

Research on Design Model Based on Maximum Output Power of Wave Energy

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Abstract: The purpose of this study is to explore the maximum output power of the oscillating buoy type wave energy generation device, and provide theoretical support for the optimal design. Through force analysis and Newton's second law, the differential equation of the heave motion of the float and the vibrator is established. Considering the constant damping coefficient, the Laplace transform is used to solve the differential equation, and the motion of the float and vibrator of the power generation device under the heave excitation force is obtained. With the help of the electromechanical analogy model, the problem is transformed into a circuit problem, and a circuit model of the wave energy generation system is established. Through the vector method, the maximum output power of the device is determined to be $P_{max}=229.334W$, and the damping force coefficient of the damper is equivalent to $R_2=37193.81\Omega$. The research results show that the maximum output power is closely related to the damping coefficient. Within a certain range of the damping coefficient, the energy conversion efficiency can be improved by setting the damper reasonably, and the large-scale utilization of wave energy can be promoted.

Keywords: Wave Energy, Laplace Transformation, Electromechanical Analogy

1. Introduction

Wave energy is a high-grade offshore renewable energy with large reserves, green, clean, and sustainable use [1]. The world's wave energy resource reserves are 29,500 TWh/a [2], which can meet the current global power demand. With the development of economy and society, human beings are faced with the dual challenges of energy demand and environmental pollution, and the development of renewable energy industry has become the consensus of all countries in the world. As an important marine renewable energy, wave energy is widely distributed and abundant, and has considerable application prospects. The energy conversion efficiency of wave energy devices [2-3] is one of the key issues in the large-scale utilization of wave energy.

2. Questions and data

The oscillating buoy type wave energy generating device [4] consists of four parts: a float, a vibrator, a central axis, and a power output system (PTO). The float consists of a cylindrical shell and a conical shell with uniform mass distribution. The interlayer between the two housings is used as a support surface for the installation of the central axis. The PTO system [5] connects the center axis base and the vibrator. Its working principle is that waves apply a simple harmonic excitation force to the oscillating buoy type wave energy generating device, so that the buoy drives the vibrator to shake through the PTO system, and the relative motion between the buoy and the vibrator drives the damper to do work and output energy. The parameters in the question are shown in Table 1.

Table 1: Problem required parameter

Incident wave frequency (s^{-1})	Heave additional mass (kg)	Damping coefficient of heave wave ($N\cdot s/m$)	Amplitude of heave exciting force (N)
1.4005	1335.535	656.3616	6250

Problem: Consider only the case where the float only moves in heaving motion, and the central axis is fixed. The vibrator is connected to the base by the PTO and moves only along the central axis. The magnitude of the damping force it receives is positively related to the relative speed of the float. The device is balanced in still water in the initial state, and then begins to move under the influence of the

heave excitation force. A mathematical model is established to calculate the heave displacement and velocity of the float and vibrator within 40 wave cycles after the device is subjected to the heave excitation force when the damping coefficient is fixed at 10000N·s/m, and the maximum output power is calculated. The power generation device is shown in Figure 1 below.

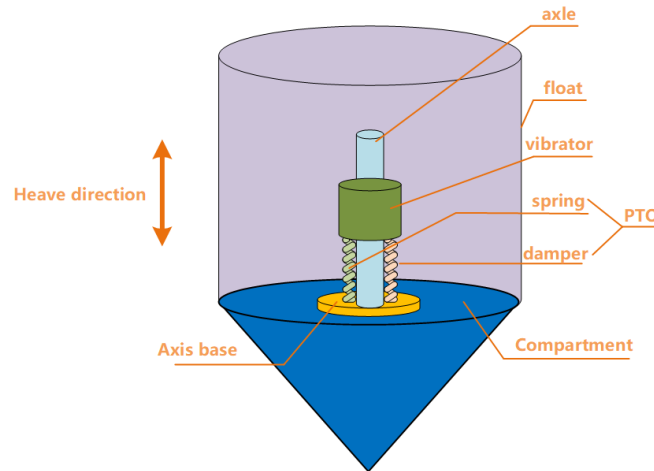


Figure 1: Diagram of a wave energy device

3. Experiments and Results

3.1 Analysis of the problem

Firstly, this essay can determine the force on the float and vibrator in the equilibrium state through force analysis. The float is subject to buoyancy and gravity, and the vibrator is subject to gravity and tension. Based on the balance condition, the balance equation of the force of the float and the vibrator is obtained; Secondly, for the heave state, the force analysis is performed again. In addition to buoyancy and gravity, the heave of the float also has resistance. When the vibrator is heaving, in addition to gravity and tension, there is also resistance. Damper power can be expressed as the product of the damping coefficient and the square of the velocity. Using the electromechanical analogy [10], the maximum power is obtained by solving the resistance value by impedance matching[6-9].

