

Fast Active Reflector Adjustment Design Based on Ideal Paraboloid Optimization Model

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Abstract: Firstly, according to the symmetry and isotropy, the three-dimensional space is transformed into a two-dimensional plane, and the equation of the ideal parabola in the two-dimensional plane is determined. Then, the factors affecting the ideal paraboloid are analyzed, and the multi-objective optimization equation of the ideal paraboloid is established according to the influencing factors; Then the entropy weight method is used to weight each factor, the multi-objective optimization problem is transformed into a single objective optimization problem, and the optimal ideal paraboloid equation is obtained by traversing the value of h . Finally, the rotation transformation of the coordinate system is carried out according to the orientation of the target celestial body, and a new ideal paraboloid equation is obtained. It is found that a total of 692 main cable nodes participate in the adjustment, of which 515 corresponding actuators are extended and 177 corresponding actuators are shortened.

Keywords: Optimization model, Spatial analytic geometry, Coordinate system transformation

1. Introduction

"Fast" celestial eye is a spherical radio telescope with an aperture of 500m. An important innovation of "fast" is that it has an active reflector system, which is mainly composed of reflector panel, main cable net, lower cable, actuator and peripheral support structure. The reflecting surface of the reflecting panel in the reference state is a reference sphere with a radius of about 300m and a diameter of 500m. During operation, part of the reflecting panel can form an approximate rotating paraboloid with a diameter of 300m according to the observation direction, so that the parallel electromagnetic wave from the target observation object can be reflected to the effective signal receiving area of the signal receiving system (feed cabin). The effective signal receiving area of the feed cabin is a disk with a diameter of 1m centered on the feed cabin. The center of the feed cabin moves on a sphere concentric with the reference sphere and is located on the focus of the rotating paraboloid formed during operation.

The process of adjusting the reflection panel to a paraboloid mainly depends on the cooperation between the lower cable and the actuator in the reflection panel system. The length of the lower cable is fixed, and the top of the actuator can expand and contract radially along the reference sphere, so as to complete the adjustment of the lower cable, so as to adjust the shape and position of the reflection panel. The radial expansion range of the top of the actuator is $-0.6\text{m} \sim 0.6\text{m}$. The requirement of this competition is to determine an ideal paraboloid, and adjust part of the reflection panel to a working paraboloid as close to the ideal paraboloid as possible by adjusting the expansion amount at the top of the actuator, so as to achieve the best receiving effect of electromagnetic wave of the target.

2. Ideal paraboloid foundation model

2.1. Convert 3D space to 2D plane

Under certain conditions, based on the characteristics of symmetry and isotropy between paraboloid and sphere, this paper adopts longitudinal section for research, that is, parabola and sphere are simplified into parabola and arc, and the coordinate system is established with the center of reference sphere as the origin, horizontal direction as x axis and vertical direction as y axis. Then the equations of circular arc and paraboloid are as follows:

$$\begin{cases} x^2 + y^2 = R^2 \\ -x^2 + 4Dfy + 4Df(R + h) = 0 \end{cases} \quad (1)$$

Where R is the radius of the reference sphere, and R = 300.4m is taken as the reference data; D is the diameter of paraboloid, D = 300m; h is the distance between the apex of the paraboloid and the original reference sphere; f is the ratio of focal length to aperture of paraboloid, $f = (F + h)/D = 0.4666 + h/300$, where F is the radius difference between two concentric spheres. It can be seen from the information that $F = 0.466R$.

2.2. Analysis on the relationship between radial displacement of main cable node and h

2.2.1. Analysis and calculation steps

By analyzing the parabolic equation, it can be found that the equation contains only one variable h. combined with the above analysis, firstly, the change of h is preliminarily studied to calculate the change of Δl_i under different h values. Considering the reflection and convergence effect of the rotating paraboloid on the electromagnetic wave, it can be known from the analysis that the vertex of the working paraboloid must be below the vertex of the original reference sphere, and the maximum length of the actuator contraction is 0.6m, so the value range of h is $0 \sim 0.6m$. In this paper, h is assigned in the range of $0 \sim 0.6m$ in the step of 0.001m, and the maximum value $\text{Max} \{\Delta l_i\}$ and minimum value $\text{min} \{\Delta l_i\}$ of the corresponding displacement of all main cable nodes to be adjusted in each case are calculated. The calculation steps are as follows:

Step 1. Write the program to assign the value of h within the range of $0 \sim 0.6m$ in steps of 0.001m.

