ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

A Resource Allocation Model for Deep Uncertainty (RAM-DU) Based on Nonlinear Optimization

Lei Yao[1,*]

[1]*Zeekr Intelligent Technology Holding Limited, Hangzhou City, China*
[*]*Corresponding author*

Abstract: *Deep uncertainty usually refers to problems with epistemic uncertainty in which the analyst or decision maker has very little information about the system, data are severely lacking, and different mathematical models to describe the system may be possible. Since little information is available to forecast the future, selecting probability distributions to represent this uncertainty is very challenging. Traditional methods of decision making with uncertainty may not be appropriate for deep uncertainty problems. This paper introduces a novel approach to allocate resources within complex and very uncertain situations. The resource allocation model, which is built by nonlinear optimization within utility function for deep uncertainty (RAM-DU) incorporates different types of uncertainty (e.g., parameter, structural, model uncertainty) and can consider every possible model, different probability distributions, and possible futures. Instead of identifying a single optimal alternative as in most resource allocation models, RAM-DU recommends an interval of allocation amounts. The RAM-DU solution generates an interval for one or multiple decision variables so that the decision maker can allocate any amount within that interval and still ensure that the objective function is within a predefined level of optimality for all the different parameters, models, and futures under consideration. RAM-DU is applied to allocating resources to prepare for and respond to a Deepwater Horizon-type oil spill. The application identifies allocation intervals for how much should be spent to prepare for this type of oil spill and how much should be spent to help industries recover from the spill.*

Keywords: *deep uncertainty, resource allocation model, nonlinear optimization, probability distribution*

1. Introduction

Uncertainty is perhaps the biggest factor that makes decision making challenging for many people, especially for complex systems. Examples of complex decisions with significant uncertainty exist in new product development [19], investments [18], climate change [20], and national security. Uncertainty can be classified into aleatory or epistemic uncertainty. Aleatory uncertainty results from natural or stochastic variation within a physical system or environment. Epistemic uncertainty results from a lack of knowledge or information about a system. Uncertainty can also be classified based on its severity [8] or the source of uncertainty [15]. In scientific research and modeling, uncertainties can include parametric uncertainty, structural or model uncertainty, algorithm uncertainty, experimental uncertainty, and interpolation uncertainty.

Probability is the most popular way to measure and model uncertainty. Paté-Cornell defines six levels of treatment for uncertainty in risk analysis where the lowest levels focus on identifying hazards and the worst cases for those hazards and the highest level models uncertainty over probability distributions. In problems with a lot of epistemic uncertainty where the analyst or decision maker has very little information about the system, the use of probabilities to measure that uncertainty can pose challenges. This type of uncertainty is known as deep uncertainty. According to Walker et al., uncertainty about the future can be divided into five categories. Increasing levels signifies increasing uncertainty, from a fairly certain future in level 1 to a completely unknown future in level 5. Deep uncertainty refers to uncertainty at level 4 (multiple possible futures with several system models) and level 5 (unknown or unidentified futures, unknown unknowns). This article will treat deep uncertainty at level 4, in which multiple plausible futures exist and multiple system models can represent those future scenarios, and an analyst or decision maker does not know which model is most appropriate or which futures are more likely. In this context, choosing a probability distribution to represent the uncertainty in model parameters or the different futures and selecting value functions to represent the desirability of different outcomes can be very challenging.

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

Several decision-making methods have been proposed to handle deep uncertainty. Robust decision framework was first proposed by Rosenhead. Robust decision making (RDM) is perhaps one of the most widely used. A robust strategy is an alternative that performs well under many or even all possible futures, and RDM helps a decision maker identify robust strategies, characterize the vulnerabilities of such strategies, and evaluate the trade-offs among strategies [9]. RDM allows for the possibility that stakeholders in a problem do not know or cannot agree on the systems model or the probabilities that should be used in the models (i.e., deep uncertainty). Information-gap theory [3][4] seeks to identify the optimal alternative that performs well as the uncertainty around a parameter grows. Probability boxes, or p-boxes, calculate every possible probability distribution that could fit within a predefined bound around that uncertainty [14]. Exploratory modeling and analysis [2] copes with deep uncertainty by calculating model outcomes across a large group of plausible representations of the future. The uncertainty can exist due to unknown external scenarios, model parameters, and problem structure [1]. Model uncertainty can also result from large amounts data because it is not clear which model is suitable for such a large data set. Adaptive boosting addresses that model uncertainty by weighting different models based on a training set [7]. Other methods for deep uncertainty include the adaptive decision-making framework [19] and using real options to hedge against uncertain futures [17-18].

A Bayesian approach to uncertainty and decision making can also address deep uncertainty through the use of probabilities. According to Bayesians, probability represents an individual's subjective degree of belief about the future, and an individual can always assign a probability for an uncertainty [10-12]. If the individual has very little information about the uncertainty, he or she should select a very diffuse or uninformative probability distribution. If several probability distributions are possible to describe a single uncertainty, Bayesians can also assign probabilities for each of these distributions, which is similar to the level 6 treatment of uncertainty in Paté-Cornell.

Despite this wealth of proposed solutions, making decisions with deep uncertainty still represents challenges. Selecting robust strategies tends to emphasize worst-case scenarios, which may be very unlikely. Using subjective probabilities for deep uncertainty can be subject to individual biases, especially the tendency to be overconfident about the future. This can result in disastrous outcomes. For complex problems with parameter uncertainty, structural and model uncertainty, and uncertainty over possible futures, it is not clear if different methods should address each type of uncertainty.

