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Rogue Wave Solutions of the (3+1)-Dimensional Generalized Kadomtsev-Petviashvili Equation

Fu-Fu Ge

Foundation Department, Xuzhou Vocational College of Industrial Technology, Xuzhou 221140, People's Republic of China
gefufu@163.com

Abstract: *The rogue wave solutions of the (3+1)-dimensional generalized Kadomtsev-Petviashvili (gKP) equation are constructed by Hirota bilinear method and symbolic computation approach. By choosing proper polynomial function, the first, the second and the third order rogue wave solutions are systematically obtained. The maximum amplitude and the minimum amplitude of the first order rogue wave solution are given. Moreover, the first three order generalised rogue wave solutions are also explicitly presented. Finally, some features of rogue wave solutions are graphically discussed.*

Keywords: *The generalized Kadomtsev-Petviashvili equation, Symbolic computation approach, Rogue wave solution, Generalised rogue wave solution*

1. Introduction

Extreme wave events occurring in seas and oceans almost every week are reported. There are a number of physical mechanisms that focus the water wave energy into a small area and produce the occurrence of extreme waves called freak or rogue waves [1]. This kinds of waves are localized in both space and time, which can describe many significant nonlinear phenomena, such as ocean's waves [2], optical fibers [3-5], Bose-Einstein condensates [6-8], financial markets [9-10], etc.

The characteristic of rogue waves is that "they appear from nowhere and disappear without a trace" [11-12]. It is believed that the modulation instability is the fundamental mechanism for the generation of the rogue waves. And one of the most important models for modulation instability is the nonlinear SchrÖdinger equation (NLSE) [13]. The first order rogue wave solution of NLSE in mathematical form is given by Peregrine [14]. There are many other nonlinear evolution equations (NLEEs) possessing rouge waves, such as the Gerdjikov-Ivanov equation [15], the generalized NLSE [16-17], the Hirota equation [18], the Fokas-Lenells equation [19], the coupled SchrÖdinger equations [20-21], the (2+1)-dimensional B-type Kadomtsev-Petviashvili equation [22-23], the (3 + 1)-dimensional generalized Kadomtsev-Petviashvili (gKP) equation [24], etc.

At present, there exist many approaches to solve these NLEEs, such as inverse scattering method [25-26], Hirota bilinear method [27], Darboux transformation [28], Bäcklund transformation [29], the variable separation method [30], and Lie group method [31]. Based on these methods, the Hirota bilinear method is an effective way to find soliton solutions. Recently, the rogue wave solutions of many NLEEs have been constructed based on the Hirota bilinear method and symbolic computation approach, such as the Boussinesq equation [32], the (2+1)-dimensional gKP equation [33-34], the (3+1)-dimensional gKP equation [35], the (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation [36], the (3+1)-dimensional potential Yu-Toda-Sasa-Fukuyama equation [37], the (3+1)-dimensional Kadomtsev-Petviashvili-Boussinesq equation [38], and so on.

Different from the above equations, in this paper, we focus on the (3+1)-dimensional gKP equation [39]

$$(u_t + h_1 u_{xxx} + h_2 u u_x)_x + h_3 u_{xx} + h_4 u_{yy} + h_5 u_{zz} + h_6 u_{xy} + h_7 u_{xz} + h_8 u_{yz} = 0, \qquad (1.1)$$

Where $u = u(x; y; z; t), h_i (i = 1,2,3,4,5,6,7,8)$ are arbitrary constants? The gKP equation (1.1) can be reduced to some other form KP equation by taking appropriate parameters for h_i.

The primary purpose of this paper is to construct the rogue wave solutions of the (3+1)-dimensional gKP equation (1.1) through symbolic computation approach. By giving the polynomial function f

with the form

$$f=\sum_{k=0}^{n(n+1)/2}\sum_{i=0}^{k}a_{n(n+1)-2k,2i}X^{2i}z^{n(n+1)-2k}, \tag{1.2}$$

We get the rogue wave solutions of the gKP equation (1.1). Further more, we add two parameters α and β to the polynomial function f, then we get the generalised rogue wave solutions of the gKP equation (1.1).

The structure of this paper is as follows. In Section 2, we present the bilinear form equation by variable substitution. In Section 3, we derive the first, the second and the third order rogue wave solutions of the gKP equation (1.1), respectively. In Section 4, we derive the first three order generalised rogue wave solutions of the gKP equation (1.1).

2. The Bilinear form Equation of the Gkp Equation

In order to obtain the rogue wave solutions of the gKP equation (1.1) we set $X=x+ay-bt$ equation (1.1) can be rewritten as follows

$$(-bu_X+h_1u_{XXX}+h_2uu_X)_X+(h_3+ah_6+a^2h_4)u_{XX}+h_5u_{zz}+(h_7+ah_8)u_{Xz}=0, \tag{2.1}$$

Where a and b are two real parameters. Under the transformation

$$u=12\frac{h_1}{h_2}(\ln f)_{XX}, \tag{2.2}$$

Equation (2.1) is equivalent to

$$(-bf_{XX}+h_1f_{XXXX}+mf_{XX}+h_5f_{zz}+sf_{Xz})f^3+(bf_X^2-4h_1f_Xf_{XXX}-3h_1f_{XX}^2+h_2f_{XX}^2$$
$$-mf_X^2-h_5f_z^2)f^2+(12h_1f_X^2f_{XX}-2h_2f_X^2f_{XX}-sf_Xf_z)f+(h_2-6h_1)f_X^4=0, \tag{2.3}$$

Where $m=h_3+ah_6+a^2h_4$ and $s=h_7+ah_8$.

For the sake of simplicity, we set $h_2=6h_1$ and set $s=0$ to eliminate the impact of mixed partial derivative u_{Xz}. And then the equation (2.3) becomes

$$(-bf_{XX}+h_1f_{XXXX}+mf_{XX}+h_5f_{zz}+sf_{Xz})f+(bf_X^2-4h_1f_Xf_{XXX}-3h_1f_{XX}^2+h_2f_{XX}^2-mf_X^2-h_5f_z^2)=0, \tag{2.4}$$

Where $m=h_3+ah_6+a^2h_4$.