Step 2. Calculate the radial displacement of the main cable node to be adjusted corresponding to each h value. Since the main cable nodes are not necessarily evenly distributed on the selected parabolic equation, by consulting relevant literature [1], considering the symmetry of the parabola and the diameter of the paraboloid is 300m, the variable range of the x-coordinate axis is $0 \sim 150m$, and 1m is used as the analysis step to calculate the radial displacement corresponding to the selected point. Assuming that the coordinate origin is O and the selected point is M, the radial displacement of the point is calculated as follows:

$$\Delta l_M = R - OM = R - \sqrt{x_M^2 + y_M^2} \quad (2)$$

Step 3. Calculate the maximum and minimum values of radial displacement of all points corresponding to each H value, and calculate the maximum value of absolute radial displacement after taking the absolute value of radial displacement. And save the results.

2.2.2. Calculation results and result analysis

Some calculated results are shown in Table 1:

Table 1: Relationship between radial displacement of main cable node and h.

h/(m)	Minimum radial displacement (/m)	Maximum radial displacement (/m)
0	0.0001	0.6962
0.1	-0.0999	0.5892
0.2	-0.1999	0.4821
0.3	-0.2999	0.3752
0.4	-0.3999	0.2683
0.5	-0.5107	0.1614
0.6	-0.6221	0.0545

It can be seen from the observation table that with the increase of the distance h between the apex of the paraboloid and the original reference sphere, the maximum distance of the positive movement of the main cable node in the working paraboloid becomes smaller and smaller, and the maximum distance of the negative movement becomes larger and larger, which shows that with the increase of h, the whole paraboloid is farther and farther away from the center of the reference sphere, which is consistent with common sense; In addition, the maximum value of the absolute value of the main cable node displacement first decreases and then increases with the increase of H, and reaches the minimum value when $h = 0.336m$.

Since $|\Delta l_i| \leq 0.6m$ needs to be met, it can be obtained from the calculation results combined with the

limiting conditions given in the title, and the value range of H is 0.09m ~ 0.58m.

2.3. Analysis of smooth transition between paraboloid edge and original datum sphere

2.3.1. Analysis and calculation steps

Considering the integrity of the forming of the working paraboloid and the original reference sphere, so that the stress at the edge will not be too large for a long time, which will reduce the fatigue resistance of the cable network and affect the service life, the paraboloid edge and the original reference sphere should transition smoothly, that is, the absolute value $|\Delta l_{edge}|$ of the radial displacement of the main cable node at the edge should be as small as possible, And the difference Δd between the derivatives at the intersection of the paraboloid and the reference sphere shall be as small as possible. The derivative of the original reference sphere at the edge of the paraboloid can be obtained directly from the equation of the arc, and the result is 0.5774; Combined with the equation of parabola, the derivative value of parabola at the edge of paraboloid can be obtained by formula.

$$y' = \frac{x}{2Df} \quad (3)$$

H is still assigned in the range of 0 ~ 0.6m in the step of 0.001m, and the radial displacement of the edge main cable node and its corresponding derivative value are calculated in each case. The calculation steps are as follows:

Step 1. Write the program to assign the value of h within the range of 0 ~ 0.6m in steps of 0.001m.

Step 2. Calculate the radial displacement of the paraboloid edge main cable node corresponding to each h value. That is, take $x = 150m$, calculate the absolute value of the corresponding radial displacement according to formula 2, and save the results.

Step 3. Calculate the derivative value at the paraboloid edge corresponding to each h value, i.e. take $x = 150m$, calculate the derivative value according to formula 3, and save the results.