This paper offers a novel approach to decision making with deep uncertainty, specifically for a problem in which a decision maker is allocating resources in a complex, uncertain situation. The resource allocation model is constructed by nonlinear optimization model within utility function, which is used to describe the effects of resource allocation from human-being's perspective. The resource allocation model for deep uncertainty (RAM-DU) incorporates all of the different types of uncertainty and can consider every possible model, different probability distributions, and possible futures. Similar to RDM, RAM-DU identifies allocation strategies that perform well across the possible parameters, models, and future outcomes. However, RAM-DU is unique because it recommends an interval of possible allocation strategies rather than a single optimal alternative. This paper focuses on allocating infinitely divisible resources, such as money, as opposed to discrete resources, such as the number of people or trucks. The RAM-DU solution generates an interval for the decision variable. Allocating any amount within that interval ensures that the objective function is within a predefined optimality gap for all the different parameters, models, and futures under consideration.

An interval solution is also beneficial because mathematical models are abstractions of the real world and cannot capture every possible factor. A decision maker may have other considerations that are not captured in the models but that should also influence his or her decision. By providing an interval rather than a point solution, RAM-DU gives the decision maker flexibility to select a resource allocation strategy within that interval. The decision maker can more easily incorporate other considerations not captured by the model but still follow recommendations of the model. For example, Floricel and Miller [13] argue that strategies for large-scale engineering projects turbulent environments should include flexibility. RAM-DU can also consider multiple stakeholders with different assumptions or opinions about the model by incorporating those different factors within the interval solution.

This paper introduces the methodology of RAM-DU and applies the method to a real-world application of an oil spill. Section 2 introduces the general model structure for RAM-DU and examines interval solutions for a single decision variable and for multiple decision variables. Section 3 applies RAM-DU to the problem of allocating resources to prevent and respond to a Deepwater Horizon-type oil spill. Concluding remarks and possible future extensions of this methodology appear in Section 4.

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

2. Methodology And Modeling

A resource allocation model seeks to optimally distribute resources in order to minimize or maximize an objective. Resources can be discrete or continuous. Discrete resources are represented by integers such as the number of people, trucks, or equipment. Continuous resources are infinitely divisible, such as money or time, and are represented by positive real numbers. RAM-DU assumes continuous resources, and the decision variables $z_1, z_2, \ldots, z_n$ are non-negative real numbers and $\mathbf{z} = (z_1, z_2, \ldots, z_n)^T$ is a vector of length n. A decision maker seeks to allocate each z_i in order to minimize a real-valued function $f(\mathbf{z}, \boldsymbol{\theta})$ where $\boldsymbol{\theta}$ denotes a vector of exogenous parameters. Constraints are represented by $\mathbf{h}(\mathbf{z}, \boldsymbol{\theta})$, a vector-valued function of the decision variables z and exogenous parameters $\boldsymbol{\theta}$. The resource allocation model can be expressed as an optimization problem in (1):

$$\begin{aligned} \min \quad & f(\mathbf{z}, \boldsymbol{\theta}) \\ \text{s.t.} \quad & \mathbf{h}(\mathbf{z}, \boldsymbol{\theta}) \leq \mathbf{0} \end{aligned} \tag{1}$$

As discussed in the Introduction, a decision maker may be uncertain about many aspects of this resource allocation model. Parameter uncertainty may exist in some, or even all, of the parameters represented by $\boldsymbol{\theta}$, and a decision maker may not be comfortable assigning probabilities to represent the uncertainty in $\boldsymbol{\theta}$. Model uncertainty may exist with the constraints $\mathbf{h}(\mathbf{z}, \boldsymbol{\theta})$ and the objective function $f(\mathbf{z}, \boldsymbol{\theta})$. Different functions could represent the decision maker's objective because the objective is difficult to model or the decision maker is unsure of his or her true objective. Given this parameter and model uncertainty and the difficulty of assigning probabilities to represent the uncertainty, the resource allocation model becomes a problem with deep uncertainty.

We assume that every possible value for the parameter $\boldsymbol{\theta}$ and every possible function for $f(\mathbf{z}, \boldsymbol{\theta})$ and $\mathbf{h}(\mathbf{z}, \boldsymbol{\theta})$ can be identified. We assume m unique versions of the optimization problem in (1) exist where the jth version of the problem is denoted by $\boldsymbol{\theta}_j$, $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$, and $\mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j)$ for $j = 1, \ldots, m$. For example, $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ could differ from $f_{j'}(\mathbf{z}, \boldsymbol{\theta}_{j'})$ because the objective function in j is exponential and the objective function in j' is logarithmic. Or, $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ could differ from $f_{j'}(\mathbf{z}, \boldsymbol{\theta}_{j'})$ because $\boldsymbol{\theta}_j = [1, 2]$ and $\boldsymbol{\theta}_{j'} = [1, 3]$. It could also be true that $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ and $f_{j'}(\mathbf{z}, \boldsymbol{\theta}_{j'})$ are identical but that $\mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j)$ differs from $\mathbf{h}_{j'}(\mathbf{z}, \boldsymbol{\theta}_{j'})$ for at least one of the constraints.

The goal of RAM-DU is to find an interval for a single decision variable or multiple intervals for multiple decision variables that account for the m unique optimization problems so that every solution within the interval guarantees that every objective function is within a predefined optimality gap of its true minimum. We initially address a situation for a single interval and let z_i be the decision variable around which we want an interval. Identifying an interval means that the objective function $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ is close to its minimum value for all z_i between a_i and b_i where $b_i > a_i \geq 0$. Let f_j* be the minimum value for the jth optimization problem. The function $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ can be considered close to its minimum f_j*, if $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ is within δ_j of f_j* where $\delta_j \geq 0$. We define $\alpha_j \equiv f_j^* + \delta_j$, where α_j denotes the maximum acceptable threshold and δ_j denotes the difference between the optimal value and the maximum acceptable threshold for the jth optimization problem. We seek to maximize the interval width $b_i - a_i$ such that $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j$ for all z_i within the interval $[a_i, b_i]$:

$$\begin{aligned} \max \quad & b_i - a_i \\ \text{s.t.} \quad & f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j; \quad \forall z_i \in [a_i, b_i], \quad j = 1, \ldots, m \\ & \mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \mathbf{0}; \quad \forall z_i \in [a_i, b_i], \quad j = 1, \ldots, m \end{aligned} \tag{2}$$

The other decision variables $z_{i'}$ $i' \neq i$ are chosen in order that the constraints in (2) are satisfied.