3. The Rogue Wave Solutions of the Gkp Equation

In this section, we assume that the function f has the following form [32,35]

$$f=F_n(z,X)=\sum_{k=0}^{n(n+1)/2}\sum_{i=0}^{k}a_{n(n+1)-2k,2i}X^{2i}z^{n(n+1)-2k}. \tag{3.1}$$

Then we get the first, the second and the third order rogue wave solutions of the gKP equation (1.1), respectively. Without losing generality, we set $a_{n(n-1),0}=1$.

Case 1: $n=1$

$$f=F_1(z,X)=\sum_{k=0}^{1}\sum_{i=0}^{k}a_{2-2k,2i}X^{2i}z^{2-2k}=z^2+a_{0,0}+a_{0,2}X^2. \tag{3.2}$$

And then substituting equation (3.2) into equation (2.4), we have

$$a_{0,2}=-\frac{h_5}{b-m},\quad a_{0,0}=-\frac{3h_1h_5}{(b-m)^2}. \tag{3.3}$$

We get the first order rogue wave solution of the gKP equation (1.1) as

$$u_{1r}=2(\ln f_{1r})_{XX}, \tag{3.4}$$

Where

$$f_{1r} = z^2 - \frac{h_5}{b-m} X^2 - \frac{3h_1 h_5}{(b-m)^2}. \tag{3.5}$$

Case 2: $n = 2$

$$f = F_2(z, X) = \sum_{k=0}^{3} \sum_{i=0}^{k} a_{6-2k,2i} X^{2i} z^{6-2k}$$

$$= z^6 + (a_{4,0} + a_{4,2} X^2) z^4 + (a_{2,0} + a_{2,2} X^2 + a_{2,4} X^4) z^2 +$$
$$(a_{0,0} + a_{0,2} X^2 + a_{0,4} X^4 + a_{0,6} X^6). \tag{3.6}$$

Substituting equation (3.6) into equation (2.4), we get

$$a_{0,0} = -\frac{1875 h_1^3 h_5^3}{(b-m)^6}, \quad a_{0,2} = \frac{125 h_1^2 h_5^3}{(b-m)^5}, \quad a_{0,4} = -\frac{25 h_1 h_5^3}{(b-m)^4}, \quad a_{0,6} = -\frac{h_5^3}{(b-m)^3},$$

$$a_{2,0} = \frac{475 h_1^2 h_5^2}{(b-m)^4}, \quad a_{2,2} = \frac{90 h_1 h_5^2}{(b-m)^3}, \quad a_{2,4} = \frac{3 h_5^2}{(b-m)^2}, \quad a_{4,0} = -\frac{17 h_1 h_5}{(b-m)^2}, \tag{3.7}$$

$$a_{4,2} = -\frac{3h_5}{b-m}.$$

Then we get the second order rogue wave solution of the gKP equation (1.1) as

$$u_{2r} = 2(\ln f_{2r})_{XX}, \tag{3.8}$$

Where f_{2r} is derived from equation (3.6) with parameters in equation (3.7).

Case 3: $n = 3$

$$f = F_3(z, X) = \sum_{k=0}^{6} \sum_{i=0}^{k} a_{12-2k,2i} X^{2i} z^{12-2k}$$

$$= z^{12} + (a_{10,0} + a_{10,2} X^2) z^{10} + (a_{8,0} + a_{8,2} X^2 + a_{8,4} X^4) z^8$$
$$+ (a_{6,0} + a_{6,2} X^2 + a_{6,4} X^4 + a_{6,6} X^6) z^6$$
$$+ (a_{4,0} + a_{4,2} X^2 + a_{4,4} X^4 + a_{4,6} X^6 + a_{4,8} X^8) z^4 \tag{3.9}$$
$$+ (a_{2,0} + a_{2,2} X^2 + a_{2,4} X^4 + a_{2,6} X^6 + a_{2,8} X^8 + a_{2,10} X^{10}) z^2$$
$$+ a_{0,0} + a_{0,2} X^2 + a_{0,4} X^4 + a_{0,6} X^6 + a_{0,8} X^8 + a_{0,10} X^{10} + a_{0,12} X^{12}.$$

Substituting equation (3.9) into equation (2.4), we get

$$a_{0,0} = \frac{878826025 h_1^6 h_5^6}{9(b-m)^{12}}, \quad a_{0,2} = \frac{159786550 h_1^5 h_5^6}{3(b-m)^{11}}, \quad a_{0,4} = -\frac{5187875 h_1^4 h_5^6}{3(b-m)^{10}},$$

$$a_{0,6} = \frac{75460 h_1^3 h_5^6}{3(b-m)^9}, \quad a_{0,8} = \frac{735 h_1^2 h_5^6}{(b-m)^8}, \quad a_{0,10} = \frac{98 h_1 h_5^6}{(b-m)^7},$$

$$a_{0,12} = \frac{h_5^6}{(b-m)^6}, \quad a_{2,0} = -\frac{300896750 h_1^5 h_5^5}{3(b-m)^{10}}, \quad a_{2,2} = -\frac{565950 h_1^4 h_5^5}{(b-m)^9},$$

$$a_{2,4} = -\frac{220500 h_1^3 h_5^5}{(b-m)^8}, \quad a_{2,6} = -\frac{18620 h_1^2 h_5^5}{(b-m)^7}, \quad a_{2,8} = -\frac{690 h_1 h_5^5}{(b-m)^6},$$