2.3.2. Calculation results and result analysis

With the increase of the distance h between the apex of the paraboloid and the original reference sphere, the absolute radial displacement of the edge node first decreases and then increases, and the minimum value is taken when $h = 0.042m$. The derivative value of the original reference sphere at the edge of the paraboloid is 0.5774. It can be found that the derivative value of the paraboloid at the edge of the paraboloid is always less than that of the original reference sphere, and the derivative difference increases with the increase of h.

2.4. Analysis of smooth transition between paraboloid edge and original datum sphere

Through the above analysis, the advantages and disadvantages of the ideal paraboloid are related to three factors: the maximum value $\max\{|\Delta l_i|\}$ of the absolute value of the radial displacement of the main cable node, the absolute value $|\Delta l_{edge}|$ of the radial displacement of the paraboloid edge node, and the difference Δd of the derivatives at the intersection of the paraboloid edge and the reference sphere, and the corresponding H values are different when the three factors reach the optimum respectively, Therefore, it is necessary to establish an optimization model to determine the ideal paraboloid.

According to the above analysis, the changes of these three factors are only related to the value of h, so h is the only decision variable, and the value range of H is 0.09m ~ 0.58m. In order to achieve the optimal ideal paraboloid, the smaller the three factors, the better. Therefore, the following multi-objective optimization model is established:

$$\begin{cases} \min & \max\{|\Delta l_i|\} \\ \min & |\Delta l_{edge}| \\ \min & \Delta d \end{cases} \quad (4)$$

s.t. $0.09m \leq h \leq 0.58m$

In order to weigh these three factors, first normalize each factor and set the weight for each factor, so that the multi-objective optimization problem can be transformed into a single objective optimization

problem. The single objective optimization equation is as follows:

$$\min Y = \lambda_1 \max\{|\Delta l_i|\}' + \lambda_2 |\Delta l_{\text{edge}}|' + \lambda_3 \Delta d' \quad (5)$$

2.5. Solution of optimization model

Through the above analysis, it can be seen that the change of h only affects the size of the above three factors. Take 0.001m as the step, assign a value to h within the range of 0.09 ~ 0.58m, calculate the Y value in each case, and find the h value corresponding to the minimum y value, so as to obtain the optimal ideal paraboloid.

After calculation, the result is that the h corresponding to the optimal score Y value is 0.336m, the absolute value of the maximum radial displacement of all corresponding nodes is 0.3368m, the absolute value of the displacement of edge nodes is 0.3368m, and the difference between the derivatives at the intersection of paraboloid edge and reference sphere is 0.0422.

All results are shown in Table 2:

Table 2: Optimal result.

index	result
Optimum h value	0.3360m
Best score Y value	0.3313
Maximum absolute radial displacement of node	0.3368m
Absolute displacement of edge node	0.3368m
Derivative difference at edge	0.0422
Parabolic equation	$-x^2 + 561.264y + 168792.290 = 0$
Parabolic equation	$-(x^2 + y^2) + 561.264z + 168792.290 = 0$

3. Optimization and improvement model

3.1. Solution of optimization model

Taking the line between the position of the observation point and the center of the reference sphere as the z'' axis, we establish the coordinate system c-x''y''z'', which is obtained by two rotations of the original coordinate system c-xyz, as shown in the figure below.

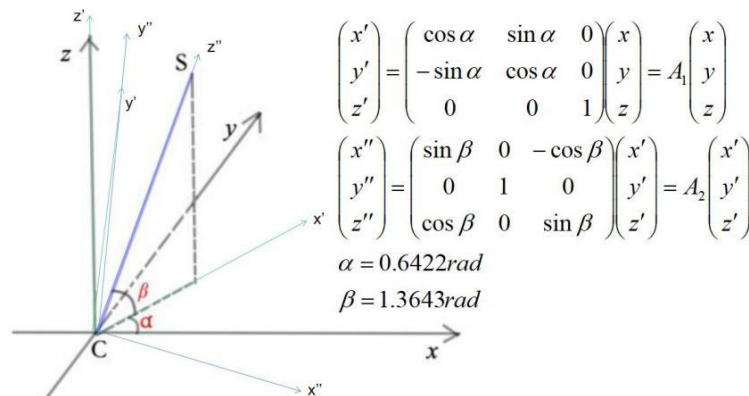


Figure 1: Schematic diagram of coordinate system rotation transformation.