3.2 Establishment of motion model of float and vibrator

This essay set up the float coordinate system as shown in Figure 2.

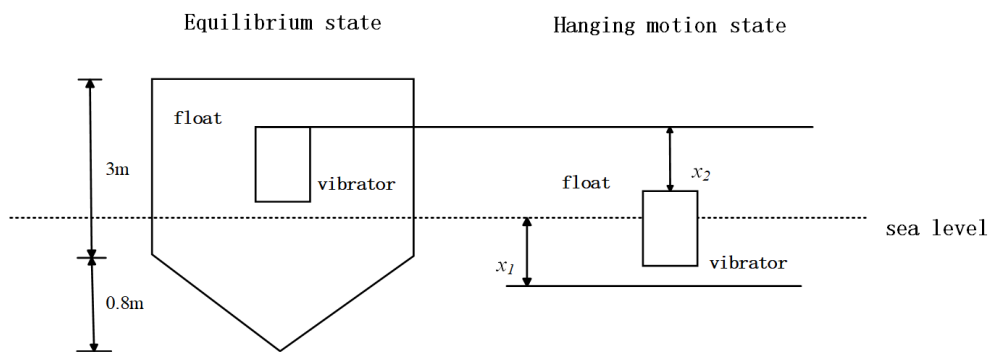


Figure 2: Float coordinate diagram

Step 1: Analyzing the initial state at equilibrium, the following expression can be obtained:

$$F_{buoyancy} = G_{total} \quad (1)$$

buoyancy equals gravity at this time.

$$\rho g V_{row} = (m_1 + m_2)g \quad (2)$$

Among them, the mass of the float is m_1 , the vibrator mass is m_2 , and V_{row} is the volume of drainage, g is the acceleration due to gravity.

Step 2: Analyze the influencing factors of various forces:

- Hydrostatic resilience F_1

The static water restoring force is caused by the buoyancy change of the floating body during the heaving motion. Assuming that the distance of the float from the equilibrium position is x_1 , F_1 can be calculated as follows:

$$F_1 = \begin{cases} -\rho g \pi R^2, & x_1 \leq -1 \\ -\rho g \pi R^2 x_1, & -1 \leq x_1 < 2 \\ \text{None}, & 2 \leq x_1 \end{cases} \quad (3)$$

- Spring force F_2 : Proportional to motion displacement.

When the floating body is in a vertical motion, set the distance of the oscillator from the equilibrium position to x_2 , and set the vertical up to be positive.

$$F_2 = -k_2(x_1 - x_2) \quad (4)$$

Among them, k_2 is a constant.

- Damping force F_3 , F_4 Damper damping force

Damper damping force F_3 : Proportional to its own speed. Wave damping force F_4 : proportional to the relative velocity, so:

$$F_3 = -k_3(x_1' - x_2') \quad (5)$$

k_3 is the damping coefficient of the damper.

The same can be obtained:

$$F_4 = -k_4 x_1' \quad (6)$$

F_4 is proportional to the speed and in the opposite direction.

- Wave motivation F_5

Force analysis is performed on floats in the vertical motion state. Let the initial direction of action of the F_5 on the float be vertical upwards, and obtain:

$$F_5 = f \cos(\omega t) \quad (7)$$

Where f is the amplitude of the wave excitation force and ω is the wave frequency.

Step3: Draw a force analysis diagram and list the equations for the float and oscillator. Figure 3 shows the force analysis of the float and Figure 4 shows the force analysis of the oscillator.

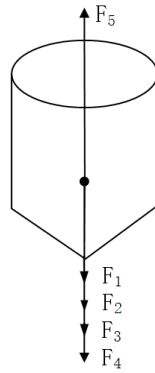


Figure 3: Float force analysis.

