

Optimal Production Quantity and Quality Choice in Agricultural Supply Chain

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ABSTRACT. *Agricultural production is susceptible to a series of uncertainties, such as the weather, edaphic condition, water resource and pest disasters. Therefore, the supply and demand matching problem is always significant to the companies in the agricultural supply chain. This paper tries to find the solution of this problem. There are one supplier and one manufacturer, where the former decides the acreage, and the latter decides the selling prices and product quality. We establish a mathematical model and obtain some conclusions. Our main findings are as follows. Firstly, when the perfect buyer's market emerges, the supply chain is hard to run. Then, when the perfect seller's market emerges, the companies in the supply chain can try their best to produce products and earn excess profits.*

KEYWORDS: *agricultural supply chain, quality choice, production quantity, selling price, stochastic productivity*

1. Introduction

Agriculture is a basic industry, which is seriously related to people's livelihood. Governments around the world are paying close attention to the development of the agriculture. Affected by various uncertain factors, agriculture is always fragile and the output is uncertain.

Depending on the order from normal social production, the demand of agricultural products is always relatively stable. At this point, the contradiction between supply and demand is prominent and it has a huge impact on the profits of the company in the agricultural supply chain. Therefore, how to decide the production quantity is a key problem for the enterprise managers.

Slow sale of agricultural products happens to us all the time. Recently, under the influence of the new coronary pneumonia, agricultural products have been slow to sell locally. Fortunately, pinduoduo establishes an agricultural products unsalable exclusive feedback channel to alleviate local supply and demand mismatch situation [1]. However, things are not always so lucky. How to decide the output is still a key problem for the company. Taking Ghana agriculture supply chain as an example,

Yeboah discusses the seriousness of the risks associated with supply chain and the ability level to control them based on engineering judgment and history [2]. The other instance is about pork supply chain in Scotland. Leat P and Revoredo - Giha conduct in-depth interviews with seven people and discuss the risk of all the participants [3].

The quality is another key issue for agricultural supply chain manager. With the improvement of living standard, the quality is more and more concerned by consumers. The specter of clenbuterol poisoning still lingers [4]. Taking Scottish agricultural food industry as an example, Leat summarizes the farm development and quality assurance activities since the early 1990s, and generalizes the European third-party certification system of quality assurance plan [5]. The other one is the Ukrainian agricultural products under Eastern Europe business environment. Through 15 in-depth interview, Hanf reveals the chain (quality) management mechanism in the early stages of the transition economies [6].

The remainder of this paper is organized as follows. In section 2, we describe the related symbols and formulate the problems as a mathematical optimization model. In section 3, we solve the model and obtain some propositions. In section 4, we conclude this paper.

2. Symbols and model

2.1 Symbols description

Q_m : The quality level per product

X_s : The cultivated area

π : The profits of the companies in the supply chain

2.2 Related assumptions

The supplier plants the crops and the manufacturer processes and sells the product in the agricultural supply chain. For convenience, the supplier and the manufacturer make the centralized decisions at the same time to maximize the overall profitability of the supply chain.

For convenience, we assume that the selling price is decided by the quality level. The manufacturer sells the product to the consumers with the price, αQ_m , where α is a technical parameter related to the quality level. By this time, the market demands are $D = a + bQ_m - c\alpha Q_m$, where a represents the basal demand, and b and c respectively represent the coefficient of quality level and selling price to the market demand.

A series of uncertain factors seriously affect the production of the crop, such as the climate, soil and water. Therefore, the land productivity, β , is random, which is

subject to a uniform distribution form λ_1 to λ_2 . The total land area owned by the supplier is w . The cost of planting is θX_s , where θ represents the marginal costs. Where supply exceeds demand, the rest is considered worthless. In the meantime, where supply cannot meet demand, only part of the demand is meet. Finally, the processing cost is ρQ_m , where $\rho < \alpha$ is a technical parameter related to the quality level.