Algorithm 1 illustrates a method to solve the optimization problem in (2) if $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ is squasiconvex and $\mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j)$ is convex in z_i for all $j = 1, \ldots, m$. A function $f(y)$ is quasiconvex if and only if $f(\lambda y_1+(1-\lambda)y_2) \leq \max\{f(y_1), f(y_2)\}$ for all y_1 and y_2 in the domain of f and $\lambda \in [0, 1]$ [6]. Quasiconvexity ensures that f decreases and then increases. According to Algorithm 1, minimizing z_i subject to $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j$ returns a_{ij} , which is a candidate for the lower bound of the interval. Similarly, maximizing z_i subject to $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j$ returns b_{ij}, which is a candidate for the upper bound of the interval. After minimizing and maximizing z_i for each of the m possible optimization problems, a_i equals the largest a_{ij} and b_i equals the smallest b_{ij} for all $j = 1, \ldots, m$.

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

Algorithm 1 Maximize interval width for one decision variable z_i

Inputs: $f_j(\mathbf{z}, \boldsymbol{\theta}_j), \mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j), \boldsymbol{\theta}_j, \delta_j, j = 1, ..., m$
Results: a_i, b_i

for $j \leftarrow 1$ to m **do**
  Solve min $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$, subject to $\mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \mathbf{0} \longmapsto f_j^*$
  $\alpha_j = f_j^* + \delta_j$
  Solve min z_i, subject to $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j$, $\mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \mathbf{0} \longmapsto a_{ij}$
  Solve max z_i, subject to $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j$, $\mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \mathbf{0} \longmapsto b_{ij}$
end for
$a_i = \max\{a_{ij}\}$
$b_i = \min\{b_{ij}\}$

A pictorial representation of the interval endpoints a_i and b_i appears in Fig. 1 for three possible optimization models, i.e. $m = 3$. Three objective functions $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$ are drawn in dotted, solid, and dashed lines as a function of z_i, and each corresponding maximum acceptable threshold α_j is depicted as a horizontal line. Objective functions $f_1(\mathbf{z}, \boldsymbol{\theta}_1) < \alpha_1$ and $f_2(\mathbf{z}, \boldsymbol{\theta}_2) < \alpha_2$ when $z_i = 0$; however, $f_3(\mathbf{z}, \boldsymbol{\theta}_3) > \alpha_3$ when $z_i = 0$. The value of a_i corresponds to the smallest value of z_i when $f_3(\mathbf{z}, \boldsymbol{\theta}_3) = \alpha_3$ (the dashed curve and line). For b_i, the largest value of z_i at which $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j$ corresponds to the largest value of z_i at which $f_1(\mathbf{z}, \boldsymbol{\theta}_1) \leq \alpha_1$ (the dotted curve and line). For all values of z_i such that $a_i \leq z_i \leq b_i$, the objective function for each of the three possible models is less than or equal to the maximum acceptable threshold a_j. The interval $[a_i, b_i]$ also represents the largest interval because at least one of the objective functions is greater than the maximum acceptable threshold for $z_i < ai$ and $z_i > b_i$.

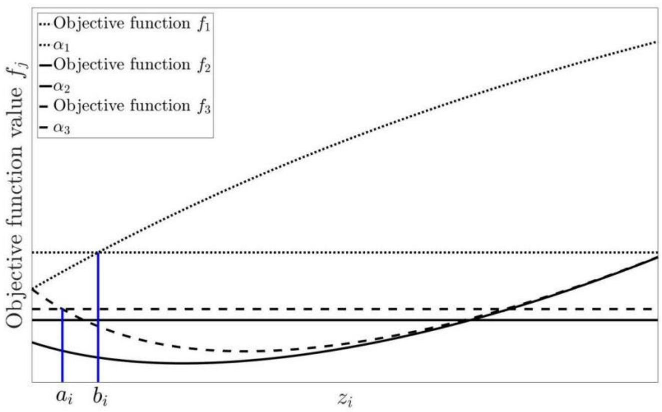

Figure 1: Interval with 3 possible optimization problems

The prior discussion focused on finding an interval for a single decision variable zi even if several decision variables exist in the resource allocation model. It might be desirable to have intervals around multiple decision variables to provide the decision maker with greater flexibility than a single interval. Instead of maximizing the width of a single interval $b_i - a_i$, the objective function in (2) becomes a multi-objective optimization problem. In this case, RAM-DU finds intervals for $\tilde{n}$ decision variables where $\tilde{n} \leq n$. The optimization problem in (3) seeks to maximize the interval widths for decision variables $z_1, z_2, \ldots, z_{\tilde{n}}$:

$$\begin{aligned} \max \quad & (b_1 - a_1, b_2 - a_2, \ldots, b_{\tilde{n}} - a_{\tilde{n}}) \\ \text{s.t.} \quad & f_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \alpha_j \\ & \mathbf{h}_j(\mathbf{z}, \boldsymbol{\theta}_j) \leq \mathbf{0} \\ & \forall z_i \in [a_i, b_i] \\ & i = 1, \ldots, \tilde{n} \\ & j = 1, \ldots, m \end{aligned} \tag{3}$$

Since (3) is a multi-objective optimization problem, it is necessary to identify a set of Pareto optimal solutions in order to create a hypervolume in $\tilde{n}$ dimensions such that any set of solutions ($z_1, z_2, \ldots,$

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

$\tilde{z}_n$) contained within that hypervolume will ensure that $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \le \alpha_j$ for all $j = 1, \ldots, m$. The application in Section 3 provides an example in two dimensions, i.e. $\tilde{n} = 2$.