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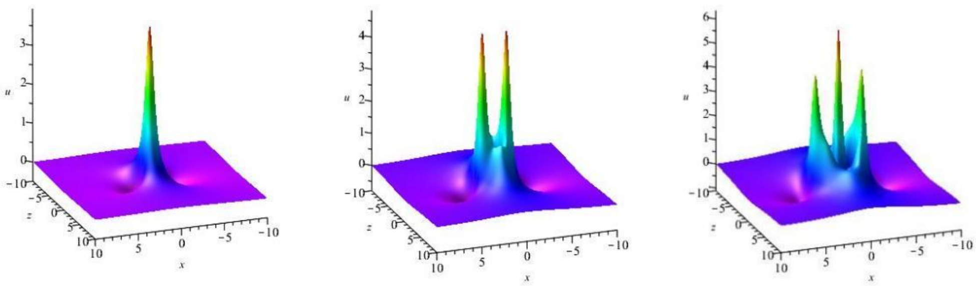
$$a_{2,10}=-\frac{6h_5^5}{(b-m)^5},\quad a_{4,0}=\frac{16391725h_1^4h_5^4}{3(b-m)^8},\quad a_{4,2}=-\frac{14700h_1^3h_5^4}{(b-m)^7},$$
$$a_{4,4}=\frac{37450h_1^2h_5^4}{(b-m)^6},\quad a_{4,6}=\frac{1540h_1h_5^4}{(b-m)^5},\quad a_{4,8}=\frac{15h_5^4}{(b-m)^4},$$
$$a_{6,0}=-\frac{798980h_1^3h_5^3}{3(b-m)^6},\quad a_{6,2}=-\frac{35420h_1^2h_5^3}{(b-m)^5},\quad a_{6,4}=-\frac{1460h_1h_5^3}{(b-m)^4},$$
$$a_{6,6}=-\frac{20h_5^3}{(b-m)^3},\quad a_{8,0}=\frac{4335h_1^2h_5^2}{(b-m)^4},\quad a_{8,2}=\frac{570h_1h_5^2}{(b-m)^3},$$
$$a_{8,4}=\frac{15h_5^2}{(b-m)^2},\quad a_{10,0}=-\frac{58h_1h_5}{(b-m)^2},\quad a_{10,2}=-\frac{6h_5}{b-m}. \tag{3.10}$$

Then we get the third order rogue wave solution of the gKP equation (1.1) as

$$u_{3r}=2(\ln f_{3r})_{XX}, \tag{3.11}$$

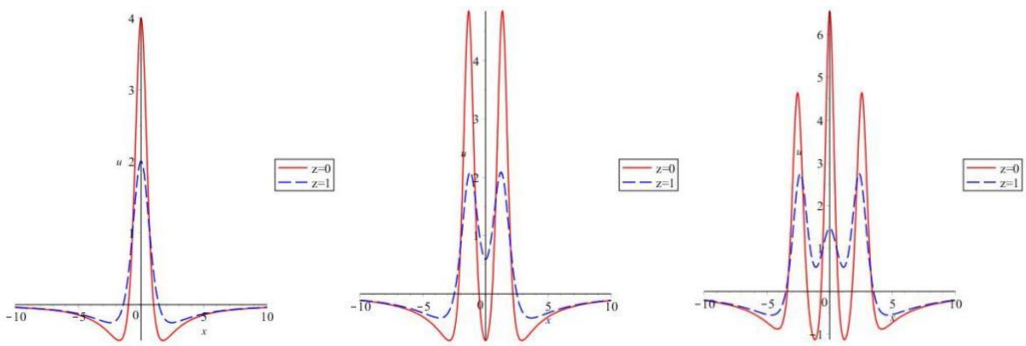
Where f_{3r} is derived from equation (3.9) with parameters in equation (3.10).



(a) u_{1r} (b) u_{2r} (c) u_{3r}

Figure 1: The rogue wave solutions u_{1r}, u_{2r}, u_{3r} *of the gKP equation (1.1) with*

$a=1, b=7, h_1=1, h_3=2, h_4=1, h_5=-3, h_6=1.$



(a) u_{1r} *(b)* u_{2r} *(c)* u_{3r}

Figure 2: The cross-sectional views of Fig.1 respectively.

Fig.1 shows the rogue wave solutions u_{1r}, u_{2r}, u_{3r} of the gKP equation (1.1). Fig.2 are the cross-sectional views of Fig.1 respectively. Fig.1 (a) shows that the first order rogue wave solution u_{1r} has one maximum 4 at (0,0) and one minimum $-1/2$ at $(-\sqrt{3},0)$ and $(\sqrt{3},0)$, in addition the maximum and the minimum values of the first order rogue wave solution lie on the same line $z=0$.

4. The Generalised Rogue Wave Solutions of the Gkp Equation

In this section, we assume that the function f has the following form [32, 35]

$$f = F_{n+1}(z,X) + 2\alpha X P_n(z,X) + 2\beta z Q_n(z,X) + (\alpha^2+\beta^2)F_{n-1}(z,X), \tag{4.1}$$

Where

$$\begin{aligned} F_n(z,X) &= \sum_{k=0}^{n(n+1)/2}\sum_{i=0}^{k} a_{n(n+1)-2k,2i} X^{2i} z^{n(n+1)-2k}, \\ P_n(z,X) &= \sum_{k=0}^{n(n+1)/2}\sum_{i=0}^{k} b_{n(n+1)-2k,2i} X^{2i} z^{n(n+1)-2k}, \\ Q_n(z,X) &= \sum_{k=0}^{n(n+1)/2}\sum_{i=0}^{k} c_{n(n+1)-2k,2i} X^{2i} z^{n(n+1)-2k}, \end{aligned} \tag{4.2}$$

$F_0 = 1, F_{-1} = P_0 = Q_0 = 0,$ Where $a_{m,l}, b_{m,l}, c_{m,l}$ $(m,l = 0,2,4,6,\cdots,n(n+1))$ and α, β are real parameters.

Substituting equation (4.1) into (2.4), we get the generalised rogue wave solutions of the gKP equation (1.1).

Case 1: $n = 0$

We choose

$$f = F_1(z,X) = a_{2,0}z^2 + a_{0,0} + a_{0,2}X^2. \tag{4.3}$$

In this case, the first order generalised rogue wave u_{1gr} is the same as the first order rogue wave u_{1r} in equation (3.4).

