

Topology Optimization of a Chain Drive's Sprocket of Aprilia RS 125 Sport Bike

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ABSTRACT: *In order to reach the aim of lightweight designing of the sprocket of Aprilia RS 125 sport bike, the topology optimization methodology is introduced to optimize the sprocket. Firstly, the chain transmission part has been optimized and replaced with a new chain, the sprockets are redesigned based on the new chain. Then the sprocket model is created in creo. Finally, the sprocket is optimized twice in Ansys Workbench and the final sprocket model's total deformation and equivalent stress are checked, the values are all in the safe range. The results show that the optimized sprocket meets the working requirement. The topology optimization design enables sprocket reducing 27% of its mass. The lightweight design has been finished successfully.*

KEYWORDS: *Topology Optimization, Structural Optimization, Sprocket Design*

1 Introduction of the Topology Optimization Methodology and Its Applications

1.1 Introduction

Topology optimization is a new optimization method that optimizes parameters with multiple use conditions as targets. It can improve the real use efficiency of parts and more accurately reflect the design optimization process. Optimized design can be greatly improved and improve the performance of castings, forgings, and stampings, and reduce product weight. Through systematic and scientific design ideas and methods, engineering designers achieve the goals of improving product development efficiency, saving raw materials, reducing costs, and improving product quality.

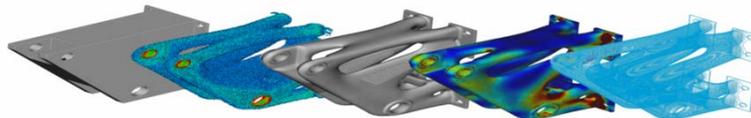


Figure 1. Structural bracket design space and topology optimization result [1]

1.2 Structural Optimization and Topology Optimization

Structural optimization has been greatly developed in recent decades and now the optimization design theory and its methods has been studied deeply and widely used in structural design. The so-called optimization design is based on the specific actual problems, a mathematical model for optimal design is established. And a certain optimization design is use to find out a design that satisfied the constraints and optimizes the objective functions.

According to the design condition of the optimization problems, nowadays there are four major area of structural optimization technology: (1) Size optimization, (2) Shape optimization, (3) Topology optimization, (4) Structural type optimization.

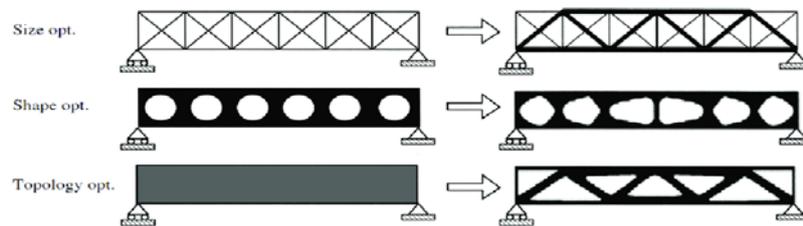


Figure 2. Comparative illustration of size, shape and topology optimization[2]

(1)Size optimization: From the known conditions, we can determine the information of material, type, geometrical outlook and topology layout and so on. Under these circumstances, through the specific procedures of structural optimization to determine the sectional size parameters of the structure. In this way to reach the goal of minimizing the structure's cost, volume or weight. This is the size optimization, it is also the basic optimization method, and applied widely in structural design. During the process of size optimization, the topology shape and boundary shape will not change, only the specific size will change. There are many design variables, such as then moment inertia and cross-sectional area of the rod, the directional angle and layer thickness of new composite materials, the thickness, width, and length of the plate, etc.

(2) Shape optimization: It is part of the field of optimal control theory. The typical problem is to find the shape which is optimal in that it minimizes a certain cost functional while satisfying given constraints. In many cases, the functional being solved depends on the solution of a given partial differential equation defined on the variable domain [3]. During the optimization process, the shape of internal and external boundary can be changed, but the number of boundaries cannot be changed. Which means that the topology relationship of design area is fixed.

(3)Topology optimization: Giving the property of material and design area, Through computer optimization program, the structure is optimized and calculated, and then the size and layout form of the structural part that can not only be the optimal

value of the objective function but also meet the constraint conditions are obtained, this is topology optimization and layout optimization.

This kind of structural optimization defines fewer initial constraints. Designers do not need to know the specific structural topology of a structure, but only need to propose the design domain of the structure. Topology optimization is an innovative structural design method, it can provide a novel and unique structural topology form. The main task of topology and layout optimization is to study how the structural members are connected to each other. The purpose is to make the structure play its proper role under the constraints of strength, rigidity, and stability.

