ISSN 2616-5872 Vol.4, Issue 6: 28-34, DOI: 10.25236/AJEE.2022.040604

Maximum benefits for water resources and hydropower based on linear programming

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Abstract: *With the increasingly serious development of greenhouse effect, hydropower, as a clean and environmentally friendly energy technology, is attracting more and more attention. On the Colorado River, the Glen Canyon Dam and the Hoover Dam simultaneously supplies water and electricity to Arizona (AZ), California (CA), Wyoming (WY), New Mexico (NM) and Colorado State (CO). However, nowadays, the two dams cannot coordinate with each other well to exert their effectiveness, and the water resources cannot be fully utilized, which makes us upset. At the same time, we also succeeded to find the solution, when there is not enough water to meet all hydropower needs [1.2]. We calculate the gap between water supply and electricity supply. Based on the Supply and demand models for water and hydropower, when there is not enough water, we deduced how much additional water needs to be injected into the dams could fill these gaps [3.4.5]. Besides, we analyzed how our model would respond, when many other supply and demand conditions changed.*

Keywords: *Profit maximization; hydropower needs; scarcity of water resource*

1. Introduction

With the increasingly serious development of greenhouse effect, hydropower, as a clean and environmentally friendly energy technology, is attracting more and more attention. Moreover, it has the characteristics of renewable and non-polluting, which can promote the development of agriculture and aquaculture at the same time. Therefore, it is a very economical and efficient choice. A long time ago, the Glen Canyon Dam and the Hoover Dam continuously supply water and hydropower to Arizona (AZ), California (CA), Wyoming (WY), New Mexico (NM) and Colorado State (CO). However, due to the scarcity of water resources, past agreements are gradually no longer suitable for today's situation, which leads to allocate water inefficiently. Therefore, we hope to assist natural resource officials better manage the use of water and hydropower production by establishing models.

2. Assumptions and explanations

(1) We assume that the dam discharges water evenly in a day. We analyze the changes of water in the dam in days, ignoring the specific situations in a day, such as the time of lifting gate and generator safety detection.

(2) During hydropower generation, the gravitational energy of water converts not only into electric energy and kinetic energy of water, but also some losses, such as Joule heat. In order to facilitate the analysis, the kinetic energy of water is not considered in the analysis, and the efficiency of converting gravitational potential energy into electric energy is regarded as a constant.

(3) Although there are many uses of water resources in Lake Powell and Lake Mead, we ignore the uses irrelevant to the topic and consider only two ways, to supply water to five states and to meet the electricity demand.

(4) Besides the direct supply of water from the lake, there will be additional water supplies, such as rain and groundwater, among which rain is the main source. In the analysis, other water sources will be ignored and rainwater is regarded as the only source of additional water.

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3. Model Preparation

3.1 Representation of the demands

As we all know, water resources and electricity resources are inseparable. Therefore, if planning the water use of Glen Canyon and Hoover Dam, it is necessary to clarify the demand for water resources and electricity resources in the involved areas.

Assuming that the average daily water demand of Arizona, California, Wyoming and Colorado is a_1, a_2, a_3, a_4, a_5 ; and that the average daily power consumption is b_1, b_2, b_3, b_4, b_5 .

The total power generation of each state is $b = \sum_{i=1}^{5} b_i$;

The total water consumption of each state is $a = \sum_{i=1}^{5} a_i$.

3.2 Classification of water

The water resources are classified according to different dams and purposes.

(1) Glen Canyon Dam

For the Glen Canyon Dam, the daily water resources are divided into four parts: the portion that goes directly to the 5 states is m_1 , that goes to the dam to generate electricity is m_2 , that flows into the Hoover Dam is m_3 , and that goes to reservoir storage is m_4 .

(2) Hoover Dam

For Hoover Dam, the daily water resources are divided into three parts: the portion that goes directly to the 5 states is n_1 , that goes to the dam to generate electricity is n_2 , and that goes to reservoir storage is n_3 .