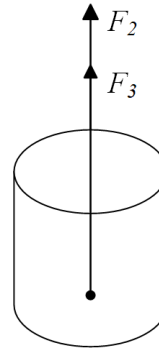


Figure 4: Oscillator force analysis

From Newton's second law and Figure 3, this essay get:

$$F_1 + F_2 + F_3 + F_4 + F_5 = (m_1 + m_2)x_1'' \quad (8)$$

Similarly, from Newton's second law and Figure 4, this essay obtain:

$$-(F_2 + F_3) = m_2x_2'' \quad (9)$$

In summary, the system of equations for float and oscillator is obtained:

$$\left\{ \begin{array}{l} (m_1 + m_c)x_1'' + (k_3 + k_4)x_1' - k_3x_2' + (k_1 + k_2)x_1 - k_2x_2 - f \cos(\omega t) = 0, \\ m_2x_2'' - k_3x_1' + k_3x_2' - k_2x_1 + k_2x_2 = 0, \\ x_1(0) = x_2(0) = 0, \\ x_1'(0) = x_2'(0) = 0. \end{array} \right. \quad (10)$$

3.3 Experiment and analysis of motion models of floats and oscillators.

When the damping coefficient is constant 10000 N·s/m, it is a linear differential equation.

Here this essay mainly use the Lass transform method: the main idea is to convert the time domain calculation to the frequency domain calculation, and then convert the result of the frequency domain calculation back to the time domain.

Step1: Do the Laplace transform on both sides of the differential equation at the same time.

According to the time-domain differential properties of the Laplace transform:

$$L\left\{\frac{df(t)}{dt}\right\} = sF(s), L\left\{\frac{d^2f(t)}{dt^2}\right\} = s^2F(s). \quad (11)$$

And the pull transformation of $\cos(\omega t)\varepsilon(t)$ is:

$$L\{\cos(\omega t)\varepsilon(t)\} = \frac{s}{s^2 + \omega^2}. \quad (12)$$

Step2: Evaluate the image function according to the algebraic method.

Step3: The Laplace inverse transformation is done on the solved image function, the time domain solution is obtained and the data is substituted to obtain the following calculus equation system model:

$$\begin{cases} X_1(t) = -7.578 \times 10^{-3} \sin(-6.247t - 1.008)e^{-2.871t} - 0.429 \sin(-1.881t - 1.505)e^{-0.043t} \\ \quad + 0.435 \sin(-1.4005t - 1.503) \\ X_2(t) = 1.789 \times 10^{-2} \sin(-6.247t - 1.081)e^{-2.871t} - 0.478 \sin(-1.881t - 1.483)e^{-0.043t} \\ \quad + 0.462 \sin(-1.4005t - 1.492) \\ X_1'(t) = 0.123 \sin(6.247t - 5.858 \times 10^{-2})e^{-2.871t} - 0.899 \sin(1.881t - 6.452 \times 10^{-2})e^{-0.043t} \\ \quad + 0.647 \sin(1.4005t - 7.860 \times 10^{-2}) \\ X_2'(t) = -5.21 \times 10^{-2} \sin(6.247t - 0.132)e^{-2.871t} - 0.807 \sin(1.881t - 4.276 \times 10^{-2})e^{-0.043t} \\ \quad + 0.610 \sin(1.4005t - 6.785 \times 10^{-2}) \end{cases} \quad (13)$$

$X_1(s), X_2(s), sX_1(s), sX_2(s)$ are image functions of Float displacement, Vibrator displacement Float velocity, and Vibrator velocity, respectively.