Comparing to size optimization and shape optimization, topology optimization saves more materials, more design freedom and greater design space. It is the most promising methods for structural optimization in the future.

1.3 Topology Optimization Applications in Automobile Industry

With the rapid development of the automotive industry and increasingly prominent energy issues, the challenges and competitive environment facing the automotive industry are becoming more and more fierce, and new requirements have been placed on automotive products to reduce their manufacturing costs and improve fuel economy. Traditional size, shape optimization and other design methods seem more and more unable to improve the overall performance and quality of the product. Topology optimization technology completely changes the optimization deficiencies of traditional design. Through topology optimization, the weight of the whole vehicle can be reduced, the design cost can be reduced, the passenger space can be increased, and so on. Audi used to optimize the topological structure of the six-cylinder engine(V6) of the A8 model. The final structural quality was reduced by 20%, the energy consumption of the engine and transmission system was reduced by 40%, and the stable operation of the engine was guaranteed.

In military industry:

When military industrial enterprises design the flight jet or any aircraft, they also apply the topology optimization to their products in order to reduce the weight, get the best structures and improve its performance.

In the latest airframe design project of Sukhoi Design Bureau aired by the Russian Red Star TV station, the 3D printed model after the topology optimized design of the Su-57 stealth fighter to be installed by the Russian Air Force was first publicly exposed [4-6].

2 Lightweight Design of the Chain Transmission Part

2.1 Parameters Calculation Based on 520 Chain

From the measurement of the current drive and driven sprocket, the tooth number of drive(front) sprocket $Z_1=14$, the tooth number of driven(rear) sprocket $Z_2=40$, the number of chain links are 106 links. By checking scientific materials, the pitch for 520 chain is 15.875mm [7-9].

1). The drive sprocket (front)

•Pitch circle diameter:

$$D_{p1} = \frac{p}{\sin\left(\frac{180^\circ}{Z_1}\right)} = \frac{15.875}{\sin\left(\frac{180^\circ}{14}\right)} = 71.34\text{mm} \quad (1)$$

2). The driven sprocket (Rear)

•Pitch circle diameter:

$$D_{p2} = \frac{p}{\sin\left(\frac{180^\circ}{Z_2}\right)} = \frac{15.875}{\sin\left(\frac{180^\circ}{40}\right)} = 202.33\text{mm} \quad (2)$$

3). Calculation of center distance between two sprockets

•Center distance Z_1 equal to Z_2 :

$$a = \frac{X - z}{2} * p \quad (3)$$

•Center distance Z_1 not equal to Z_2 :

$$a = [2 * X - (Z_1 + Z_2)] * C * p = (2 * 106 - 54) * 0.24856 * 15.875 \\ = 623.45\text{mm}$$

X: number of links; C: Factor of center distance; p: Pitch.

2.2 Redesign the sprockets with 415 chain

The concept of Lightweight design originally established from motor sports. It is obvious that Lightweight design contains many advantages, lighter weight can bring better control and handling and the power output by the engine can produce better acceleration. Since the vehicle is light, the acceleration performance is better when starting, also it will have a shorter brake distance. According to aim of lightweight design of transmission part of Aprilia RS 125, a lighter 415 chain can be chosen according to this aim. Comparing to 520 chain, the 415 chain has a lighter weight, the weight per meter is 0.55kg, but weight per meter of 520 chain is 0.8kg.

According to introduction of technical parameters Aprilia RS 125 motor bike, the following data can be found:

- Torque: 19N.m
- The maximum rotational speed: 11000+-1000rpm,
- Maximum power: 21KW

According to the parameters of chain 415, these data can be found:

- Pitch(P)=12.7mm
- Roller diameter max $d_1=7.92\text{mm}$
- Width between inner plates $b_1=7.85\text{mm}$

In order to get best performance of the motor bike, a proper transmission ratio should be selected, according to calculation. The ratio should be set:2.61, which means that when the transmission ratio is 2.61, the motor bike has highest velocity.

