}$$

D and α_s are both functions of β.

According to the function, the maximum and minimum widths can be found.

$$W_{\min}(0) = W[D_{\min}(0)], W_{\max}(0) = W[D_{\max}(0)] \tag{17}$$

According to the definition of overlap rate, the maximum and minimum overlap rates are calculated. According to the above conclusion, the deepest overlap rate corresponds to the highest overlap rate and the shallowest overlap rate is the lowest for each line, which only need to satisfy the minimum overlap rate is more than 10% and the maximum overlap rate is less than 20% [4].

$$\eta_{\min}(i) = 1 - \frac{d_i}{W_{\min}(0)\cos\alpha_s}, \eta_{\max}(i) = 1 - \frac{d_i}{W_{\max}(0)\cos\alpha_s} \tag{18}$$

When $x_i > Y - \Delta$, according to the geometric relationship, the deepest depth is DMAX constant. At this time, the maximum coverage width is WMAX constant, calculate the length of the line at this time.

$$L_i = \frac{X - x_i}{\cos\beta}\ x_i < X \tag{19}$$

Then we calculate L_0 the eastern line, where the minimum depth is D_0. The y-coordinates of the starting point are y_i,

$$y_i = \frac{\sum d_i}{\cos\beta}\ i = -1, -2, \ldots \tag{20}$$

At this point the corresponding line length is,

$$Li = \frac{Y - y_i}{\sin\beta}\ i = -1, -2, \ldots\ y_i < Y \tag{21}$$

Total length of the line,

$$L = \Sigma L_i \tag{22}$$

Based on the previous equations, for the line length with angle β and line spacing d and line spacing, the problem of designing the distribution of lines is given as the following optimization model.

$$\min \textstyle\sum_i L_i \tag{23}$$

$$\text{s.t.}\begin{cases}\eta_{\min}(i) > 10\% \\ \eta_{\max}(i) < 20\%\end{cases} \tag{24}$$

Finally, the optimal wiring strategy is calculated by genetic algorithm [6], 36 lines are arranged at different distances from south to north, and the total distance of the lines is 133344m. The distribution is shown in Table. 3 below.

Table 3: Distribution of shortest lines

Line distance	-3680.9	-3084.7	-2533.6	-2024.2	-1553.3	-1118.0	-715.7	-343.8	0.0	317.8	611.5	883.0
Sea depth	206.4	190.8	176.3	163.0	150.7	139.3	128.7	119.0	110.0	101.7	94.0	86.9
Coverage	716.7	662.5	612.3	566.0	523.2	483.6	447.0	413.2	382.0	353.1	326.4	301.7
Overlap rate	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Line distance	1134.0	1366.0	1580.4	1778.6	1961.8	2131.2	2287.7	2432.5	2566.2	2689.9	2804.2	2909.8
Sea depth	80.3	74.2	68.6	63.4	58.6	54.2	50.1	46.3	42.8	39.6	36.6	33.8
Coverage	278.9	257.8	238.3	220.2	203.6	188.2	173.9	160.8	148.6	137.4	127.0	117.4
Overlap rate	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Line distance	3007.5	3097.7	3181.2	3258.3	3329.6	3395.5	3456.4	3512.7	3564.7	3612.8	3657.3	3698.4
Sea depth	31.2	28.9	26.7	24.7	22.8	21.1	19.5	18.0	16.7	15.4	14.2	13.2
Coverage	108.5	100.3	92.7	85.7	79.2	73.2	67.7	62.6	57.8	53.5	49.4	45.7
Overlap rate	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

4.2 Model application

Firstly, we analyze the seawater depth of different sea locations, and then we obtain more data by spline interpolation, and then we model the seabed of the sea area. 3D After modeling the seabed model, the data are drawn into isobaths by MATLAB as shown in Figure 6 below, and the isobaths divide the 3D model into 12 different slopes, as shown in Figure 7.