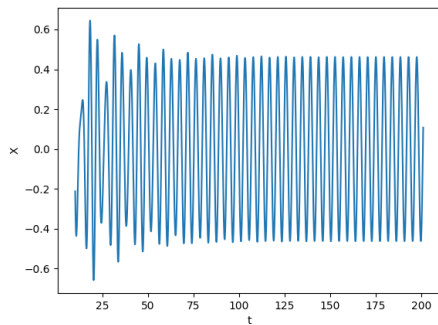


Figure 5: Float displacement

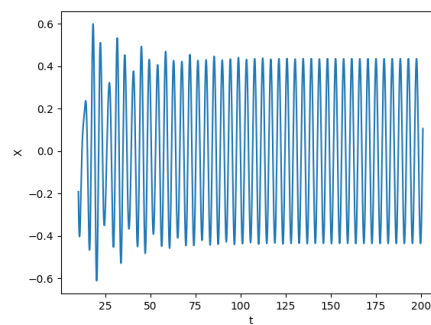


Figure 6: Vibrator displacement

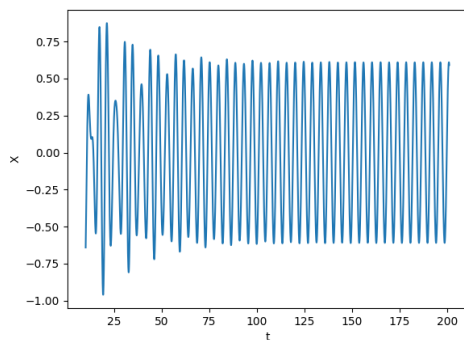


Figure 7: Float speed

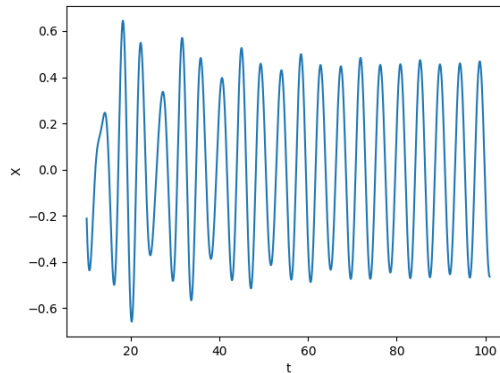


Figure 8: Vibrator speed

Figure 5, Figure 6, Figure 7 and Figure 8 is the change of speed and displacement when the damping coefficient is fixed in the first 40 cycles of the float and oscillator, it can be seen from the above figure that the absolute speed and displacement of the float and the oscillator are very close, and the motion of the float and the oscillator reaches stability after a certain period of time.

Table 2: Partial results are indicated

Time (s)	Float displacement (m)	Float speed (m/s)	Vibrator displacement (m)	Vibrator speed (m/s)
10	-0.191116	-0.693437	-0.211682	-0.641002
20	-0.590861	-0.272008	-0.634529	-0.240441
40	0.284987	0.333633	0.296331	0.313828
60	-0.314572	-0.51544	-0.331597	-0.479527
100	-0.08366	-0.643032	-0.084056	-0.604652

In order to more clearly observe the state of the two vertical motions, Table 2 reflects the comparison of the velocity displacement parameters of the float and the oscillator at a specific moment.

3.4 Establishment of electromechanical analogy model.

Step1: Calculate the average power.

The power of the translational motion of an object can be expressed as:

$$P_h = FV \tag{14}$$

From equation (5), the simultaneous integration of $(x_1 - x_2)$ on both sides of the equation yields:

$$P = \int k_3(x'_1 - x'_2)d(x_1 - x_2) \tag{15}$$

From $dx = vdt$, this essay get:

$$P = \int k_3(x'_1 - x'_2)^2 dt \tag{16}$$

From this, get:

$$\bar{P} = \frac{\int_{t_0}^{t_0+T} k_3(x'_1 - x'_2)^2 dt}{T} \tag{17}$$

Step2: Equation conversion

The problem requires the solution of the maximum power, and the problem of maximum power is obviously closely related to the knowledge of circuit theory, so this essay can transform the mechanical problem into an electrical problem. If two systems satisfy the constraints of the second-order homogeneous linear constant coefficient differential equation, i.e. the precondition of this model is that the damping coefficient k_3 is constant, the problem obviously meets this condition and its range is $[0,100000]$, so the two systems have a dual relationship.

$$mx'' + \rho x' + kx = F \Leftrightarrow Lq'' + Rq' + C^{-1}q = U \tag{18}$$

Converting the system of original mechanics equations into a system of electrical equations yields:

$$k_3 = \alpha: \begin{cases} L_1 \frac{dI_1}{dt} + \frac{1}{C_1} \int (I_1 - I_2)dt + R_2(I_1 - I_2) + R_1 I_1 = A \cos(\omega t) \\ I_2 \frac{dI_2}{dt} - \frac{1}{C_2} \int (I_1 - I_2)dt - R_2(I_1 - I_2) = 0 \\ q_1(0) = q_2(0) = 0, I_1(0) = I_2(0) = 0 \end{cases} \tag{19}$$

Step3: Draw the circuit diagram after the comparison

An equation in the system of equations obtained in the previous step corresponds to a loop in the circuit. Therefore, the following figure 9 is obtained.