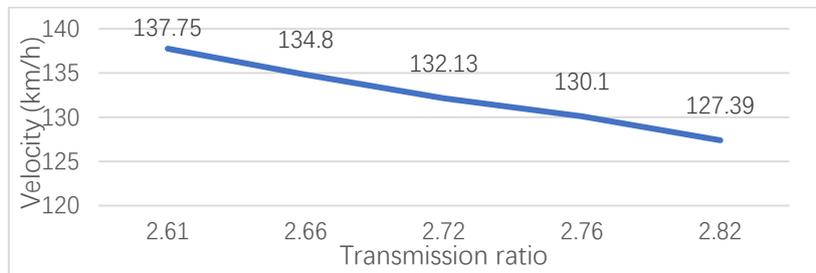


Figure 3. Velocity-Transmission ratio diagram

By checking scientific materials, mostly the sprocket teeth number for main drive(front) should ≥ 17 , so I choose 18.

As shown in Figure 4, D_p =pitch diameter, D_f =root diameter, D_e =tip diameter

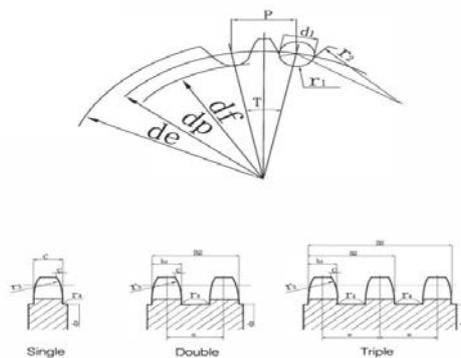


Figure 4. Tooth profile of sprocket

(1) The calculation of sprocket:

Table 1. Calculation formula of sprocket

Name	notation	unit	formula
Pitch circle diameter	D_p	mm	$D_p = \frac{p}{\sin(\frac{180^\circ}{Z})}$
Tip diameter	D_e	mm	$D_e = D_p + p * (1 - \frac{1.6}{Z}) - d_1$
Root diameter	D_f	mm	$D_{f1} = D_{p1} - d_1$

1) The drive sprocket(front):

•Pitch circle diameter:

$$D_{p1} = \frac{p}{\sin(\frac{180^\circ}{Z_1})} = \frac{12.7}{\sin(\frac{180^\circ}{18})} = 73.13\text{mm}$$

Z_1 : the teeth number of drive sprocket(front)

•Addendum circle diameter (Tip diameter):

$$D_{e1} = D_{p1} + p * \left(1 - \frac{1.6}{Z_1}\right) - d_1 = 73.13 + 12.7 * \left(1 - \frac{1.6}{18}\right) - 7.92 = 76.78\text{mm} \quad (4)$$

•Dedendum circle diameter (Root diameter):

$$D_{f1} = D_{p1} - d_1 = 73.13 - 7.92 = 65.21\text{mm} \quad (5)$$

$$\bullet \text{Tooth width: if } P \geq 12.5, b_{f1} = 0.93 * b_1 = 0.93 * 7.85 = 7.3\text{mm} \quad (6)$$

Transmission ratio: $i=2.61$

So that the driven sprocket (Rear) teeth number will be:

$$Z_2 = 18 * 2.61 = 46.98 \approx 47$$

2) The driven sprocket (Rear):

•Pitch circle diameter:

$$D_{p2} = \frac{p}{\sin(\frac{180^\circ}{Z_2})} = \frac{12.7}{\sin(\frac{180^\circ}{47})} = 190.14\text{mm}$$

•Addendum circle diameter (Tip diameter):

$$D_{e2} = D_{p2} + p * \left(1 - \frac{1.6}{Z2}\right) - d1 = 190.14 + 12.7 \left(1 - \frac{1.6}{47}\right) - 7.92 = 194.34\text{mm}$$

•Dedendum circle diameter (Root diameter):

$$D_{f2} = D_{p2} - d_1 = 190.14 - 7.92 = 182.22\text{mm}$$

•Tooth width: $b_{f2} = 0.93 * b1 = 0.93 * 7.85 = 7.3$

(2). The calculation of the power for sprocket of simplex chain:

From the scientific material, $K_a=1.0$, $K_s=1.5$, sprocket of simplex chain, then:

$$P_{cs} = K_a * K_s * P = 1.5 * 21 = 31.5\text{KW} \quad (7)$$

(3). Calculation of the links number and center distance:

Based on the calculation of 520 chain, the center distance can be determined and $a=623.45\text{mm}$.