Case 2: $n = 1$

$$\begin{aligned} f &= F_2(z,X) + 2\alpha X P_1(z,X) + 2\beta z Q_1(z,X) + (\alpha^2+\beta^2)F_0 \\ &= z^6 + (a_{4,0} + a_{4,2}X^2)z^4 + (a_{2,0} + a_{2,2}X^2 + a_{2,4}X^4)z^2 + (a_{0,0} + a_{0,2}X^2 + a_{0,4}X^4 + a_{0,6}X^6) \\ &\quad + 2\alpha X(b_{0,0} + b_{0,2}X^2 + b_{2,0}z^2) + 2\beta z(c_{0,0} + c_{0,2}X^2 + c_{2,0}z^2) + \alpha^2 + \beta^2. \end{aligned} \tag{4.4}$$

Substituting equation (4.4) into equation (2.4), we get

$$\begin{aligned} a_{0,0} &= -\frac{9(b-m)^6(\alpha^2+\beta^2)h_5^3 + 9(b-m)^9\alpha^2 b_{0,2}^2 - (b-m)^8\beta^2 h_5 c_{0,2}^2 + 16875 h_1^3 h_5^6}{9(b-m)^6 h_5^3}, \\ a_{0,2} &= \frac{125 h_1^2 h_5^3}{(b-m)^5}, \quad a_{0,4} = -\frac{25 h_1 h_5^3}{(b-m)^4}, \quad a_{0,6} = -\frac{h_5^3}{(b-m)^3}, \\ a_{2,0} &= \frac{475 h_1^2 h_5^2}{(b-m)^4}, \quad a_{2,2} = \frac{90 h_1 h_5^2}{(b-m)^3}, \quad a_{2,4} = \frac{3h_5^2}{(b-m)^2}, \\ a_{4,0} &= -\frac{17 h_1 h_5}{(b-m)^2}, \quad a_{4,2} = -\frac{3h_5}{b-m}, \quad b_{0,0} = -\frac{b_{0,2}h_1}{b-m}, \\ b_{2,0} &= -\frac{3b_{0,2}(b-m)}{h_5}, \quad c_{0,0} = \frac{5c_{0,2}h_1}{3(b-m)}, \quad c_{2,0} = \frac{c_{0,2}(b-m)}{3h_5}, \\ b_{0,2} &= b_{0,2}, \quad c_{0,2} = c_{0,2}, \end{aligned} \tag{4.5}$$

Where $b_{0,2}, c_{0,2}$ are arbitrary constants? Then we get the second order generalised rogue wave solution of the gKP equation (1.1) as

$$u_{2gr} = 2(\ln f_{2gr})_{XX}, \tag{4.6}$$

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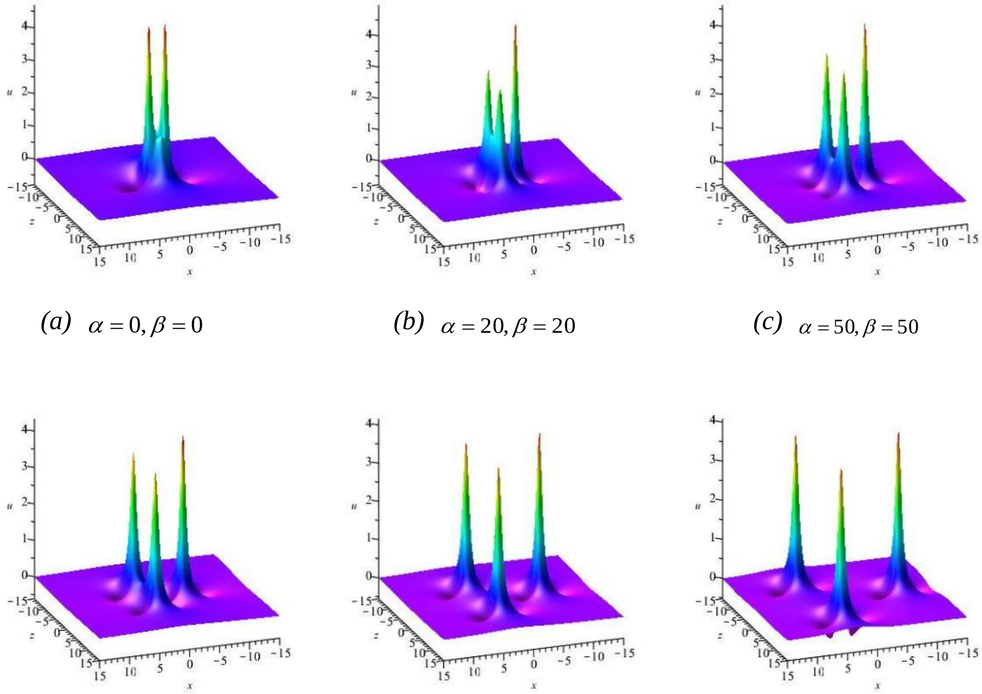
Where f_{2gr} is derived from equation (4.4) with parameters in equation (4.5).

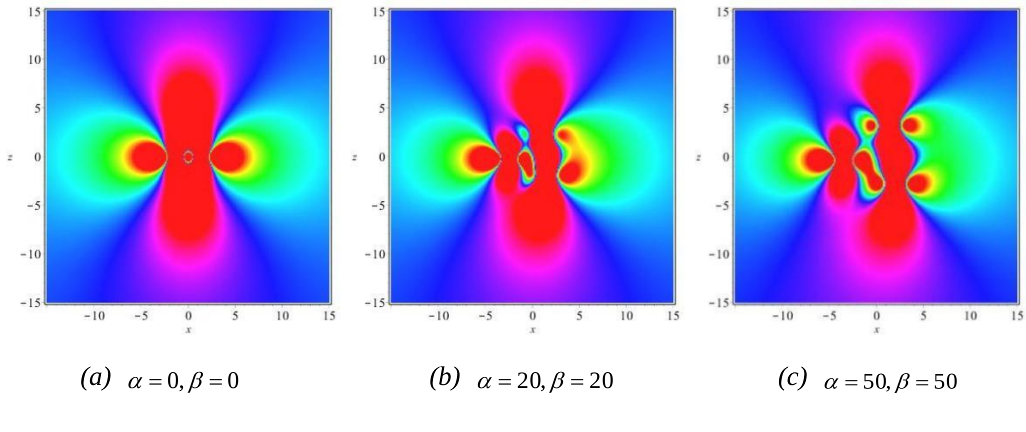
Fig.3 shows the second order generalised rogue wave solution u_{2gr} of the gKP equation (1.1) with different values of the parameters α and β. And Fig.4 is the density figures of Fig.3 respectively. Fig.3 (a) shows that the second order generalised rogue wave solution u_{2gr} is the second order rogue wave solution u_{2r} in Fig.1 (b) when $\alpha = \beta = 0$. With the increase of parameters α and β, the second order generalised rogue wave solution is decomposed into three first order rogue wave solutions and form into a triangle.