To sum up, the daily water supply directly used in the five states are only $m_1 + n_1$,regardless of other uses.

4. Supply and demand models for water and hydropower

4.1 How much water should be drown

Since the total daily water demand of the five states cannot be more than $m_1 + n_1$, it can be obtained:

$$\sum_{i=1}^{5} a_i \le m_1 + n_1 \tag{1}$$

By consulting the literature, it can be known that the water resources used for power generation are related to two factors: water volume and water level. Therefore, water volume and water level are independent variables in the above formula.
Make the following assumptions:

The water consumption of Powell lake after discharge is w_P , the water level is P , and the water volume before discharge is $w_P^{'}$;

The water consumption of Lake Mead after discharge is w_M , the water level is M , and the water consumption before discharge is $w_M^{'}$.

Then,

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$$w_P = w_P^{'} - m_3 \tag{2}$$

$$w_M = w_M^{'} + m_3 \tag{3}$$

According to the law of conservation of mass,

$$W_P = \sum_{i=1}^{4} m_i \tag{4}$$

$$W_M = n_1 + n_2 + n_3 \tag{5}$$

From the conservation of energy, there will be energy loss when the potential energy of water is converted into electrical energy. We assume that the loss rate is μ, then W_E is the product of the loss rate and the change in gravitational potential energy.

For Glen Canyon Dam and Hoover Dam, the electricity generated is as the following:

$$W_{E1} = m_2 \rho_M (P + h_1) g \times \mu \tag{6}$$

$$W_{E1} = m_2 \rho_M (P + h_1) g \times \mu \tag{7}$$

Among them, h1 and h2 represent the heights of the generators in Lake Powell and Lake Mead from the ground, respectively. Then the total potential energy of water converting into electrical energy is:

$$W_E = \mathrm{m}_2 \rho (P + h_1)\mu + n_2 \rho (m + h_2)\mu \tag{8}$$

The remaining water volume and of Lake Powell and Lake Mead, m_4 and n_3, must be within the specified safe water level. Set the safe water level of Lake Powell as $\left[u_1 \quad u_2\right]$ and Lake Mead as $\left[u_3 \quad u_4\right]$,

$$u_3 \le n_3 \le u_4 \tag{9}$$

For each state, the greater the power generation, the higher the benefit, and the more water allocation, the better. Therefore, the model is based on this standard:

$$\max y = \frac{m_2 \rho (P + h_1) + n_2 \rho (m + h_2) - \sum_{i=1}^{i=5} b_i}{\sum_{i=1}^{i=5} b_i} + \frac{m_1 + n_1 - \sum_{i=1}^{i=5} a_i}{\sum_{i=1}^{i=5} a_i} \tag{10}$$

The decision conditions are as follows,

$$s.t. \begin{cases} g m_2 \rho (P + h_1)\mu + n_2 \rho (m + h_2) g \mu > \sum_{i=1}^{5} b_i \\ w_P^{'} - m_3 = m_1 + m_2 + m_3 + m_4 \\ w_M^{'} + m_3 = n_1 + n_2 + n_3 \\ \sum_{i=1}^{5} a_i \le m_1 + n_1 \\ u_1 \le m_4 \le u_2 \\ u_3 \le n_3 \le u_4 \\ m_i \ge 0, n_i \ge 0 \end{cases} \tag{11}$$

4.2 How long will it take and how much additional water must be supplied

Besides the direct supply of water from the lake, there will be additional water supplies, such as rain and groundwater, among which rain is the main source. In the analysis, other water sources are ignored and rainwater is regarded as the only source of additional water. By investigating the relevant data, we set the rainfall in each state as Q_i, then the average daily water demand of the five states is the sum of the water in the dam and the extra water,

$$a=\sum_{i=1}^{5}a_i+\sum_{i=1}^{5}Q_i \tag{12}$$

The power demand is still as,

$$b=\sum_{i=1}^{5}b_i \tag{13}$$

In the above model, synthesizing the law of conservation of mass and conservation of energy, the model can be established based on the following constraints.

