

Mathematical Modeling and Analysis of COVID-19 Epidemic Situation Based on Markov Chain

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Abstract: Based on the statistical data of COVID-19 from January 22, 2020 to June 7, 2020 in Hubei, China, the states are divided according to the standard deviation, and based on the Markov chain theory, the mathematical analysis model of daily confirmed, dead and cured number of COVID-19 in Hubei, China was established, and the probability of steady distribution of each state and the average return time were used to analyze the prevention and control of COVID-19 epidemic situation in Hubei, China. The analysis shows that by June 2020, the daily confirmed number of COVID-19 in Hubei, China is basically in state I, the number of daily deaths is basically in state I-II, and the number of people cured per day is basically in state I-II.

Keywords: Markov chain, COVID-19 epidemic situation, Mathematical modeling

1. Introduction

COVID-19 (Corona Virus Disease 2019 (COVID-19), abbreviated as "COVID-19". The World Health Organization named it "2019 coronavirus disease", which refers to pneumonia caused by 2019-nCoV infection. Since December 2019, some hospitals in Wuhan City, Hubei Province have successively found a number of cases of unexplained pneumonia with a history of exposure to South China seafood market, which were confirmed to be acute respiratory infectious diseases caused by 2019-nCoV infection [1].

2. Markov chain calculation model of the number of people diagnosed

The daily diagnosed data of COVID-19 from January 22, 2020 to June 7, 2020 in Hubei, China were analyzed [2]. The highest number of confirmed cases per day is 14840, and the minimum number of confirmed cases is 0. All the confirmed cases are divided into 5 status intervals, as shown in Table 1.

Table 1: Status classification of the number of diagnosed persons

Status of confirmed population	I (Less)	II (few)	III(some)	IV(many)	V (more)
Number of diagnosed persons interval	[0 500)	[500 1500)	[1500 4000)	[4000 8000)	[8000 15000]

The frequency of diagnosed persons on all dates is calculated based on statistical data, as shown in Table 2.

Table 2: Frequency of confirmed cases

$j \backslash i$	1	2	3	4	5	Total
1	110	1	2	1	1	115
2	1	0	1	0	0	2
3	3	1	8	2	0	14
4	1	0	2	0	0	3
5	0	0	1	0	2	3
Total	115	2	14	3	3	137

During the 138-day period(2020.1.22-2020.6.7), the number of confirmed cases in the range of 5 grades $n_i (i = 1, 2, 3, 4, 5)$ is available. $P_j = P(x_0 = j)$

As a result, the state transition probability matrix is

$$P = \begin{bmatrix} \frac{n_{11}}{n_1} & \frac{n_{12}}{n_1} & \frac{n_{13}}{n_1} & \frac{n_{14}}{n_1} & \frac{n_{15}}{n_1} \\ \frac{n_{21}}{n_2} & \frac{n_{22}}{n_2} & \frac{n_{23}}{n_2} & \frac{n_{24}}{n_2} & \frac{n_{25}}{n_2} \\ \frac{n_{31}}{n_3} & \frac{n_{32}}{n_3} & \frac{n_{33}}{n_3} & \frac{n_{34}}{n_3} & \frac{n_{35}}{n_3} \\ \frac{n_{41}}{n_4} & \frac{n_{42}}{n_4} & \frac{n_{43}}{n_4} & \frac{n_{44}}{n_4} & \frac{n_{45}}{n_4} \\ \frac{n_{51}}{n_5} & \frac{n_{52}}{n_5} & \frac{n_{53}}{n_5} & \frac{n_{54}}{n_5} & \frac{n_{55}}{n_5} \end{bmatrix} = \begin{bmatrix} \frac{110}{115} & \frac{1}{115} & \frac{2}{115} & \frac{1}{115} & \frac{1}{115} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{3}{14} & \frac{1}{14} & \frac{8}{14} & \frac{2}{14} & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix}$$

$n_{ij}(i, j = 1, 2, 3, 4, 5)$ Indicates the number of diagnosed people who have transferred from state to state j.

Using matlab programming to find out that the daily one-step transfer matrix is

$$P^{(2)} = \begin{bmatrix} 0.9259 & 0.0096 & 0.0396 & 0.0108 & 0.0141 \\ 0.5854 & 0.0401 & 0.2944 & 0.0758 & 0.0043 \\ 0.4108 & 0.0427 & 0.4612 & 0.0835 & 0.0019 \\ 0.4617 & 0.0505 & 0.3867 & 0.0981 & 0.0029 \\ 0.0714 & 0.0238 & 0.4127 & 0.0476 & 0.4444 \end{bmatrix}$$

From the discrimination condition of the ergodicity of the Markov process, it can be known that the process has ergodicity. $\{\pi_j, j = 1, 2, 3, 4, 5\}$ is the stationary distribution of the Markov chain.

$\{\mu_j, j = 1, 2, 3, 4, 5\}$ Average return time for each state.

$$\begin{cases} \pi_1 = 0.9259\pi_1 + 0.5\pi_2 + 0.2143\pi_3 + 0.3333\pi_4 \\ \pi_2 = 0.0087\pi_1 + 0.0714\pi_3 \\ \pi_3 = 0.0174\pi_1 + 0.5\pi_2 + 0.5714\pi_3 + 0.6667\pi_4 + 0.3333\pi_5 \\ \pi_4 = 0.0087\pi_1 + 0.1429\pi_3 \\ \pi_5 = 0.0087\pi_1 + 0.6667\pi_5 \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1 \end{cases}$$

The interval distribution of the number of people diagnosed in the stationary state is obtained by solving the equation set.

$$(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = (0.8394, 0.0146, 0.1022, 0.0219, 0.0219) \tag{1}$$

The average return time for each state is

$$(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5) = \left(\frac{1}{\pi_1}, \frac{1}{\pi_2}, \frac{1}{\pi_3}, \frac{1}{\pi_4}, \frac{1}{\pi_5} \right) = (1.191, 68.493, 9.785, 45.662, 45.662) \tag{2}$$

It can be seen from the formula (1) that the probability of the daily diagnosed number of COVID-19 in Hubei, China is 0.8394, and the probability sum of the other four states is 0.1606, indicating that the daily confirmed number of COVID-19 in Hubei, China is basically in the first state.