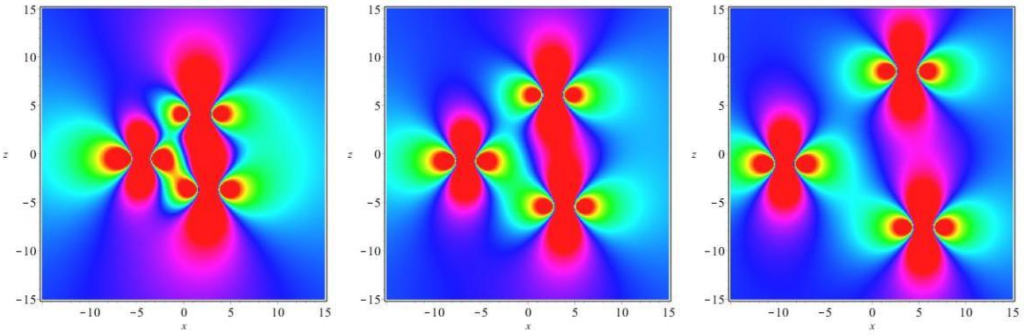
(a) $\alpha = 0, \beta = 0$ (b) $\alpha = 20, \beta = 20$ (c) $\alpha = 50, \beta = 50$

(d) $\alpha = 100, \beta = 100$ (e) $\alpha = 300, \beta = 300$ (f) $\alpha = 800, \beta = 800$

Figure 3: The second order generalised rogue wave solution u_{2gr} *of the gKP equation (1.1) with* $a=1, b=7, h_1=1, h_3=2, h_4=1, h_5=-3, h_6=1, b_{0,2}=c_{0,2}=1,$ *for different values of the parameters* α *and* β.



(a) $\alpha = 0, \beta = 0$ (b) $\alpha = 20, \beta = 20$ (c) $\alpha = 50, \beta = 50$



(d) $\alpha=100, \beta=100$ (e) $\alpha=300, \beta=300$ (f) $\alpha=800, \beta=800$

Figure 4: The density figures of Fig.3 respectively.

Case 3: $n=2$

$$
\begin{aligned}
f &= F_3(z,X)+2\alpha X P_2(z,X)+2\beta z Q_2(z,X)+(\alpha^2+\beta^2)F_1 \\
&= z^{12}+(a_{10,0}+a_{10,2}X^2)z^{10}+(a_{8,0}+a_{8,2}X^2+a_{8,4}X^4)z^8+(a_{6,0}+a_{6,2}X^2+a_{6,4}X^4+a_{6,6}X^6)z^6 \\
&\quad +(a_{4,0}+a_{4,2}X^2+a_{4,4}X^4+a_{4,6}X^6+a_{4,8}X^8)z^4 \\
&+(a_{2,0}+a_{2,2}X^2+a_{2,4}X^4+a_{2,6}X^6+a_{2,8}X^8+a_{2,10}X^{10})z^2 \\
&+a_{0,0}+a_{0,2}X^2+a_{0,4}X^4+a_{0,6}X^6+a_{0,8}X^8+a_{0,10}X^{10}+a_{0,12}X^{12} \\
&+2\alpha X(b_{6,0}z^6+(b_{4,0}+b_{4,2}X^2)z^4+(b_{2,0}+b_{2,2}X^2+b_{2,4}X^4)z^2+b_{0,0}+b_{0,2}X^2+b_{0,4}X^4+b_{0,6}X^6) \\
&+2\beta z(c_{6,0}z^6+(c_{4,0}+c_{4,2}X^2)z^4+(c_{2,0}+c_{2,2}X^2+c_{2,4}X^4)z^2+c_{0,0}+c_{0,2}X^2+c_{0,4}X^4+c_{0,6}X^6) \\
&+(\alpha^2+\beta^2)F_1.
\end{aligned} \tag{4.7}
$$

where

$$F_1 = z^2-\frac{h_5}{b-m}X^2-\frac{3h_1h_5}{(b-m)^2}. \tag{4.8}$$

Substituting equation (4.7) into equation (2.4), we get

$$a_{0,0}=\frac{675\alpha^2h_1b_{0,6}^2(b-m)^{17}-27\beta^2h_1h_5c_{0,6}^2(b-m)^{16}+675h_1h_5^7(\alpha^2+\beta^2)(b-m)^{10}+21970650625h_1^6h_5^{12}}{225(b-m)^{12}h_5^6},$$

$$a_{0,2}=\frac{75\alpha^2b_{0,6}^2(b-m)^{17}-3\beta^2h_5c_{0,6}^2(b-m)^{16}+75h_5^7(\alpha^2+\beta^2)(b-m)^{10}+3994663750h_1^5h_5^{12}}{75(b-m)^{11}h_5^6},$$

$$a_{0,4}=-\frac{5187875h_1^4h_5^6}{3(b-m)^{10}},\quad a_{0,6}=\frac{75460h_1^3h_5^6}{3(b-m)^9},\quad a_{0,8}=\frac{735h_1^2h_5^6}{(b-m)^8},$$

$$a_{0,10}=\frac{98h_1h_5^6}{(b-m)^7},\quad a_{0,12}=\frac{h_5^6}{(b-m)^6},$$

$$a_{2,0}=\frac{-75\alpha^2b_{0,6}^2(b-m)^7+3\beta^2h_5c_{0,6}^2(b-m)^{16}-h_5^7(\alpha^2+\beta^2)(b-m)^{10}-7522418750h_1^5h_5^{12}}{75(b-m)^{10}h_5^7},$$