The optimization problems in (2) and (3) may not have feasible solutions. If δ_j is very small for several $f_j(\mathbf{z}, \boldsymbol{\theta}_j)$, there might not be any z_i that can satisfy $f_j(\mathbf{z}, \boldsymbol{\theta}_j) \le \alpha_j$. If this occurs, it is necessary to increase δ_j.

The use of intervals in RAM-DU enables resources to be allocated to account for situations where deep uncertainty exists. Any allocation within the interval or hypervolume guarantees that each objective function is less than or equal to an acceptable threshold. The interval also provides flexibility for the decision maker because the decision maker can choose to allocate any z_i within $[a_i, b_i]$. If the decision maker prefers to allocate less—perhaps because there are other demands for resources that have not been modeled—he or she should select an allocation closer to a_i. If the decision maker prefers to allocate more—perhaps because these resources will have additional benefits that are not modeled—he or she should select an allocation closer to b_i.

3. Illustrative Example of The Deepwater Horizon Oil Spill

On April 20, 2010, an explosion occurred on the *Deepwater Horizon* semi-submersible mobile offshore drilling unit, which led to 11 dead workers, 17 injured workers, the loss of almost 5 million barrels of oil, and enormous environmental damage. This section applies RAM-DU to a large oil spill in the Gulf of Mexico similar to that of the *Deepwater Horizon* oil spill. The resource allocation model is derived from MacKenzie in which a decision maker allocates resources to prevent and prepare for an oil spill and then allocates resources to help the Gulf region recover if an oil spill occurs. This section first presents the resource allocation model for the oil spill and then demonstrates how RAM-DU can be applied to this situation to help a decision maker determine how much should be spent to prevent and prepare for an oil spill and how much should be spent to help the region recover from a large oil spill.

3.1. Resource Allocation Model for an Oil Spill

Although the *Deepwater Horizon* oil spill resulted in fatalities, injuries, environmental damage, and lost business, the resource allocation model focuses exclusively on economic losses. The economic losses from an oil spill result from less drilling for oil, decreased demand for seafood and real estate, and a drop in tourism. Economic losses for this model are calculated for the five U.S. states touching the Gulf of Mexico: Florida, Alabama, Mississippi, Louisiana, and Texas. A single decision maker in the model controls resources that can help prevent an oil spill and limit the economic losses if a spill occurs. In reality, resources to prepare for and respond to an oil spill are controlled by federal, state, and local governments and the private sector.

The objective function seeks to minimize the expected economic loss from an oil spill. The oil spill directly impacts $\bar{l} = 5$ industries (fishing and forestry, real estate, amusement, accommodations, and oil and gas) out of a total of $l = 63$ industries in the economy. The Inoperability Input-Output Model translates these direct impacts into total production losses in all industries, and the total economic loss equals $\mathbf{x} > \mathbf{D}\mathbf{c}*$ where $\mathbf{x}$ is a vector of length l representing normal production for each industry and $\mathbf{D}$ is an $l \times \bar{l}$ interdependency matrix that translates direct losses to direct and indirect losses. The vector $\mathbf{c}^*$ is of length $\bar{l}$ where $c*i$ measures the direct impacts, in proportional terms, to industry i. Economic data to populate $\mathbf{x}$ and $\mathbf{D}$ are collected by the U.S. Bureau of Economic Analysis.

The decision maker can allocate resources before the oil spill for prevention and preparedness, z_1, to help all industries recover after the oil spill z_2, and help each of the $\bar{l} = 5$ directly impacted industries recover $z_3, \ldots, z_7$. The total number of decision variables is $n = 7$. The parameter $\hat{p}$ is the probability of a spill if no money is spent to prevent a spill. Allocating $z1$ helps prevent an oil spill by reducing the initial probability from $\hat{p}$ to p, where $p \le \hat{p}$ and can also help to reduce the direct impacts. The direct impacts, $c*i$, for industry i are a function of money allocated before an oil spill z_1, money allocated to help all industries simultaneously z_2, and money allocated to industry i z_i, where $i = 3, \ldots, n = 7$. Based on these decisions, the direct impacts from an oil spill are reduced from $\hat{\mathbf{c}}_i^*$ to $\mathbf{c}_i^*$. The total available budget is Z. The minimum expected economic losses can be calculated by solving (4) in which allocating resources reduces the probability and impacts according to an exponential function [5][11][16].

$$\begin{aligned} \min \quad & p\mathbf{x}^\top \mathbf{D}\mathbf{c}^* - (1-p)g(z_1, Z) \\ \text{s.t.} \quad & p = \hat{p}e^{-k_p z_1} \\ & c_i^* = \hat{c}_i^* e^{-(k_q z_1 + k_0 z_2 + k_i z_i)}; \quad i = 3, \ldots, n \\ & \sum_{i=1}^{n} z_i \leq Z \\ & z_i \geq 0, \quad i = 1, \ldots, n \end{aligned} \tag{4}$$

where k_p denotes the effectiveness of allocating z_1 to reduce the probability of an oil spill; k_q describes the effectiveness of allocating z_1 to reduce the impacts; k_0 describes the effectiveness of allocating z_2 to help all industries recover; and k_i denotes the effectiveness of allocating z_i to help industry i recover, where $i = 3, \ldots, n$.