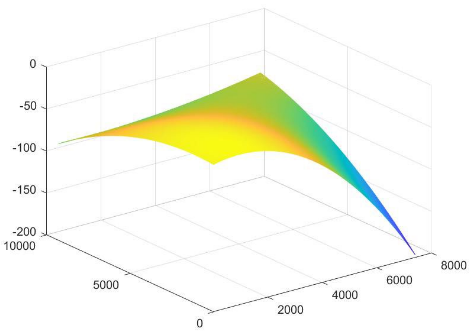

Figure 6: 3D modeling of the seafloor in the area to be measured

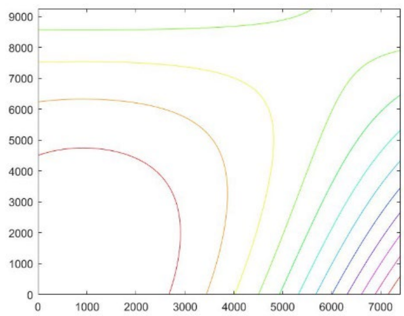

Figure 7: Bathymetry of the sea area to be measured

According to the wiring plan model, the line parallel to the isobath is the shortest, and the seabed of the given sea area is simplified by linking the northeast and southwest corners, and divided into two triangular areas, as shown in Figure 8, the west is high and the east is low, and the south is high, and the north is low.

The angle between $\triangle ABC$ and the horizontal plane is α_1, the angle between $\triangle ADC$ and the horizontal plane is α_2 , the total length of the survey line for the line planning scheme parallel to the isobath is the shortest, as shown by the parallel lines at the top of Figure 8.

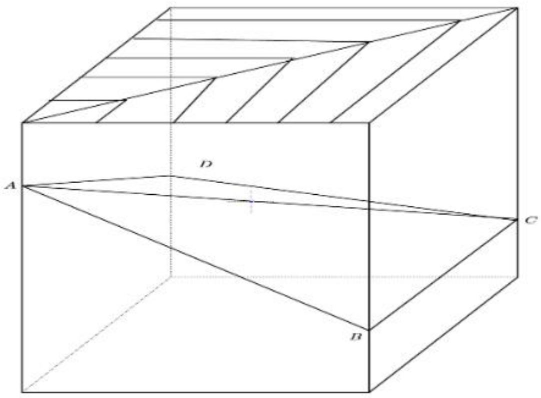

Figure 8: Schematic diagram of the distribution of survey lines

According to the wiring plan model, given the direction of the survey line parallel to the seawater isobath, the distribution pattern of the survey line and the length of the survey line, as well as the corresponding coverage rate can be calculated directly.

5. Conclusions

Based on the multibeam measurement technology, this study is carried out to address the line measurement problem in seabed topographic survey by establishing an optimization model and scheme. By analyzing the distance of the survey line from the center, the seawater depth and the coverage width, and combining the geometric theorem and algebraic operation, a linear model and a mathematical model are established, which effectively solve the key problems of depth measurement and survey line wiring. The total length of the survey line was optimized by genetic algorithm to ensure the optimal strategy of survey line wiring. Finally, we successfully established the optimization model and realized the effective adjustment of depth measurement and line routing in the sea area to be measured, which provides theoretical support and practical guidance for marine topographic survey and provides important technical support for China's sea area exploration and geological exploration work.

References

[1] Lecours V, Dolan M F J, Micallef A, et al. A review of marine geomorphometry, the quantitative study of the seafloor [J]. Hydrology and Earth System Sciences, 2016, 20(8): 3207-3244.

[2] Ferreira I O, Andrade L C, Teixeira V G, et al. State of art of bathymetric surveys[J]. Boletim de Ciências Geodésicas, 2022, 28: e2022002.

[3] Cheng D, Zhang Y, Liu B et al. Comprehensive evaluation of bathymetric performance of deepwater multibeam bathymetric system [J]. Marine Surveying and Mapping, 2021, 41(01):51-55.

[4] Oktaviani N, Hartanto P, Dewi R S, et al. Coastline Accuracy Assessment Developed by Using Multi Data Source[J]. Journal of Applied Geospatial Information, 2023, 7(1): 762-768.

[5] Wang Y, Li Z, Zhang Y. Multibeam Line Finding Problem Based on Geometric Analysis and Genetic Algorithm [J]. Transactions on Computational and Applied Mathematics, 2023, 3(1): 40-51.

[6] Freitas S, Serra F, Brito M C. PV layout optimization: String tiling using a multi-objective genetic algorithm [J]. Solar Energy, 2015, 118: 562-574.