$$a_{2,2}=-\frac{565950h_1^4h_5^5}{(b-m)^9},\quad a_{2,4}=-\frac{220500h_1^3h_5^5}{(b-m)^8},\quad a_{2,6}=-\frac{18620h_1^2h_5^5}{(b-m)^7},$$

$$a_{2,8}=-\frac{690h_1h_5^5}{(b-m)^6},\quad a_{2,10}=-\frac{6h_5^5}{(b-m)^5},\quad a_{4,0}=\frac{16391725h_1^4h_5^4}{3(b-m)^8},$$

$$a_{4,2}=-\frac{14700h_1^3h_5^4}{(b-m)^7},\quad a_{4,4}=\frac{37450h_1^2h_5^4}{(b-m)^6},\quad a_{4,6}=\frac{1540h_1h_5^4}{(b-m)^5},$$

$$a_{4,8}=\frac{15h_5^4}{(b-m)^4},\quad a_{6,0}=-\frac{798980h_1^3h_5^3}{3(b-m)^6},\quad a_{6,2}=-\frac{35420h_1^2h_5^3}{(b-m)^5},$$

$$a_{6,4}=-\frac{1460h_1h_5^3}{(b-m)^4},\quad a_{6,6}=-\frac{20h_5^3}{(b-m)^3},\quad a_{8,0}=\frac{4335h_1^2h_5^2}{(b-m)^4},$$

$$a_{8,2}=\frac{570h_1h_5^2}{(b-m)^3},\quad a_{8,4}=\frac{15h_5^2}{(b-m)^2},\quad a_{10,0}=-\frac{58h_1h_5}{(b-m)^2},$$

$$a_{10,2}=-\frac{6h_5}{b-m},\quad b_{0,0}=\frac{12005b_{0,6}h_1^3}{3(b-m)^3},\quad b_{0,2}=-\frac{245b_{0,6}h_1^2}{(b-m)^2},\tag{4.9}$$

$$b_{0,4}=\frac{13b_{0,6}h_1}{b-m},\quad b_{2,0}=-\frac{535b_{0,6}h_1^2}{h_5(b-m)},\quad b_{2,2}=\frac{230b_{0,6}h_1}{h_5},$$

$$b_{2,4}=\frac{9b_{0,6}(b-m)}{h_5},\quad b_{4,0}=\frac{45h_1b_{0,6}(b-m)}{h_5^2},\quad b_{4,2}=-\frac{5b_{0,6}h_1(b-m)^2}{h_5^2},$$

$$b_{6,0}=-\frac{5b_{0,6}(b-m)^3}{h_5^3},\quad c_{0,0}=\frac{3773c_{0,6}h_1^3}{3(b-m)^3},\quad c_{0,2}=-\frac{133c_{0,6}h_1^2}{(b-m)^2},$$

$$c_{0,4}=\frac{21h_1c_{0,6}}{b-m},\quad c_{2,0}=\frac{49c_{0,6}h_1^2}{h_5(b-m)},\quad c_{2,2}=\frac{38c_{0,6}h_1}{h_5},$$

$$c_{2,4}=\frac{c_{0,6}(b-m)}{h_5},\quad c_{4,0}=-\frac{7c_{0,6}h_1(b-m)}{5h_5^2},\quad c_{4,2}=-\frac{9c_{0,6}(b-m)^2}{5h_5^2},$$

$$c_{6,0}=-\frac{c_{0,6}(b-m)^3}{5h_5^3},\quad b_{0,6}=b_{0,6},\quad c_{0,6}=c_{0,6}.$$

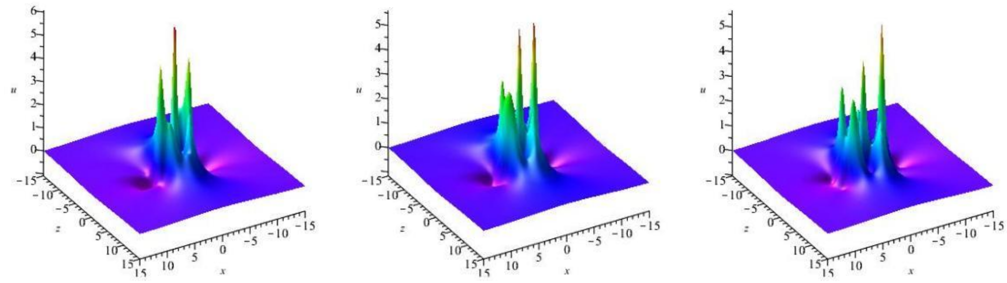
Where $b_{0,6}$ and $c_{0,6}$ are arbitrary constants. Then we get the third order generalised rogue wave solution of the gKP equation (1.1) as

$$u_{3gr}=2(\ln f_{3gr})_{XX},\tag{4.10}$$

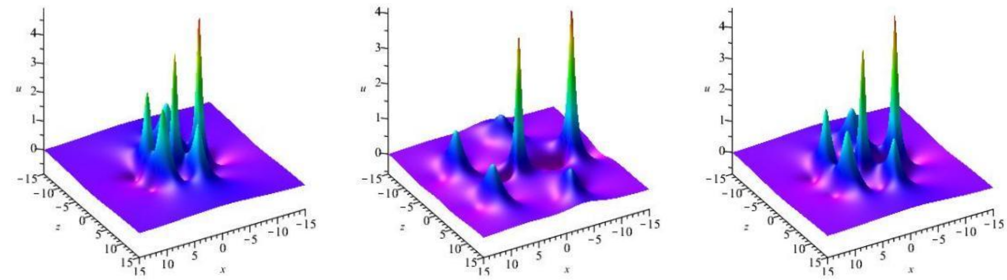
Where f_{3gr} is derived from equation (4.7) with parameters in equation (4.9).

Fig.5 shows the third order generalised rogue wave solution u_{3gr} of the gKP equation (1.1) with different values of the parameters α and β. And Fig.6 is the density figures of Fig.5 respectively. Fig.5 (a) shows that the third order generalised rogue wave solution u_{3gr} is the third order rogue wave solution u_{gr} in Fig.1 (c) when $\alpha=\beta=0$. With the increase of parameters α and β, the third order generalised rogue wave solution is decomposed into six first order rogue wave solutions, one of which is in the center, and the other five form a pentagon around it.