The right-hand part of the objective function $g(z_1, Z)$ in Eqs. (4) is the opportunity cost and represents what could be done with the resources to increase regional production if no oil spill occurs. If there is no oil spill, the amount $Z - z_1$ could be spent on other projects or returned to taxpayers which should increase production in the region. The function $g(z_1, Z)$ is strictly decreasing in z_1, increasing in Z, and non-negative for $z_1 \leq Z$. Since a decision maker desires to minimize the expected economic losses if an oil spill occurs and maximize the expected production gain if the oil spill does not occur, minimizing the objective function requires inserting a negative sign before the expected gain in production $(1 - p)g(z_1, Z)$ where $1 - p$ denotes the probability of no spill. When the objective function in (4) is negative, it means the region experiences expected production gains because the expected production gains from $(1 - p)g(z_1, Z)$ are larger than the expected production losses from the spill, $p\mathbf{x}^\mathrm{T} > \mathbf{D}\mathbf{c}^*$. If the objective function is positive, the region has expected production losses.

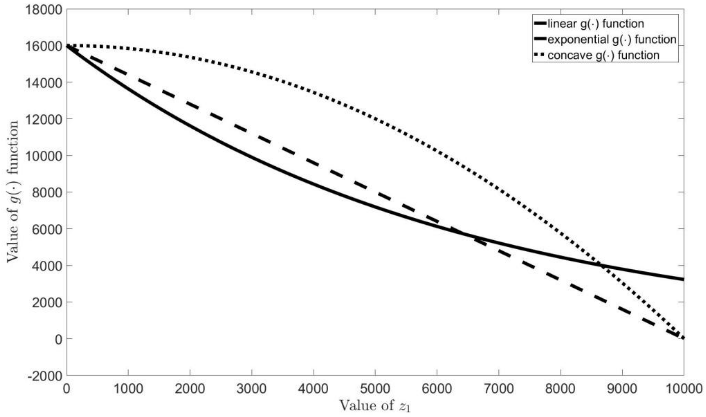

Figure 2: Types of g(·) function

The function to describe production gains $g(z_1, Z)$ can take on different forms to describe how resources impact regional production. Three possible functions are proposed in this article: linear, exponential, and concave as described in Eqs. (5)-(7). Fig. 2 depicts the linear, exponential, and concave $g(\cdot)$ functions. A linear $g(\cdot)$ function indicates that every dollar not spent on the oil spill increases regional production by a constant amount. An exponential $g(\cdot)$ indicates that smaller values of z_1 have a much bigger impact on production than larger values of z_1. A concave $g(\cdot)$ function is opposite to the exponential function. Larger values of z_1 have a larger impact on production than smaller values of z_1. The concave function indicates that decision maker is using resources more efficiently because he or she is initially removing money from projects that that are less effective in increasing regional production.

$$g_1 = q_1 + \gamma_1(Z - z_1) \tag{5}$$

$$g_2 = \gamma_2 Z e^{-q_2 z_1} \tag{6}$$

$$g_3 = Z\gamma_3 - q_3 z_1^2 \tag{7}$$

where q_i and γ_i , $i = 1, 2, 3$ are parameters of the $g(\cdot)$ function. In this oil spill case study, $\gamma_i = 1.6$ and $q_i = 1.6 \times 10^{-4}$.

Table 1: Initial Parameter Values for Deepwater Horizon Oil Spill.

Probability of oil spill		$\hat{p} = 0.045$	
Prevention effectiveness		$k_p = 2.8 \times 10{-}4$	
Preparedness effectiveness		$k_q = 1.6 \times 10{-}4$	
Recovery for all industries effectiveness		$k_0 = 1.0 \times 10{-}5$	
i	Industry	k_i (per \$1 million)	$\hat{c}_i^*$
1	Fishing and forestry	0.074	0.0084
2	Real estate	0	0.047
3	Amusements	0.0038	0.21
4	Accommodations	0.0027	0.16
5	Oil and gas	0.0057	0.079

Table 2: Optimal Allocation Amounts for Deepwater Horizon Oil Spill for Different Opportunity Cost Functions (Millions of \$)

	Linear g(·) function	Exponential g(·) function	Concave g(·) function
Objective function (f_j^*)	-14,383	-14,383	-14,730
Pre oil spill allocation amount	0	0	1,205
Post oil spill allocation amount	5,982	5,982	4,777
Fishing & Forestry	46	46	46
Real Estate	0	0	0
Amusements	1,209	1,209	1,209
Accommodations	1,752	1,752	1,752
Oil & Gas	1,011	1,011	1,011

3.2. Optimal Allocation Results

As depicted in Table 1, the parameters for the *Deepwater Horizon* oil spill are explained in MacKenzie et al. and MacKenzie and are based on public economic data, government reports, and journal articles. The five directly impacted industries are fishing and forestry (which reflects the lack of seafood), real estate, amusements, accommodations, and oil and gas. We choose a budget of Z = \$10 billion, which is a little less than what BP spent to stop the *Deepwater Horizon* oil spill.

With these parameter values, the optimal allocation from the budget is calculated for each of the three possible $g(\cdot)$ functions as illustrated in Eqs. (5)-(7). All the parameters and functions are considered as known. Table 2 depicts the optimal amount to spend to prevent and prepare for an oil spill, to help all industries recover, and to help each individual industry recovery.

If $g(\cdot)$ is linear or exponential, the decision maker should not allocate any money to prevent or prepare for an oil spill. Since the probability of an oil spill is small, it is better to spend the budget to increase regional production via the opportunity cost function than prevent and prepare for an oil spill. If an oil spill occurs, the decision maker should spend z_2 = \$5.98 billion to help all industries recover and distribute the remainder of budget among four industries (fishing and forestry, amusements, accommodations, and oil and gas). If the opportunity cost function is concave, the decision maker should spend z_1 = \$1.20 billion before a spill. The amounts allocated for the five industries remain the same, but the money to help all industries recover simultaneously is reduced to z_2 = \$4.78 billion. The concave $g(\cdot)$ function should induce a positive z_1 because the concave function indicates that production does not decrease as much as in the linear and exponential functions for small values of z_1. Thus, it is rational to spend that money to prevent and prepare for an oil spill.