Number of chain links:

$$X = 2 * \frac{a}{p} + \frac{Z1 + z2}{2} + \frac{A * p}{a} = 131.11 \rightarrow 132 \text{ links} \quad (8)$$

A: Compensating factor= $\left(\frac{Z2-Z1}{2*\pi}\right)^2$; a: center to center distance.

So, the actual chain length links number: 132 links

The Chain speed calculation can also be calculated:

$$V = \frac{Z * n * P}{60 * 1000} = \frac{11000 * 18 * 12.7}{60 * 1000} = 41.91\text{m/s} \quad (9)$$

2.3 Determination of the Load Acting on Chain 415

Chain drive is a meshing drive with intermediate flexible parts. The chain receives tension and dynamic load during transmission. Here it will focus on tension acting on chain drive, the dynamics load can be neglected.

The tension acting on chain drive:

Excluding various additional dynamics loads, the tight side tension F of the drive chain is composed of the effective circumferential force F_1 , the tension F_e caused by centrifugal force, and the tension F_f caused by the sag of the loose side. Each links of chain drive, when it is in loose edge position, its tension is $F_e + F_f$; when it is in tight side, its tension is $F_1 + F_e + F_f$.

(1) Effective circumferential force (F_1)

According to the Torque which needed,

$$F_1 = \frac{M_2}{r_1} = \frac{M_1}{r_2} \quad (10)$$

In this equation, M_1 , M_2 : drive, driven sprocket torque. (N/m); r_1 , r_2 : The pitch diameter of drive, driven sprocket. (m)

$$F_1 = \frac{M_1}{r_2} = \frac{19}{(0.19418/2)} = 195.69N$$

(2) Tension caused by centrifugal force (F_e)

All the chain links located on the sprocket are subjected to centrifugal force, and each chain rings passes the sprocket along the circular track of the radius r at the circumferential speed when rotating. For all the chain links surrounding the sprocket, the tension generated is:

$$F_e = q * v^2 \quad (11)$$

q : chain weight per meter (kg/m) , v : chain speed;

From scientific material, chain weight per meter of chain 415 is 0.55kg

$$F_e = 0.55 * 39.58 = 861.617 N$$

(3) Tension caused by loose edge sag (F_f):

$$F_f = g * \sqrt{\left(\frac{q * a^2}{8 * f} + \left(\frac{q * a}{2}\right)^2\right)} \approx 9.8 * \frac{q * a^2}{8 * f} = 9.8 * K_f * q * a \quad (12)$$

In this equation: q : chain weight per meter(kg/m); a : the center distance of chain. K_f : coefficient, it can be determined by the angle (β) between the center line of two sprocket and horizontal line.

Table 2. Value of coefficient K_f

β	0°(horizontal)	0–40°	40 – 90°	90°(vertical)
K_f	6-7	4	2	1

The angle is 0°, so $K_f=6$

$$\begin{aligned} F_f &= g * \sqrt{\left(\frac{q * a^2}{8 * f} + \left(\frac{q * a}{2}\right)^2\right)} \approx 9.8 * \frac{q * a^2}{8 * f} = 9.8 * K_f * q * a \\ &= 9.8 * 6 * 0.55 * 0.46 = 14.876 N \end{aligned}$$

3 Topology Optimization of the Sprockets to Reduce the Inertia

3.1 The Method and Process of Topology Optimization

Topology optimization is one of the three types of structural optimization. It is mainly used to study the optimal layout of material structures. It is widely used in the

lightweight design of product structures. For example, in the design of auto parts, which areas can be removed to reduce the quality, and the structures of the parts is safe and reliable in the process of transferring loads.

At present, structural topology optimization mainly includes variable thickness method, homogenization method, variable density method, etc. Among them, the variable density method is used more in the topology optimization design of parts. Changes between variables, where the material distribution is empty and defined as 0, and the distribution is completely defined as 1, which can be understood as the optimization of the combination of 0-1 discrete variables. The deletion of a unit set is determined according to a certain criterion. Optimization iteratively, keep the structural elements that has great use to the structure's transfer force path, and delete those that are not useful. In essence, the structural topology optimization problem based on the variable density method is a discrete optimization problem that includes unit increase and decrease. The corresponding mathematical model is:

$$\begin{aligned} & \text{Find } \rho = \{\rho_1, \rho_2, \dots, \rho_n\}^T \\ & \min C(\rho) = F^T U \\ & \text{s.t } V(\rho)/V_0 \leq f \\ & KU=F \\ & \rho_i = \{0,1\} (i = 1,2, \dots, n) \end{aligned}$$

In these equations, $C(\rho)$ is structural compliance, the objective function is the maximum stiffness of the structure; ρ_1 is design variable of the unit; F is the load matrix; U is displacement matrix; K is the overall stiffness matrix; $V(\rho)$ is the structural volume under the design variable state; V_0 is the initial volume of the structure; f is volume constraint parameters.