ISSN 2706-6827 Vol. 3, Issue 13: 95-105, DOI: 10.25236/IJFS.2021.031315



(a) $\alpha = 0, \beta = 0$ *(b)* $\alpha = 300, \beta = 300$ *(c)* $\alpha = 1000, \beta = 1000$

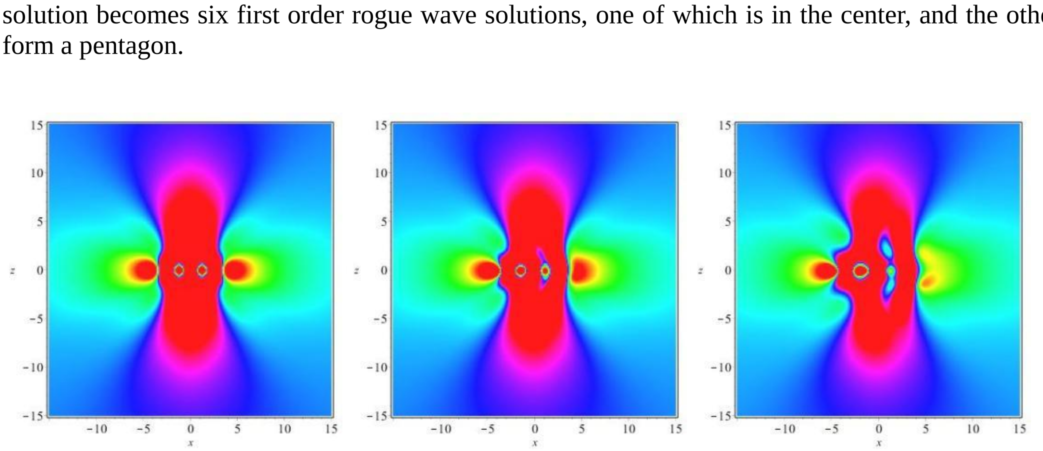


(d) $\alpha = 3000, \beta = 3000$ *(e)* $\alpha = 10000, \beta = 10000$ *(f)* $\alpha = 100000, \beta = 100000$

Figure 5: The third order generalised rogue wave solution u_{3gr} *of the gKP equation (1.1) with* $a=1, b=7, h_1=1, h_3=2, h_4=1, h_5=-3, h_6=1, b_{0,2}=1, c_{0,2}=1,$ *for different values of the parameters* α *and* β.

5. Conclusions

In this paper, we have investigated the rogue wave solutions of the gKP equation (1.1) based on symbolic computation approach. By choosing proper polynomial function, the first, the second and the third order rogue wave solutions are systematically obtained. We show that the maximum and the minimum values of the rogue wave solutions lie on the same line $z=0$. Further more, we add two parameters α and β to the polynomial function, then we get the first three order generalised rogue wave solutions of the gKP equation (1.1). With the increase of parameters α and β, the second order generalised rogue wave solution is decomposed into three first order rogue wave solutions and form into a triangle. When the parameters α and β are large enough, the third order generalised rogue wave solution becomes six first order rogue wave solutions, one of which is in the center, and the other five form a pentagon.



(a) $\alpha = 0, \beta = 0$ *(b)* $\alpha = 300, \beta = 300$ *(c)* $\alpha = 1000, \beta = 1000$

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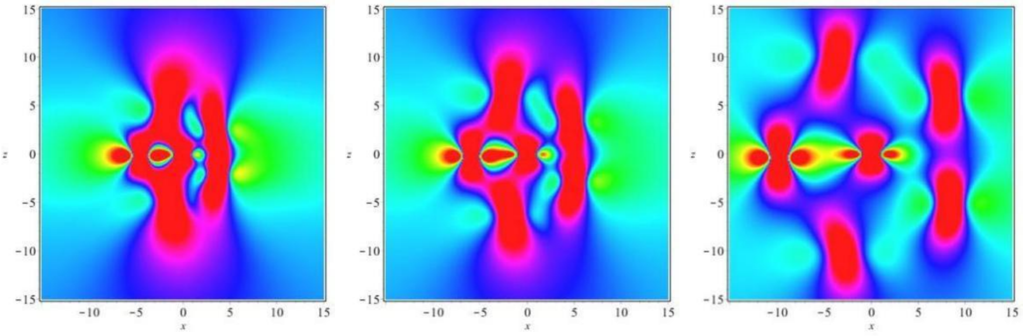

(d) $\alpha=3000, \beta=3000$ *(e)* $\alpha=10000, \beta=10000$ *(f)* $\alpha=100000, \beta=100000$

Figure 6: The density figures of Fig.5 respectively.

