

Reflecting the Basic Idea of Mathematics in Mathematics Education

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Abstract: *The three core elements of abstraction, reasoning and model constitute the basic idea of mathematics. The basic idea of mathematics has the following two principles: one is the ideas that mathematics must rely on for its production and development; the second is the basic thinking characteristics that people who have studied mathematics should have.[1] Mathematical education that embodies the basic ideas of mathematics should at least have the following characteristics: pay attention to the degree of consistency between school mathematics tasks and real situations outside school; pay attention to students' emotional fields; pay attention to students' individual socialization; provide adaptive thinking guidance.*

Keywords: *Basic idea of mathematics, Mathematics education*

1. Introduction

Freidenthal believes that the greatest advantage of mathematics is its generality. The magic of mathematics is that it can remove the context and convert the rest into mathematical forms that can be used repeatedly.[2]The discovery of mathematical conclusions often relies on subjective personal intuition, In order to facilitate the interpretation of knowledge and elevate communication to a sub-meaningful community with a considerable degree of autonomy, we need to use a mathematical language achieved through social processes independent of subjective perception, and to obey the principle of mathematical rigor. However, being too obsessed with symbols and logical terminology in mathematics education, and sticking to the rigor of mathematics, is not conducive to the purpose of educating people. A good mathematics education should help students truly understand, think independently, and acquire real knowledge.

Dewey once said in his article that for students, the biggest waste of school education is to waste their lives. On the one hand, students cannot use the experience they have gained outside the school in any complete and free way within the school; On the other hand, he cannot apply what he has learned in school to his daily life. So the school is isolated from the life of the students.[3] This implies that if mathematics is taught in a way that is divorced from the students' existing experience, the students will not be able to apply it, and such mathematics learning is useless. Many application questions that are often accompanied by existing mathematics education are originally intended to make abstract mathematics concrete with real situations as carriers. However, school education is mostly based on explanations. And many of the current mathematics problems are "essentially school problems covered with a thin layer of reality connection", which leads to the mainstream of inauthenticity and repetition in school education.[4] This makes students' attitudes and beliefs about mathematics become negative, and many students feel confused in the process of mathematics learning, "Why should I learn this?". Therefore, many researchers began to think about how to improve mathematics education.

2. Reflecting the basic idea of mathematics in mathematics education

In the process of reading and thinking, I believe that mathematics education that embodies the basic ideas of mathematics has at least the following characteristics:

2.1. Pay attention to the degree of consistency between school mathematics tasks and real-world situations outside of school

Mathematics is a highly theoretical subject, and one of the ways to connect mathematics with life is through word problems. Historically, word problems appeared more than 2,000 years ago. At the time,

thinking about word problems was a recreational activity for adults. As a man-made question type with realistic situation as the background, word problems mainly have two functions: to train students' computing ability; to show students the application situation of mathematical skills. However, there are also many shortcomings in the word problems: ignoring students' common sense, ignoring other simple non-algebraic methods in order to embody modern algebraic methods and so on. Therefore, many educators began to think about how to improve the shortcomings of word problems.

In view of the shortcomings of word problems, mathematical modeling came into being. Studies have shown that students who learn mathematics through traditional methods have no significant difference in performance compared to students who learn mathematics through modeling methods, but when using mathematical methods, the latter outperforms the former. This shows that mathematical modeling helps learners to transfer knowledge. As the important subject of mathematical modeling has entered people's field of vision, many educators have also improved the teaching of mathematical modeling by analyzing the process of mathematical modeling activities. For example, Borromeo Ferri adopts the individual modeling route to study the construction of mathematical models from a cognitive perspective. She refers to "visible" information such as oral expressions or external representations, and adopts the form of arrow diagrams imposed on the research framework to complete her research. [5] Årlebäck developed a modeling activity diagram to document the details of building a mathematical model from the mathematical thought process used to create the mathematical model. [6]

The definition of mathematical literacy in PISA2021 adjusts the "different situations" in the definition of mathematical literacy in PISA2012 to "different real-world situations", and adds the purpose of "solving problems", it is further clarified that mathematical literacy is intended to reflect the ability a person uses to solve problems in real-world situations. [7] On the one hand, learners' learning in real-world situations can stimulate their learning motivation and curiosity, and on the other hand, they can enhance learning transfer by demonstrating the connection between school mathematics and real-world problems. [8] Niss distinguishes between the real world and the mathematical world, and defines real mathematical problems as embedded in the real world and involving some mathematical concepts, methods and results, Learners need to deal with objects, phenomena, and problems that are truly relevant to the field and recognized by those working in the field. [9] Students can make judgments in open-ended, ambiguous questions through mathematical problems that simulate the challenges and performance standards typically faced by social roles such as writers, businessmen, scientists, community leaders, designers, or historians. Students' mastery of knowledge should not only be reflected in making simple oral answers, but also mean that they can make effective, transformative or novel responses to questions or complex situations. [10]

There are also scholars who believe that in education, not everything that is a mathematical task needs to be real, but that real mathematics education is to provide students with an experience that allows them to "feel" how experts handle real mathematics in the real world. To serve this purpose, the task designer can reduce certain aspects of the real to the unreal - shrinking or simplifying activities from the real world to match the educational environment framed by time constraints, cognitive constraints, organizational constraints, etc. Authenticity can relate to different aspects of the task, such as real contextual situations, real questions, real research objects, real research experiences, students' real answers, and the real applicability of mathematical tasks to real problems. The real question does not refer to the question that only has the real situation, but the question that the real protagonist in the real situation will really raise. [4] In the teaching process, taking into account the learners' understanding of mathematical tasks, the limitations of classroom teaching and other factors, educators can transform the tasks on the basis of real life.