Table 3: Input Values for Deepwater Horizon Oil Spill.

Category	Initial value	Range
Preparedness	$k_q = 1.6 \times 10^{-4}$	$k_q \in [1.0 \times 10^{-5}, 0.1]$
Prevention	$k_p = 2.8 \times 10^{-4}$	$k_p \in [1.0 \times 10^{-5}, 0.1]$
	$k_p \in [1.0 \times 10^{-5}, 0.1]$	$k_0 \in [6.67 \times 10^{-7}, 6.67 \times 10^{-3}]$
	$\hat{p} = 0.045$ / year	$\hat{p} \in [0.01, 0.08]$

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

3.3. Interval for allocating resources before an oil spill

The preceding illustration demonstrates that a different function for $g(\cdot)$ can result in a different allocation amount to spend before an oil spill. This is an example of model uncertainty. Many of the parameters used in the model also have considerable uncertainty in part because the rarity of really big oil spills limits the availability of historical data. In particular, the probability of an oil spill $\hat{p}$ and the effectiveness parameters k_p, k_q, and k_0 are very difficult to estimate.

Given the uncertainty in the function $g(\cdot)$ and the parameters $\hat{p}$, k_p, k_q, and k_0, RAM-DU is applied to the *Deepwater Horizon* oil spill and we want to find an interval around the amount of money that should be spent before an oil spill $z1$. The resource allocation model in (4) is extended to RAM-DU to find the maximum interval $[a_1, b_1]$ for z_1 where the superscript $j = 1, \ldots, m$ refers to the different optimization problems based on uncertainty in the parameters and the $g(\cdot)$ function:

$$\begin{aligned}
\max \quad & b_1 - a_1 \\
\text{s.t.} \quad & p\mathbf{x}^\top \boldsymbol{D}\boldsymbol{c}^* - (1-p)g^{(j)}(z_1, Z) \le \alpha_j; \;\; j = 1, \ldots, m \\
& p = \hat{p}^{(j)} e^{-k_p^{(j)} z_1}; \;\; j = 1, \ldots, m \\
& c_i^* = \hat{c}_i^* e^{-(k_q^{(j)} z_1 + k_0^{(j)} z_2 + k_i z_i)}; \;\; \begin{array}{l} i = 3, \ldots, n \\ j = 1, \ldots, m \end{array} \\
& \sum_{i=1}^{n} z_i \le Z \\
& z_i \ge 0; \;\; i = 1, \ldots, n
\end{aligned} \tag{8}$$

Table 3 depicts the ranges for $\hat{p}$, k_p, k_q, and k_0. MacKenzie argues that $k_0 = k_q/15$, and this relationship is preserved in this application. These ranges of parameters are combined with the three different $g(\cdot)$ functions in order to establish hundreds of possible optimization problems. The value of α_j for each of these optimization problems is selected as a percentage of the minimum value of each objective function. Algorithm 1 is used to maximize the interval width for z_1.

Figs. 3 and 4 show the objective function value as a function of z_1 for several of the possible optimization problems when α_j corresponds to 91% and 93%, respectively, of the minimum objective function value. Although hundreds of optimization problems are considered, the figures only display six of these objective functions in order to depict these results on a graph. Each figure shows two objective functions with a linear $g(\cdot)$ function, two objective functions with an exponential $g(\cdot)$ function, and two objective functions with a concave $g(\cdot)$ function. The horizontal lines represent the maximum acceptable threshold α_j for each of the six optimization problems. In Fig. 3, when α_j is 91% of the minimum objective function, a_1 = \$244 million is determined by the intersection of α_6 and objective function f_6, which corresponds to a concave $g(\cdot)$ function. The upper bound of the interval b_1 = \$529 million is determined by the intersection of α_4 and objective function f_4, which corresponds to an exponential $g(\cdot)$ function. When α_j is 93% of the minimum objective function, a_1 = \$387 million and b_1 = \$408 million are again determined by objective functions f_6 and f_4, respectively. But the interval is much narrower because the maximum acceptable threshold has been tightened.