$$\max y=\frac{m_2\rho\left(P+h_1\right)g\mu+n_2\rho\left(m+h_2\right)g\mu-\sum_{i=1}^{5}b_i}{\sum_{i=1}^{5}b_i}+\frac{m_1+n_1-\left(\sum_{i=1}^{5}a_i+\sum_{i=1}^{5}Q_i\right)}{\sum_{i=1}^{5}a_i+\sum_{i=1}^{5}Q_i} \tag{14}$$

$$s.t.\begin{cases}gm_2\rho\left(P+h_1\right)\mu+n_2\rho\left(m+h_2\right)g\mu>\sum_{i=1}^{5}b_i\\ w_P^{'}-m_3=m_1+m_2+m_3+m_4\\ w_M^{'}+m_3=n_1+n_2+n_3\\ \sum_{i=1}^{5}a_i\le m_1+n_1\\ u_1\le m_4\le u_2\\ u_3\le n_3\le u_4\\ m_i\ge 0,n_i\ge 0\end{cases} \tag{15}$$

Make the volume of water drained off from Lake Mead and Lake Powell respectively V_M ,V_P .

$$\mathrm{t}_1=\frac{m_1}{V_P} \tag{16}$$

$$\mathrm{t}_2=\frac{n_1}{V_M} \tag{17}$$

Then:

$$\mathrm{t}=\begin{cases}\mathrm{t}_1,\mathrm{t}_1\ge\mathrm{t}_2\\ \mathrm{t}_2,\mathrm{t}_1<\mathrm{t}_2\end{cases} \tag{18}$$

$$= \begin{cases} \frac{m_1}{V_P}, t_1 \geq t_2 \\ \frac{n_1}{V_M}, t_1 < t_2 \end{cases} \tag{19}$$

Discuss *t* in categories:

If *t* ≤ 1, the drainage can meet the water demand of five states, and it may take *t* to meet the demands.

If *t* > 1, the drainage cannot meet the water demand of five states.

The actual drainage is $V_P + V_M$, while the total demand is, the daily additional water demand is $m_1 + n_1$:

$$Q_a = m_1 + n_1 - (V_P + V_M) \tag{20}$$

in summary,

$$Q_a = \begin{cases} 0 & t \leq 1 \\ \left(m_1 + n_1 - (V_P + V_M)\right) & t>1 \end{cases} \tag{21}$$

Then the water as much as Q_a must be supplied additionally.

4.3 Verification idea

Since the data we obtained are from real life, we can only get the theoretical value for the results of water storage and drainage operation of the two dams assumed by the model. Here, we propose to use the model to simulate the multi-year data, then the predicted results we get will be consistent with the data of each year.

Here is the detailed step. First, let the operation of the model for each year be recorded as vector $V = (m_1, m_2, m_3, m_4, n_1, n_2, n_3)$. Since there is a linear relationship in the actual situation, if it changes little for each year, we will assume its linear relationship coefficient K, and divide the obtained V: 70% as the training set and 30% as the test set. Then compare the result K' from the training set with the test set to get the difference between the data and the test set.

However, in this model, the actual water operation changes with time (the data of the water operation is regarded as one unit per year), so the model results only have reference value for similar time.

Then we propose to verify with the data of the only first two years, that is, the model simulation operation obtained in the first year is directly regarded as the expected operation in the second year.

The data of power generation prediction and water consumption prediction are compared with the actual data. After comparison, if the model simulation results of the first year can be used as the operational guidance of the second year, then for the data of recent years, the operational guidance for the future time can be found through the model.

5. Profit maximization model

5.1 Model Development

The general usage water used by Lake Powell and Lake Mead are m_1, n_1.The profit generated is $\pi_1 = k_1(m_1 + n_1)$.