The process of topology optimization:

The sprocket is a disc-shaped structure with a certain thickness and teeth on the periphery. In the addition to the tooth profile parameters being the focus of the transmission function of design, the disc-shaped internal structure is also the focus of the lightweight design. The main steps of doing topology optimization is:

1. Define the topology optimization problem.
2. Select the unit type.
3. Specify the areas to be optimized and not optimized.
4. Define and control load conditions.
5. Define and control the optimization process.
6. View the results

3.2 Applying Topology Optimization to the Initial Model

3.2.1 The First Topology Optimization

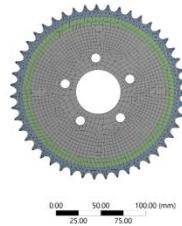


Figure 5. Meshing

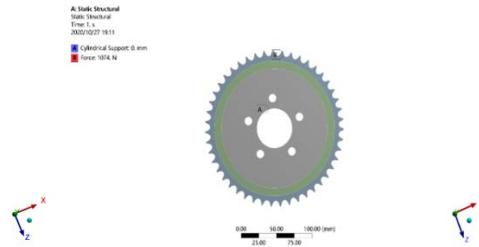


Figure 6. The initial FE model

First we browser the Creo stp.file into ANSYS workbench, open the model and select material, I select structural steel carbon 49 for sprocket. And then apply “mesh” on the geometry model, next we have to add loads and constraint on the sprocket. When I calculate the loads acting on the chain drive, the load on tight side is $F_1 + F_e + F_f = 1074\text{N}$, so adding 1074N on the teeth. Also, we insert a cylindrical support at the central hole. By calculating the initial finite element model, we can get the information that the Total deformation is 0.00039 mm, the equivalent stress is 9.9105 Mpa. Therefore, the material properties of the initial structure of the sprocket are not fully utilized.

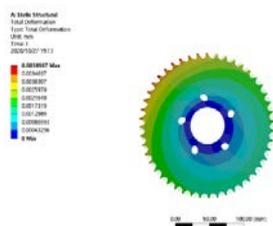


Figure 7. Total deformation of the initial model

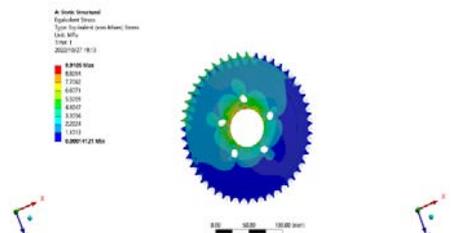


Figure 8. Equivalent stress of initial model

The initial model mass of the sprocket is 1.3869kg. Applying topology optimization to the sprocket according to the aim of lightweight, the result is showed in Figure 7. The red parts mean that these materials can be removed. Based on this, we can design and remove some materials to get a new structure of the sprocket which is showed in Figure 8. The mass of the optimized sprocket reduces to 1.166kg.



Figure 9. The result of first topology Optimization

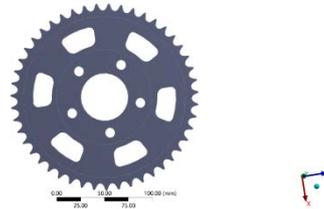


Figure 10. The geometric space after first optimization

Performing static analysis on the model after the first topology optimization. The constraints and loads applied are consistent with the initial analysis condition. In Total deformation, we can get that the maximum strain value is 0.0058957mm, in Equivalent stress, the maximum stress value is 13.653MPa. They are all in the safe range. Therefore, we can apply the second topology optimization.