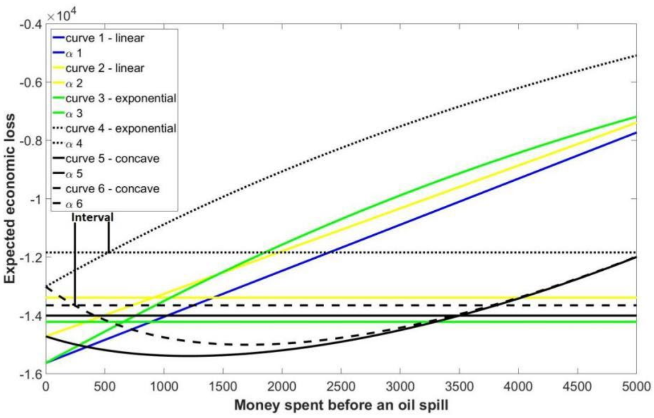

Figure 3: Pre oil spill allocation interval with acceptable threshold at 91%

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

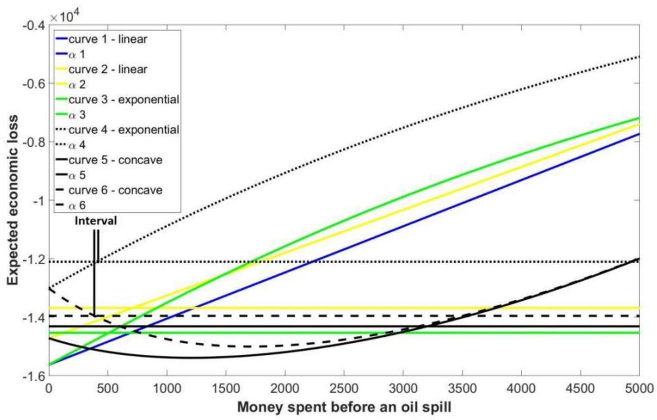

Figure 4: Pre oil spill allocation interval with acceptable threshold at 93%

Table 4 depicts the interval for z_1 for several different thresholds. As the threshold gets tighter—signifying that objective function must be closer to the optimal values—the interval gets smaller. For several of the optimization problems, the interval does not contain the optimal amount. For example, z_1 = \$0 in the base case for the linear and exponential $g(\cdot)$ functions and z_1 = \$1.2 billion for the concave $g(\cdot)$ function. However, neither of those amounts are contained within the intervals. Accounting for all of the uncertainty in the resource allocation model seems to recommend allocation amounts that are in between the optimal allocations of the various optimization problems. If the decision maker wants to guarantee that the objective function is within 91% of the minimum objective function values for the hundreds of possible models, the decision maker should allocate between \$244 and \$529 million to prevent and prepare for an oil spill in the Gulf. If the decision maker is really worried about an oil spill, he or she should spend closer to the upper bound of the interval. If the decision maker wants to spend more money on other priorities, he or she should spend approximately \$250 million. If the decision maker requires that that the interval is within more than 93% of the optimal values, the optimization problem in (8) is infeasible.

Table 4: Interval for Pre Oil Spill Allocation (Millions of \$)

Threshold	88%	89%	90%	91%	92%	93%
z_1 lower bound	\$66	\$121	\$180	\$245	\$312	\$387
z_2 upper bound	\$716	\$653	\$591	\$529	\$468	\$408

3.4. Multiple Allocation Intervals for Pre and Post Oil Spill

The application of RAM-DU in the previous subsection examined a single interval. Given the allocation of z_1, the other amounts for recovery are fixed in order to minimize the expected economic loss. However, a decision maker may want the allocation to help all industries to recover simultaneously to also consider all of the different uncertainties. In this subsection, two intervals will be generated, for the pre-oil spill amount and to help all industries after an oil spill. As depicted in the optimization problem in (3), it becomes a multi-objective optimization problem in which two interval widths $b_1 - a_1$ and $b_2 - a_2$ are maximized. As with the one interval illustration, the opportunity cost function, $\hat{p}$, k_p, k_q, and k_0 are uncertain with the ranges for those parameters depicted in Table 3.

Fig. 5 depicts the result of RAM-DU for these two intervals when the acceptable threshold is set at 91% of the minimum function value. Again, hundreds of possible optimization problems are calculated, but only three of these problems are shown in the figure. The shaded region, similar to a rectangular shape, represents the set of solutions for (z_1, z_2) that achieves objective functions less than or equal to α_j for each optimization problem. The lower bound for z_1 (a dashed line) corresponds to the concave $g(\cdot)$ function with parameters values $\hat{p} = 0.1$, $k_p = 10^{-5}$, $k_q = 10^{-2}$, and $k_0 = 6.67 \times 10^{-4}$. The upper bound for z_1 (a solid line) corresponds to the exponential $g(\cdot)$ function with $\hat{p} = 0.1$, $k_p = 10^{-5}$, $k_q = 10^{-5}$, and $k_0 = 6.67 \times 10^{-7}$.

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

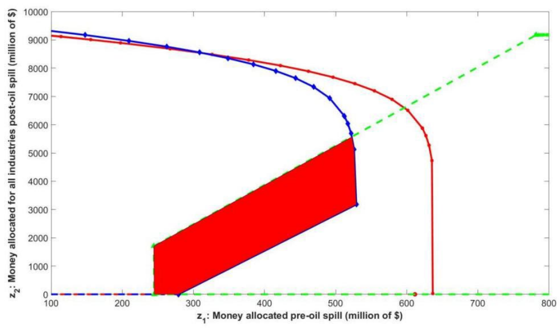

Figure 5: Allocation intervals for z1 and z2 with acceptable threshold at 91%

A decision maker should allocate between \$244 million and \$529 million for prevention and preparedness. The allocation interval is between \$0 and \$5.57 billion to help all industries recover. The remainder of budget helps individual industries recover. Fig. 5 shows that as z_1 increases, the decision maker should also increase its allocation for z_2 in order to guarantee that all objective functions are less than or equal to the threshold. If the decision maker chooses z_1 = \$244 million, then he or she can choose between \$0 and \$1.71 billion for z_2. If the decision maker chooses z_1 = \$529 million, then he or she should choose between \$3.18 and \$5.57 billion for z_2.

Fig. 6 shows the results of RAM-DU when the acceptable threshold is 93% on the minimum value-. The shaded region in Fig. 6 is much than smaller than that in Fig. 5. When a decision maker requires the objective function to be closer to its minimum value, the decision maker has less flexibility in allocating resources. If α_j corresponds to 91% of the minimum value, a decision maker can choose between \$387 and \$404 million for prevention and preparedness and between \$0 and \$2.21 billion to help all industries recover.

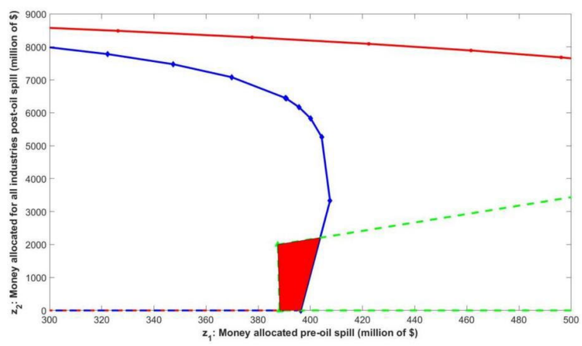

Figure 6: Allocation intervals for z1 and z2 with acceptable threshold at 93%

4. Conclusion

This article introduces RAM-DU as a solution to incorporate deep uncertainty within resource allocation models. The unique elements of RAM-DU include: (i) the incorporation of parameter, model, and structural uncertainty within the resource allocation model; (ii) the recommendation of an interval for allocation amounts rather than a point solution; and (iii) the objective function of each identified model will be no greater than the maximum acceptable threshold for every allocation amount within the interval. Extending RAM-DU to multiple decision variables generates a multidimensional hypervolume in which every set of values within that space are acceptable allocation amounts.