2.3 Mathematical Model

In the agricultural supply chain, the supplier decides the cultivated area and the manufacturer decides the quality level. According to the above hypothesis, the overall profits are described as follower.

$$\pi = (\alpha Q_m - \rho Q_m)E(\min\{\beta X_s, D\}) - \theta X_s \quad (1)$$

The equation reflects the total profits of all the companies in the agricultural supply chain. Under the assumption of uniform distribution, we update the equation.

$$\pi = \begin{cases} (\alpha Q_m - \rho Q_m) \frac{\lambda_1 + \lambda_2}{2} X_s - \theta X_s & D \geq \lambda_2 X_s \\ (\alpha Q_m - \rho Q_m) \frac{2\lambda_2 X_s D - D^2 - (\lambda_1 X_s)^2}{2(\lambda_2 - \lambda_1) X_s} - \theta X_s & \lambda_1 X_s < D < \lambda_2 X_s \\ (\alpha Q_m - \rho Q_m) D - \theta X_s & D \leq \lambda_1 X_s \end{cases} \quad (2)$$

The equation (2) is a piecewise function. In next section, we will solve it.

3. Solutions

In this section, we solve the model by the extreme value theory and derive various conclusions. Then, we discuss the extremun of each stage.

When $a + bQ_m - c\alpha Q_m \geq \lambda_2 X_s$, the demand is greater than the maximal capacity. We define it as seller's market. Firstly, we take the first partial derivative of equation (2) with respect to the area and quality.

$$\frac{d\pi}{dQ_m} = (\alpha - \rho) \frac{\lambda_1 + \lambda_2}{2} X_s > 0 \quad (3)$$

$$\frac{d\pi}{dX_s} = (\alpha - \rho) Q_m \frac{\lambda_1 + \lambda_2}{2} - \theta > 0 \quad (4)$$

The equation (4) is greater than zero, which is assumed to make sure that farmers are motivated to plant. Evidently, the profits increase with the area and the quality level.

Proposition 1: When $a + bQ_m - c\alpha Q_m \geq \lambda_2 X_s$ and $c\alpha > b$, $X_s1^* = w$ and $Q_m1^* = \frac{a - \lambda_2 w}{c\alpha - b}$.

Proof: According to equation (4) and $X_s \leq w$, we can obtain $X_s1^* = w$. According to $Q_m \leq \frac{a - \lambda_2 X_s}{c\alpha - b} = \frac{a - \lambda_2 w}{c\alpha - b}$, we can obtain $Q_m1^* = \frac{a - \lambda_2 w}{c\alpha - b}$, similarly.

The proposition 1 indicates that when the perfect seller's market emerges, the optimal decision is producing high quality products at the maximum capacity. At this moment, the market environment is extremely favorable for the company. When the company's products are highly competitive, novel and advanced, this case happens.

When $a + bQm - c\alpha Qm \leq \lambda 1Xs$, the demand is less than the minimum capacity. We define it as buyer's market. Firstly, we take the first partial derivative of equation (2) with respect to the area and quality.

$$\frac{d\pi}{dQm} = a(\alpha - \rho) + 2(\alpha - \rho)(b - c\alpha)Qm < (\alpha - \rho)(2\lambda 1Xs - a) \quad (5)$$

$$\frac{d\pi}{dXs} = -\theta < 0 \quad (6)$$

Proposition 2: When $a + bQm - c\alpha Qm \leq \lambda 1Xs$ and $c\alpha > b$, $Xs2^* = 0$ and $Qm2^* = \frac{a}{c\alpha - b}$.

Proof: According to equation (6) and $0 \leq Xs \leq w$, we can obtain $Xs2^* = 0$ and $\frac{d\pi}{dQm} < 0$. According to $Qm \geq \frac{a - \lambda 1Xs}{c\alpha - b} = \frac{a}{c\alpha - b}$, $Qm2^* = \frac{a}{c\alpha - b}$.

The proposition 2 indicates that when the perfect buyer's market emerges, the optimal decision is not carrying out production activities. At this point, the market environment is extremely harsh. In other words, the market demand can not reach the minimum level of production. When the products of the company are extremely laggard, unpractical and outdated, this case happens.