3.2. New coordinates of main cable node and calculation of expansion length

Firstly, according to the assumption that the actuator under the main cable node can only expand and contract along the radial direction of the reference sphere, we can calculate the linear equation passing through the origin c of the reference sphere and a main cable node (x₀, y₀, z₀). The parameter equation is expressed as (where t is the parameter):

$$\begin{cases} x = \frac{x_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}} t \\ y = \frac{y_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}} t \\ z = \frac{z_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}} t \end{cases} \quad (6)$$

Since the diameter of the rotating paraboloid is only 300m, only a part of the main cable nodes on the reference sphere need to be adjusted during adjustment. For the main cable nodes that are not adjusted, the coordinates when they maintain the reference state remain unchanged, and their expansion length is naturally 0. We use the following criteria to judge whether a main cable node participates in adjustment:

Assuming that the direction vector of the connecting line between the observation point and the spherical center of the reference sphere is \vec{n} , $\vec{n}_0 = -(0.1642, 0.1228, 0.9788)$ can be calculated. Consider the angle between the paraboloid boundary point and the connecting line of the spherical center, which should be the maximum angle θ_{max} . If the angle between the main cable node and the connecting line of the ball center (the direction vector is \vec{n}) is greater than this angle, it falls outside the paraboloid, that is, the node does not participate in the adjustment of the paraboloid; On the contrary, the point whose included angle is less than the critical angle meets the criterion and participates in the adjustment. The criterion is written as:

$$\cos \langle \vec{n}, \vec{n}_0 \rangle \geq \cos \theta_0 \quad (7)$$

From the solution results, it is easy to obtain by using geometric relations $\theta_{max} = 0.5228\text{rad}$. For the main cable nodes that meet the criterion and participate in the adjustment, we continue to solve the expansion distance and adjusted coordinates.

For a main cable node, the parameter equation of the line corresponding to the node and the paraboloid equation are solved simultaneously, and the intersection coordinates $(x_{new}, y_{new}, z_{new})$ are obtained as the new coordinates of the main cable node after adjustment. The corresponding actuator expansion distance D is equal to the reference spherical radius minus the distance from the new coordinate to the spherical center, which is expressed by the following formula:

$$\cos \langle \vec{n}, \vec{n}_0 \rangle \geq \cos \theta_0 \quad (8)$$

d is a positive value, indicating the elongation of the actuator relative to the ground state, and a negative value indicates the shortening of the actuator.

3.3. New coordinates of main cable node and calculation of expansion length

It is found that 692 main and branch nodes participate in this regulation, accounting for 31.1% of the total nodes, of which 515 points are elongated and 177 points are shortened.

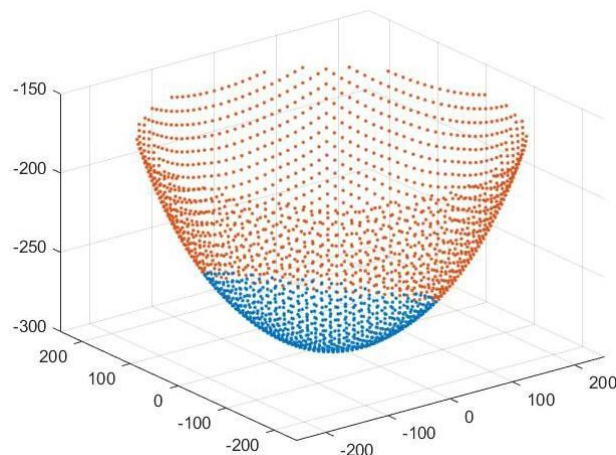


Figure 2: Paraboloid position diagram.

The parabolic equation obtained is:

$$92.041x + 68.835y + 546.659z - (0.784x + 0.586y - 0.205z)^2 - (0.599x - 0.801y)^2 + 168575.222 = 0 \quad (9)$$

References

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