Applying RAM-DU to the *Deepwater Horizon* oil spill shows that allocating between \$244 and \$529 million to prevent and prepare for an oil spill will ensure that the economic losses are close to the minimum economic loss while accounting for uncertainty in the opportunity cost function, the probability of an oil spill, and the effectiveness of allocating resources. If the decision maker requires tighter thresholds, the interval becomes narrower, and he or she should allocate between \$387 and \$408 million before the oil spill. When the amount spent to help all industries recover is also considered, the decision maker has additional flexibility in spending between \$0 and \$5.57 billion to help all industries recover. The exact interval for this recovery allocation depends on the amount spent before the oil spill.

ISSN 2616-5775 Vol. 7, Issue 2: 91-101, DOI: 10.25236/AJCIS.2024.070213

If a decision maker believes that one model or parameter value better reflects reality than other models or parameter values, this belief could be represented in the acceptable threshold. For those models that do not seem the most accurate but which the decision maker still wants to consider, the threshold could be further away from the minimum value. RAM-DU can also be applied to multiple disruptions to identify the different ranges to allocate to prevent and prepare for each one of several disruptions. RAM-DU could also consider different risk attitudes where each risk attitude represents a different objective function in the form of a utility function. This could reflect multiple stakeholders where one individual is more risk neutral and another individual is more risk averse.

The *Deepwater Horizon* case only shows an oil spill application in RAM-DU. However, RAM-DU can also be applied to other resource allocation problems with deep uncertainty, especially public sector type problems. RAM-DU is particularly suitable for allocating resources in national security, homeland security, climate change, and complex system problems that plan for the distant future because these problems typically are very uncertain and models and parameters are unknown. RAM-DU provides decision makers with flexibility when they face multiple plausible futures.

References

[1] Agusdinata, B., 2008. Exploratory modeling and analysis: a promising method to deal with deep uncertainty.
[2] Bankes, S., 1993. Exploratory modeling for policy analysis. Operations research, 41(3), pp. 435-449.
[3] Ben-Haim, Y., 2001. Information-gap decision theory: decisions under severe uncertainty. San Diego, California: Academic Press.
[4] Ben-Haim, Y., 2004. Uncertainty, probability and information gaps. Reliability engineering & system safety, 85(1), pp. 249-266.
[5] Bier, V.M. and Abhichandani, V., 2003. Optimal allocation of resources for defense of simple series and parallel systems from determined adversaries. In Risk-based decision making in water resources X (pp. 59-76).
[6] Boyd, S. and Vandenberghe, L., 2004. Convex optimization. Cambridge, U.K.: Cambridge University Press.
[7] Cox, L.A.T., 2012. Confronting deep uncertainties in risk analysis. Risk Analysis, 32(10), pp.1607-1629.
[8] Courtney, H., 2001. 20/20 foresight: Crafting strategy in an uncertain world. Boston: Harvard Business Press.
[9] Croskerry, P., 2009. A universal model of diagnostic reasoning. Academic medicine, 84(8), pp.1022-1028.
[10] DeGroot, M.H., 1988. A Bayesian view of assessing uncertainty and comparing expert opinion. Journal of statistical planning and inference, 20(3), pp.295-306.
[11] Dillon, R. L., Paté-Cornell, M. E., and Guikema, S. D. (2005). Optimal use of budget reserves to minimize technical and management failure risks during complex project development. IEEE Transactions on Engineering Management 52(3), 382-395.
[12] Etkin, D.S., 2004, April. Modeling oil spill response and damage costs. In Proceedings of the Fifth Biennial Freshwater Spills Symposium.
[13] Floricel, S. and Miller, R., 2001. Strategizing for anticipated risks and turbulence in large-scale engineering projects. International Journal of project management, 19(8), pp.445-455.
[14] Ferson, S., Kreinovich, V., Ginzburg, L., Myers, D.S. and Sentz, K., 2003. Constructing probability boxes and Dempster-Shafer structures (pp. 1896-1901). Technical report, Sandia National Laboratories.
[15] Grander, M.M. and Max, H., 1990. A guide to dealing with uncertainty in quantitative risk and policy analysis. Cambridge, U.K.: Cambridge University Press.
[16] Guikema, S.D., and Paté-Cornell, M.E. (2002). Component choice for managing risk in engineered systems with generalized risk/cost functions. Reliability Engineering and System Safety 78(3), 227-237.
[17] Hall, J.W., Lempert, R.J., Keller, K., Hackbarth, A., Mijere, C. and McInerney, D.J., 2012. Robust climate policies under uncertainty: a comparison of robust decision making and info-gap methods. Risk Analysis, 32(10), pp.1657-1672.
[18] Hallegatte, S., Shah, A., Brown, C., Lempert, R. and Gill, S., 2012. Investment decision making under deep uncertainty–application to climate change (September 1, 2012). World Bank Policy Research Working Paper No. 6193. Available at SSRN: https://ssrn.com/abstract=2143067
[19] Hamarat, C., Kwakkel, J.H. and Pruyt, E., 2013. Adaptive robust design under deep uncertainty. Technological Forecasting and Social Change, 80(3), pp.408-418.
[20] Heal, G. and Millner, A., 2014. Reflections: Uncertainty and decision making in climate change economics. Review of Environmental Economics and Policy, 8(1), pp.120-137.