It can be seen from formula (2) that the average return time of COVID-19 in state I in Hubei, China is 1.191, which is much less than that in the other four states, indicating that COVID-19 in Hubei, China has been in state I for the longest time.

The above analysis shows that the measures taken by China in the prevention and control of the

epidemic situation of COVID-19 have achieved very good results, and provide effective reference experience for the epidemic prevention and control work of countries all over the world.

3. Markov chain calculation model of death toll

The daily death toll data of COVID-19 from January 22, 2020 to June 7, 2020 in Hubei, China were analyzed. The highest daily death toll is 1290 and the lowest death toll is 0. The data of all deaths are divided into five state intervals, as shown in Table 3.

Table 3: Death toll status classification

Status of confirmed population	I (Less)	II (few)	III(some)	IV(many)	V (more)
Number of diagnosed persons interval	[0 20)	[20 50)	[50 200)	[200 500)	[500 1300]

According to the statistical data, the frequency of deaths on all dates can be obtained, as shown in Table 4.

Table 4: Frequency of deaths

$\begin{matrix} j \\ i \end{matrix}$	1	2	3	4	5	Total
1	78	3	3	0	1	85
2	3	24	1	0	0	28
3	2	1	17	1	0	21
4	1	0	0	0	0	1
5	1	0	0	0	1	2
Total	85	28	21	1	2	137

Then the state transition probability matrix is

$$P = \begin{bmatrix} \frac{78}{85} & \frac{3}{85} & \frac{3}{85} & 0 & \frac{1}{85} \\ \frac{3}{28} & \frac{24}{28} & \frac{1}{28} & 0 & 0 \\ \frac{2}{21} & \frac{1}{21} & \frac{17}{21} & \frac{1}{21} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

The daily one-step transfer matrix of the death toll calculated by matlab programming is

$$P^{(2)} = \begin{bmatrix} 0.8075 & 0.0883 & 0.0828 & 0.0030 & 0.0184 \\ 0.1936 & 0.6443 & 0.0845 & 0.0030 & 0.0029 \\ 0.2172 & 0.1100 & 0.5452 & 0.0314 & 0.0031 \\ 0.9176 & 0.0643 & 0.0622 & 0.0017 & 0.0167 \\ 0.7088 & 0.0410 & 0.0399 & 0.0008 & 0.1363 \end{bmatrix}$$

From the discrimination condition of the ergodicity of the Markov process, it can be known that the process has ergodicity. There are

$$\begin{cases} \pi_1 = 0.9176\pi_1 + 0.1071\pi_2 + 0.0952\pi_3 + \pi_4 + 0.5\pi_5 \\ \pi_2 = 0.0353\pi_1 + 0.8571\pi_2 + 0.0467\pi_3 \\ \pi_3 = 0.0353\pi_1 + 0.0357\pi_2 + 0.8095\pi_3 \\ \pi_4 = 0.0476\pi_3 \\ \pi_5 = 0.0118\pi_1 + 0.5\pi_5 \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1 \end{cases}$$

Then the interval distribution of the death toll in the steady state is

$$(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = (0.6204, 0.2044, 0.1533, 0.0073, 0.0146) \tag{3}$$

The average return time for each state is

$$(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5) = (1.612, 4.892, 6.523, 136.986, 68.493) \quad (4)$$

It can be seen from formula (3) that the probability sum of COVID-19 's daily death toll in state I and II in Hubei, China is 0.8248, and the probability sum of the other three states is 0.1752, indicating that the daily death toll of COVID-19 in Hubei, China is basically in state I-II. And mainly in the first state.

It can be seen from formula (4) that the average return time of COVID-19 's daily death toll in state I in Hubei, China is 1.612, which is less than the average return time in state II and III, and much less than that in state IV and V. It shows that the daily death toll of COVID-19 in Hubei, China has been in the first state for the longest time, followed by the second and third states.

The above analysis shows that the treatment methods adopted by the Chinese medical team for COVID-19 patients have effectively reduced the mortality rate of the epidemic.

4. Conclusions

The Markov chain mathematical analysis model of daily confirmed number and death toll of COVID-19 in Hubei Province of China is established, and the prevention and control of COVID-19 epidemic situation in Hubei Province of China is analyzed. The main conclusions are as follows:

(1) The daily confirmed number of COVID-19 in Hubei, China is basically in the first state, indicating that the measures taken by China in the prevention and control of the epidemic situation of COVID-19 have achieved very good results, and provide effective reference experience for the epidemic prevention and control work of countries all over the world.

(2) The daily death toll of COVID-19 in Hubei, China is basically in the first-second state, indicating that the treatment adopted by the Chinese medical team for COVID-19 patients has effectively reduced the mortality rate of the epidemic.

(3) The number of people cured per day of COVID-19 in Hubei, China is basically in the first-second state, indicating that by June 7, 2020, China has not completely found a way to solve the COVID-19 virus.

References

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