The actual power generation of Lake Powell and Lake Mead are

$$m_2\rho(P+h_1)g\mu \tag{22}$$

$$n_2\rho(m+h_2)g\mu \tag{23}$$

The profit generated is

$$\pi_2 = k_2[m_2\rho(p+h_1)g\mu + n_2\rho(m+h_2)g\mu] \tag{24}$$

Therefore, the total profit is the sum of the two :

$$\max y = \frac{m_2\rho(p+h_1)g\mu + n_2\rho(m+h_2)g\mu - \sum_{i=1}^{5} b_i}{\sum_{i=1}^{5} b_i} + \frac{m_1+n_2-\sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} a} \tag{25}$$

$$\max y = \frac{m_2\rho(P+h_1)g\mu + n_2\rho(m+h_2)g\mu - \sum_{i=1}^{5} b_i}{\sum_{i=1}^{5} b} + \frac{m_1+n_1-\sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} a_i} \tag{26}$$

$$\max \pi = k_1(m_1+n_2) + k_2[m_2\rho(P+h_1)g\mu + n_2\rho(m+h_2)g\mu] \tag{27}$$

The constraints are the same as Supply and demand models for water and hydropower:

$$s.t.\begin{cases} gm_2\rho(P+h_1)\mu + n_2\rho(m+h_2)g\mu > \sum_{i=1}^{5} b_i \\ w_P^{'} - m_3 = m_1+m_2+m_3+m_4 \\ w_M^{'} + m_3 = n_1+n_2+n_3 \\ \sum_{i=1}^{5} a_i \le m_1+n_1 \\ u_1 \le m_4 \le u_2 \\ u_3 \le n_3 \le u_4 \\ m_i \ge 0, n_i \ge 0 \end{cases} \tag{28}$$

By using LINGO software to solve the above model, we can get the best allocation scheme of water resources on the premise of meeting the basic demands. Our standard for resolving the competitive interests is to maximize the profits. Considering meeting all the above current used constraints, we can make a reasonable allocation for the maximum profit.

5.2 Improvement

In reality, the profit coefficient of electricity and water is not just keep constant. Instead, in order to encourage consumers to save water and electricity, the electricity price and water price of each state is always priced step by step, and the pricing system of each state is different. In order to make the model more accurate, we should classify different situations and discuss the model to distinguish different profit coefficiency in different intervals.

6. Conclusion

No matter if there's a change in the demands for water and electricity, the proportion of renewable energy technologies increases or new conservation measures are implemented, we can easily find out that there is just some part of the model should be changed, such as the demands of people in the five states. So we need to test the sensitivity of the model to find out what the model will react.

In the model, we analyze the change of daily water use. If m1 and n1 changes within a certain range and are 10% higher than the model value for a long time during the model construction, the two power generation dams should be provided additional water if they still can meet the operation. Because normalization is considered in the construction of the model, the expression of y is only related to m1 (n1 only changes its feasible region). After re-simulating its linear programming, the value of y changes by 0.95 times the original, that is, a change of 5%; After the m1 and n1 values change by more than 20%, the y value will change to 0.7 times the original value but at the same order of magnitude due to the limit of the safe water level; When m1 and n1 decrease, y basically does not change. The above points are in line with the expectations of the actual situation, and when the changes of these two parameters are small, the model has good universality; and when the range of the change becomes larger, the limit of the safe water level will be obviously reflected. If only the water level changes, the model can also be used, and the parameters affecting the model are only m_1, m_2, m_3, n_1, n_2 rimental parameters, it is found that the most influential parameters are the additional water volume and m1. After doubling m1 and substituting it into the model, the change of y value is less than 50%. The floating range of the additional water volume is small in the ordinary data, and the b change in the order of magnitude will occur only when the precipitation is very large. After changing the additional water volume to the original 1-100 times, it is found that the results tend to be the same when it is greater than 20 times, and the results change greatly when it is less than 20 times, which is also consistent with the same application under a certain precipitation model.

The results show that the sensitivity of the model is relatively low. It can also be well simulated in the face of abnormal data and the extreme data under the extreme weather conditions.

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