When $\lambda 1Xs < a + bQm - c\alpha Qm < \lambda 2Xs$, the demand is between the minimum capacity and the maximum capacity. We define it as positive market. Firstly, we take the first partial derivative of equation (2) with respect to the area and quality.

$$\frac{d\pi}{dQm} = (\alpha - \rho) \frac{4\lambda 2Xs(\frac{a}{2} + bQm - c\alpha Qm) - (\lambda 1Xs)^2 - (a + 3bQm - 3c\alpha Qm)(a + bQm - c\alpha Qm)}{2(\lambda 2 - \lambda 1)Xs} = 0 \quad (7)$$

$$\frac{d\pi}{dXs} = (\alpha Qm - \rho Qm) \frac{(a + bQm - c\alpha Qm)^2 - (\lambda 1Xs)^2}{2(\lambda 2 - \lambda 1)Xs^2} - \theta = 0 \quad (8)$$

$$\text{From equation (8), we can obtain } Xs^2 = \frac{(\alpha Qm - \rho Qm)(a + bQm - c\alpha Qm)^2}{(\alpha Qm - \rho Qm)(\lambda 1)^2 + 2\theta(\lambda 2 - \lambda 1)} \quad (9)$$

Then, we plug the equation (9) into equation (7).

$$(a + 3bQm - 3c\alpha Qm)(a + bQm - c\alpha Qm)[(\alpha Qm - \rho Qm)(\lambda 1)^2 + 2\theta(\lambda 2 - \lambda 1)] + \lambda 1^2(\alpha Qm - \rho Qm)(a + bQm - c\alpha Qm)^2 - 4\lambda 2\left(\frac{a}{2} + bQm - c\alpha Qm\right)(a + bQm - c\alpha Qm)\sqrt{(\alpha Qm - \rho Qm)[(\alpha Qm - \rho Qm)(\lambda 1)^2 + 2\theta(\lambda 2 - \lambda 1)]} = 0 \quad (10)$$

Then, we can obtain that $Qm3^*$ satisfies equation (10) and $Xs3^* = \sqrt{\frac{(\alpha - \rho)Qm^*(a + bQm^* - c\alpha Qm^*)^2}{(\alpha - \rho)Qm^*(\lambda 1)^2 + 2\theta(\lambda 2 - \lambda 1)}}$.

Proposition 3 : The solution of Q_m satisfying $Q_m > \frac{2\theta}{(\alpha-\rho)(\lambda_1+\lambda_2)}$ is one of the locally optimal solution of the profit.

Proof : By the backward reasoning and equation (9), $\lambda_1 X_s < a + bQ_m - \alpha Q_m < \lambda_2 X_s$ can be obtained obviously.

Proposition 4 : $(Q_m^*, X_s^*) = \{(Q_m, X_s) | \max(\pi(Q_m, X_s)), (Q_m, X_s) \in [(Q_{m1}^*, X_{s1}^*), (Q_{m2}^*, X_{s2}^*), (Q_{m3}^*, X_{s3}^*)]\}$ is the optimal solution of the enterprise decision.

4. Conclusion and managerial implication

In this paper, we study an agricultural supply chain consisted of one manufacturer and one supplier. They intensively make decisions to maximize the overall profits, where the former decides the quality level and the latter decides the area. We formulate this problem as a mathematical optimization problem. Because of the complexity of the solution, it is difficult for us to make theoretical analysis. This paper only gives people, who can obtain the actual date, a new way of thinking. However, there are a couple of new findings. Firstly, when the perfect buyer's market emerges, it is extremely unfavorable for the company. The supply chain is hard to run. At this point, the subsidies from the government are always necessary. Then, when the perfect seller's market emerges, the companies in the supply chain can try their best to produce products and earn excess profits.

There are some limitations in this paper. According, further researches are raised. Firstly, the demand is decided by the quality and price in this paper. We can assume the demand is random to be more realistic in the future. Secondly, only one supplier and one manufacturer are considered in this paper. Then, competitive mechanism can be introduced. Finally, the decision makers are risk neutral. The risk preference can be taken into account in the future.

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