References

[1] E. Pelinovsky and C. Kharif(Editors), "Extreme Ocean Waves," Second Edition, Springer (2016).
[2] R. Grimshaw, E. Pelinovsky, T. Taipova, A. Sergeeva, Rogue internal waves in the ocean: long wave model, Eur. Phys. J. Spec. Top. 185 (2010) 195-208.
[3] D.R. Solli, C. Ropers, P. Koonath, B. Jalali, Optical rogue waves, Nature 450 (2007) 1045-1057.
[4] B. Kibler, J. Fatome, C. Finot, et al, The Peregrine soliton in nonlinear fibre optics, Nat. Phys. 6(10) (2010) 790-795.
[5] D.W. Zuo, Y.T. Gao, L. Xue, Y.J. Feng, Y.H. Sun, Rogue waves for the generalized nonlinear SchrÖingder-Maxwell-Bloch system in optical-fiber communication, Appl. Math. Lett. 40 (2015) 78-83.
[6] P.S. Vinayagam, R. Radha, K. Porszian, Taming rogue waves in Vector BECs, Phys. Rev. A 88 (2013) 042906.
[7] L.C. Zhao, Dynamics of nonautonomous rogue waves in Bose-Einstein condensate, Ann. Phys. 329 (2013) 73-79.
[8] J.S. He, E.G. Charalampidis, P.G. Kevrekidis, D.J. Frantzeskakis, Rogue waves in nonlinear SchrÖdinger models with variable coefficients: application to Bose-Einstein condensates, Phys. Lett. A 378(5-6) (2014) 577-583.
[9] Z.Y. Yan, Financial rogue waves, Commun. Theor. Phys. 54(5) (2010) 947-949.
[10] Z.Y. Yan, Vector financial rogue waves, Phys. Lett. A 375(48) (2011) 4274-4279.
[11] N. Akhmediv, A. Ankiewicz, M. Taki, Waves that appear from nowhere and disappear without a trace, Phys. Lett. A 373 (2009) 675-678.
[12] N. Akhmediev, J.M. Soto-Crespo and A. Ankiewicz, Extreme waves that appear from nowhere: on the nature of rogue waves, Phys. Lett. A 373 (2009) 2137-2145.
[13] A. Ankiewicz, D.J. Kedziora, N. Akhmediev, Controllable rogue waves in the nonautonomous nonlinear system with a linear potential, Phys. Lett. A 375 (2011) 2782.
[14] D.H. Peregrine, Water waves nonlinear SchrÖding equations and their solutions, Math. Soc. B, App. Math. 25 (1983) 16-43.
[15] S.W. Xu, J.S. He, The rogue wave and breather solution of the Gerdjikov-Ivanov equation, J. Math. Phys. 53 (2012) 063507.
[16] L.H. Wang, K. Porsezian, J.S. He, Breather and rogue wave solutions of a generalized nonlinear SchrÖdinger equation, Phys. Rev. E 87 (2013) 053202.
[17] Zhaqilao, On Nth-order rogue wave solution to the generalized nonlinear SchrÖdinger equation, Phys. Lett. A 377(12) (2013) 855-859.
[18] A. Ankiewicz, J.M. Soto-Crespo, N. Akhmediev, Rogue waves and rational solutions of the Hirota equation, Phys. Rev. E 81 (2010) 046602.
[19] J.S. He, S.W. Xu, K. Porsezian, Rogue waves of the Fokas-Lenells equation, J. Phys. Soc. Jpn. 81 (2012) 124007.
[20] B.L. Guo, L.M. Ling, Rogue wave, breathers and Bright-Dark-Rogue solutions for the Coupled SchrÖdinger equations, Chin. Phys. Lett. 28(11) (2011) 4-7.
[21] W.Q. Peng, S.F. Tian, T.T. Zhang, Dynamics of breather waves and higher-order rogue waves in a coupled nonlinear SchrÖdinger equation. EPL, 123 (2018) 50005.
[22] L.L. Feng, S.F. Tian, X.B. Wang, T.T. Zhang, Rogue waves, homoclinic breather waves and soliton waves for the (2+1)-dimensional B-type Kadomtsev-Petviashvili equation, Appl. Math. Lett. 65 (2017) 90-97.

ISSN 2706-6827 Vol. 3, Issue 13: 95-105, DOI: 10.25236/IJFS.2021.031315

[23] W.Q. Peng, S.F. Tian, T.T. Zhang, Analysis on lump, lumpoff and rogue waves with predictability to the (2+1)-dimensional B-type Kadomtsev-Petviashvili equation, Phys. Lett. A 241 (2018) 1-8.

[24] C.Y. Qin, S.F. Tian, X.B. Wang, T.T. Zhang, J. Li, Rogue waves, bright-dark solitons and traveling wave solutions of the (3 + 1)-dimensional generalized Kadomtsev-Petviashvili equation, Comput. Math. Appl. 75 (2018) 4221–4231.

[25] M.J. Ablowitz, H. Segur, Solitons and the Inverse Scattering Transform. SIAM, USA: Philadephia, 1981.

[26] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambrige: Cambrige Univ. Press 1991.

[27] R. Hirota, Direct methods in soliton theory. Springer-Verlag Berlin Heideberg, 1980.

[28] V.B. Matveev, M.A. Salli, Darboux Transformations and Solitons, Springer Series in Nonlinear Dynamics. Berlin: Sprige Press 1991.

[29] C. Rogers, W.F. Shadwick, B$\ddot{\rm a}$cklund transformations and their applications. Academic Press, New York-London 1982.

[30] S.Y. Lou, L.L. Chen, Formally variable separation approach for nonintegrable models, J. Math. Phys. 40 (1999) 6491-6500.

[31] G.W. Bluman, S. Kumei, Symmetries and Differential Equations, New York: Springer Verlag, 1989.

[32] P.A. Clarkson, E. Dowie, Rational solutions of the Boussinesq equation and applications to rogue waves, Trans. Math. Appl. 1 (2017) 1-26.

[33] W.X. Ma, Lump solutions to the Kadomtsev-Petviashhvili equation, Phys. Lett. A 379(36) (2015) 1975-1978.

[34] J.J. Guo, J.S. He, M.H. Li, D. Mihalache, Multiple-order line rogue wave solutions of extended Kadomtsev-Petviashhvili equation, Math. Comput. Simulat. 180 (2021) 251-257.

[35] Zhaqilao, A symbolic computation approach to constructing rogue waves with a controllable center in the nonlinear systems, Comput. Math. Appl. 75(9) (2018) 3331-3342.

[36] B.Q. Li, Y.L. Ma, Multiple-lump waves for a (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation arising from incompressible fluid, Comput. Math. Appl. 76 (2018) 204-214.

[37] Z.L. Zhao, L.C. He, Multiple lump solutions of the (3+1)-dimensional potential Yu-Toda-Sasa-Fukuyama equation, Appl. Math. Lett. 95 (2019) 114-121.

[38] W.H. Liu, Y.F. Zhang, Multiple rogue wave solutions of the (3+1)-dimensional Kadomtsev-Petviashhvili-Boussinesq equation, Z. Angew. Math. Phys. 70 (2019) 112.

[39] S.F. Tian, P.L. Ma, On the Quasi-Periodic wave solutions and asymptotic ananlysis to a (3+1)-dimensional generalized Kadomtsev-Petviashhvili equation, Commun. Theor. Phys. 62 (2014) 245-258.