2.2. Focus on the student's emotional realm

Researchers have found many times that learners' interests will affect their attention, goals, learning level, etc., and thus affect their learning process.

From a theoretical point of view, the process of interest development involves two kinds of interests. The authentic learning environment can trigger learners' emotional and cognitive processing to trigger their situational interests. It is then possible to develop a lasting personal interest through the learner's in-depth participation in the problem. [11]

In constructivism, knowledge is not simply a set of facts, concepts, or rules that can be accepted and remembered, but must be constructed and given meaning through actual experience. [12] Students who are poor in mathematics-related self-concept can change their beliefs about mathematics through

authentic learning, and improve their self-efficacy, learning motivation and attitude.[13] In addition, some researchers have proposed that mathematics education can be compared to the field of performance, tracking students' performance, setting different levels for the test to cultivate students' confidence in the process of gradual development, the desire to perform well in public and positive peer pressure provide students with a powerful incentive to better bridge the gap between current abilities and ideal performance. [10]

Learners' attitudes toward mathematics are not one-dimensional, mathematics has different content, and learners have different feelings about each type of mathematics. Task designers should consider factors such as the complexity, scope, and degree of students' experience when choosing a context. Educators believe that mathematics in an "everyday" context is easier to understand than its abstract equivalent, and that learning mathematics in an everyday context ensures that students' "everyday" life, but only if the math activity is really open and allows students to move in the direction of their understanding of the problem. By encouraging and supporting students' social and cultural values in the mathematics classroom by using context or by acknowledging personal line and direction, then their learning will be more meaningful. [8]

2.3. Pay attention to the individual socialization of students

Mathematics is incorporated into basic education due to its importance as a foundation for other scientific disciplines, as an effective tool for thinking and communication, and as a fundamental tool for many professions. Mathematics can be integrated into tools, crafts, arts, everyday life, etc., it can be part of a chair or part of a computer. According to the relationship between mathematics curriculum and off-campus practice, mathematics practitioners can be divided into four categories: first, builders, whose daily work is based on the prerequisite of mathematical ability, and conducts mathematics-rich construction practice; the second is operators, where mathematics is implicitly integrated into their work environment as part of the tools or tools they operate; the third is consumers, whose mathematics is reflected in their daily activities such as trade, medical care, communication, and services, such as taxation and pension systems; fourth, one-time practitioners, who live in marginal areas of the information economy, but also participate in mathematical practices such as buying and selling.[14]

From a sociological point of view, the primary function of school education is individual socialization, that is, "the process of social interaction in which a person acquires his own personality and learns ways to participate in society or a group." [15] Many real mathematics tasks will be closely related to the work of professionals, and the work faced by true professional mathematics researchers is very different from the tasks of students, for educational purposes, educators prefer to provide students with some secure and resource-rich learning environment that de-authenticates.[4]

The purpose of education in China is "cultivating morality and cultivating people". Front-line teachers can work hard to explore the way of subject moral education and complete the training goal - cultivating people with all-round development. In the process of mathematics teaching, attention should be paid to the penetration of moral education, and teachers can achieve this goal by constructing appropriate teaching situations. For example, when learning statistics-related knowledge points, you can show students the situation of ordinary household water consumption and water waste by citing information released by various authoritative statistical bureaus. Situations related to reality can improve students' classroom participation and allow them to "grow" knowledge based on their existing experience, so that they not only acquire subject knowledge, but also realize the improvement of moral cognition.

2.4. Provide adaptive thinking guidance

Harvard University's Center for Cognitive Research once conducted an experiment to observe the process of 3-year-old, 4-year-old, and 5-year-old children forming a pyramid with a number of wooden blocks that can fit together. Initially, the children worked independently, and the teachers observed whether the children could match the wooden blocks with each other, and if not, the tutors would provide further support; children can continue to work independently if they can. Once children can operate successfully, teachers will reduce their own intervention. [16] Teacher interventions should adhere to the "principle of least help": this help minimally supports the student's personal learning and problem-solving process so that the student can work at the greatest level of independence. [17]

A prerequisite for reasonable intervention is the determination of the student's current level of ability. Peter Stender's experiments show that a greater proportion of successful interventions are adaptive, while invasive interventions initiated by teachers have little effect. In the experiment, teachers asked students

to show their work status and results, and teachers made a comprehensive diagnosis of students' work and gave them support.[18] Such measures not only aim to help the child solve problems that he/she cannot solve independently, but also hope that over time the learner will acquire the missing knowledge and skills. As a teacher, it is necessary to diagnose the students' learning first and understand the obstacles in the students' learning process, in order to better help the students' progress through tools such as "scaffolding".

3. Conclusions

The above-mentioned teacher intervention is based on the specific personal situation of the learner. Due to the heterogeneity of the learners, it is mostly one-to-one behavior. The teacher's diagnosis and the student's response not only occur in the classroom, but also can be expressed in homework, inter-class interaction, etc. [19]

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