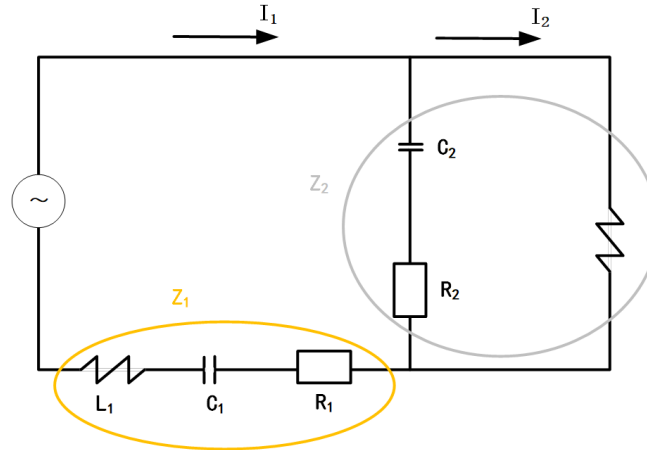


Figure 9: Circuit diagram

This step is crucial, and complex problems become simple circuit theory analysis problems. Because the damping force coefficient of the damper is compared to R2, the power of the damper is equal to the power consumed on R2.

Step 4: Vector method solving

The three devices L₁, C₁, and R₁ are regarded as a whole, and their impedance is written as Z₁, according to the series and parallel relationship of the three, this essay obtain:

$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1} + R_1 = Z_{1R} + jZ_{1X}, \quad (20)$$

The three devices L₂, C₂, and R₂ are regarded as another whole, and their impedance is written as Z₂, according to the series and parallel relationship of the three, there are:

$$Z_2 = \frac{j\omega L_2 \left(\frac{1}{j\omega C_2} + R_2 \right)}{j\omega L_2 + \frac{1}{j\omega C_2} + R_2} = Z_{2R} + jZ_{2X}, \quad (21)$$

Among them, Z_{2R} and Z_{2X} represent the real and imaginary parts of Z₂, and the formula (21) is simplified to obtain the following formula:

$$\begin{cases} Z_{2R} = \frac{C_2 L_2 R_2 \omega^2 - C_2 L_2 R_2 \omega^2 (1 - C_2 L_2 \omega^2)}{C_2^2 R_2^2 \omega^2 + (1 - C_2 L_2 \omega^2)^2} \\ Z_{2X} = \frac{C_2^2 L_2 R_2^2 \omega^3 + L_2 \omega (1 - C_2 L_2 \omega^2)}{C_2^2 R_2^2 \omega^2 + (1 - C_2 L_2 \omega^2)^2} \end{cases} \quad (22)$$

In summary, the solution is:

$$P_2 = I^2 R_2 = \frac{A^2}{2} \frac{Z_{2R}}{(Z_{1R} + Z_{2R})^2 + (Z_{1X} + Z_{2X})^2} \quad (23)$$

At this point, the expression relationship of power P₂ resistor R₂ is established.

3.5 Solution of electromechanical analogy model

Substituting the data into Equation (23) simplifies it to:

$$P_2 = \frac{1.717 \times 10^7 \times (9.450 \times 10^8 R_2 + R_2^3)}{1.307 \times 10^{18} + 4.555 \times 10^{11} R_2 + 2.328 \times 10^9 R_2^2 + 482.0 R_2^3 + R_2^4} \quad (24)$$

Derive equation (29) to R2 so that the derivative is equal to zero to get the extreme point.

$$\begin{cases} R_2 = 37193.81\Omega \\ P_{2\max} = 229.334W \end{cases} \quad (25)$$

4. Conclusions

In this paper, based on Newton's second law, in-depth analysis and processing, the heave model of the oscillating buoy type wave energy generation device was successfully established. By discretizing the function, the difficulty of calculation is subtly reduced, which facilitates the solution of the model. The obtained numerical results are more in line with the actual situation, and effectively describe the heave motion of the oscillating buoy type wave energy generation device.

Overall, this paper focuses on the maximum output power and optimal damping coefficient of the oscillatory buoy type wave energy generation system. Within a given damping coefficient range, setting the damper reasonably can optimize the energy conversion efficiency of the system, thereby realizing the effective utilization of wave energy. This study provides important theoretical support and guidance for the optimal design of wave energy devices.

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