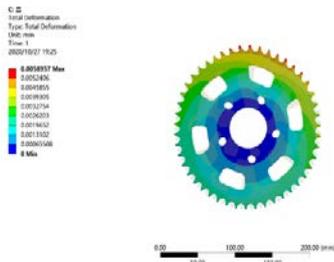


Figure 11. Total deformation of the first optimized model

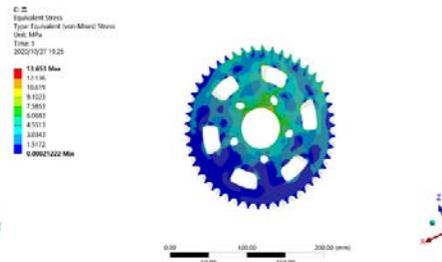


Figure 12. Equivalent stress of the first optimized model

3.3 The second topology optimization

In the second topology optimization, we apply the same process which mentioned before. In the Figure 13, the red regions mean that these parts obtain minimal impact during the motion. Thus, we can remove these materials and design a new structure which is showed in Figure 14. In this case, the mass of the sprocket reduces to 1.0121kg.

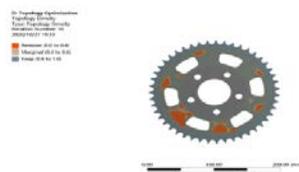


Figure 13. Second topology Optimization

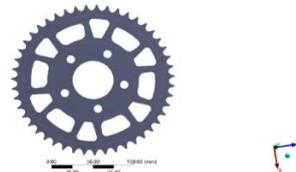


Figure 14. Geometric space after second topology optimization

Performing static analysis on the model after the second topology optimization. The constraints and loads applied are consistent with the initial analysis condition. In Total deformation, we can get that the maximum strain value is 0.0089796mm, in Equivalent stress, the maximum stress value is 20.07MPa. The values are still in the safe range.

Consequently, after first and second topology optimization, we can find the places that allow us to remove the materials, and based on that we can design the geometric space for the sprocket to reduce the mass and keep strength and stiffness. The initial mass of the sprocket is 1.3869kg, and after the second topology optimization, the mass reduces to 1.0121.

$$\text{Mass reduction percentage (\%)} = \frac{1.3869 - 1.0121}{1.3869} = 27\%$$

Therefore, topology optimization enable sprocket reducing 27% of its mass.

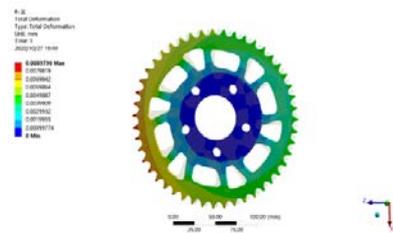


Figure 15. Total deformation of second optimized model

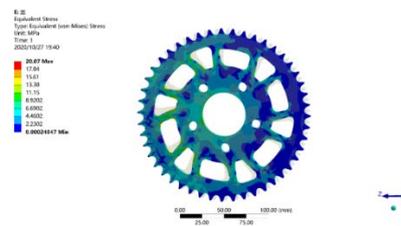


Figure 16. Equivalent stress of second optimized model

4 Conclusion

To make the lightweight design of the chain drive's sprocket of Aprilia RS 125 sport bike, the topology optimization method is introduced to the structural design of the sprocket. The topology optimization theoretical principles and its applications are introduced in the article.

Aprilia is an Italian motorcycle manufacturer and RS 125 sport bike is one of its products. All the necessary parameters are introduced in the article. Initially, this sport bike equipped with 520 chains. To make the lightweight design for its chain drive, a lighter chain 415 need to be used, which contains less weight. Therefore, a new sprocket should be designed in order to match the 415 chain. Based on designing conditions and giving data, the center distance between two sprockets, the tooth number of sprockets, the pitch diameter, root diameter and tip diameter for drive and driven sprocket, power for sprocket, links number for 415 chain can be calculated.

For the tension acting on a chain drive, excluding various additional dynamics loads, the tight side tension F of the drive chain is composed of the effective circumferential force F_1 , the tension F_e caused by centrifugal force, and the tension F_f caused by the sag of the loose side. These values are calculated.

The geometric model of new rear sprocket is made in Creo, the using of CAD design is very important for mechanical engineers. Applying two times topology optimization to reduce the mass of sprocket in ANSYS workbench. After giving the force and constraints for sprocket, generating topology optimization, the red part means these materials can be removed, and based on it, a new structure of sprocket should be designed. The final sprocket model's total deformation and equivalent stress are checked, the values are all in the safe range. The results show that the optimized sprocket meets the working requirement. The topology optimization design enables sprocket reducing 27% of its mass. The lightweight design has been finished successfully.

Topology optimization is a difficult but novel subject in the engineering field. The research on topology optimization design meets the current social and actual engineering development needs. It has important theoretical significance, academic and engineering application value. It will have broad application prospects in the future.

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