A Theoretical Justification for Using the International System of Units to Measure Navigational Speed

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Abstract. Even in countries where the International System of Units (SI, the modern metric system) has long been established, knots are used to measure navigational speed on the sea and in the air. This is surprising because a knot, which equals one nautical mile per hour, is not an SI unit of measurement. Both the International Bureau of Weights and Measures (the independent international agency charged with regulating SI) and the International Civil Aviation Organization (the United Nations agency charged with regulating international aviation) decry the use of non-SI units to measure navigational speed. So why are knots used instead of kilometers per hour, which is an SI unit? The reason is that the knot is based upon the size and shape of the Earth, and therefore this unit has natural appeal to navigators. Does a method exist to use kilometers per hour to measure navigational speed that is also based upon the size and shape of the Earth, and which would therefore provide equal appeal to navigators? Yes it does, provided that angular measurement is performed using gradians (also called gons) instead of degrees.

Key words: Aviation; Navigation; International System of Units (SI); Metric system; Degrees; Gradians; Gons; Measurement

1. INTRODUCTION

In 1795, during the upheaval of the French Revolution, the French National Assembly established the metric system to modernize the chaotic and archaic system of weights and measures that characterized the recently deposed Old Regime. In 1875, to promote international standardization of the metric system, the Conference of the Meter was held in Paris, which 17 nations attended, one of which was the United States. At this conference, the attendees signed the Meter Convention "to ensure worldwide unification of measurements." To carry out this task on an ongoing basis, the Meter Convention established a permanent International Bureau of Weights and Measures (BIPM, the widely used abbreviation following the French-language *Bureau International des Poids et Mesures*) [3]. In its most modern form, the metric system has come to be called the International System of Units, "universally abbreviated SI (from the French *Le Système International d'Unités*.)" According to the National Institute of Standards and Technology, an agency of the U.S. Department of Commerce, "long the dominant measurement system used in science, the SI is becoming the dominant measurement system used in international commerce" [15].

2. THE WORK OF THE BIPM: SI AND NON-SI UNITS

According to BIPM standards, the base unit of length is the meter and the base unit of time is the second. Note, incidentally, that the second is the only base unit not founded on a multiple or a submultiple of ten. Altogether, the BIPM lists seven base units. In addition to the meter and the second, these are the kilogram to measure mass, the ampere to measure electric current, the kelvin for thermodynamic temperature, the mole for the amount of substance, and the candela for luminous intensity [3].

Besides the base units, the BIPM permits the use of certain "derived units expressed in terms of base units," and therefore the SI prefix kilo may be used with the word meter to obtain kilometer for describing a substantial length. Similarly, to describe a passage of time so long that using seconds would be cumbersome, the BIPM defines the hour — which is "widely used . . . in matters of everyday life" — as a "non-SI unit accepted for use with the International System of Units." Finally, the BIPM allows a further derivation — kilometers per hour — to measure speed [3].

What about the knot? The BIPM defines the knot (which is commonly symbolized as 'kn', 'KN', 'kt', or 'KT') as a unit "outside the SI," and it devotes considerable attention to the matter of non-SI units. SI "is a system of units . . . which provides the internationally agreed reference in terms of which all other units are now defined," writes the BIPM. The International System of Units "is recommended for use throughout science, technology, engineering, and commerce. The SI base units, and the SI coherent derived units, including those with special names, have the important advantage of forming a coherent set. . . . Because the SI is the only system of units that is globally recognized, it also has a clear advantage for establishing a worldwide dialogue. Finally, it simplifies the teaching of science and technology to the next generation if everyone uses this system."

The BIPM goes on to state this: "Nonetheless it is recognized that some non-SI units still appear in the scientific, technical, and commercial literature, and will continue to be used for many years. Some non-SI units are of historical importance in the established literature. Other non-SI units, such as the units of time and angle, are so deeply embedded in the history and culture of the human race that they will continue to be used for the foreseeable future. . . . For these reasons it is helpful to list some of the more important non-SI units. . . . The inclusion of non-SI units in this text does not imply that [their use] is to be encouraged." The BIPM proceeds to discuss at length various "non-SI units not recommended for use" — two of which are the nautical mile as a measurement of length, and the knot as a measurement of speed. The BIPM concludes its discussion of non-SI units with these words: "There are many more non-SI units, which are too numerous to list here, which are either of historical interest, or are still used but only in specialized fields (for example, the barrel of oil) or in particular countries (the inch, foot, and yard)." But the BIPM "can see no case for continuing to use these units in modern scientific and technical work" [3].

3. MEASURING NAVIGATIONAL SPEED

Throughout much of the world, horizontal air speed in aviation is measured in knots, which we have seen the BIPM regards as a non-SI unit. The author has not been able to determine with certainty which countries measure horizontal air speed using kilometers per hour (km/h), which the BIPM accepts as a derived SI unit, again as we have seen. Apparently, however, the People's Republic of China uses km/h, and so do Russia as well as several other countries formerly part of the Soviet Union. The author is speculating, here, but probably the members of the Shanghai Cooperation Organization — China, Russia, Kazakhstan, Kyrgyzstan, Tajikistan, and Uzbekistan [11] — all use km/h to measure horizontal navigational speed. These constitute only a small percentage of the countries in the world, although they do make up a significant portion of the planet's population considering that China alone comprises more than 18 percent of it and Russia almost two percent.

4. THE INTERNATIONAL CIVIL AVIATION ORGANIZATION (ICAO) AND ITS SUPPORT OF METRIC UNITS

So, what position does the ICAO take regarding the use of knots versus kilometers per hour?

Originating in 1944, the International Civil Aviation Organization became a specialized agency of the United Nations in 1947. To cite its founding document, the ICAO was set up "in order that international aviation may be developed in a safe and orderly manner and that international air transport services may be established on the basis of equality of opportunity and operated soundly and economically." Notably, on its website next to its logo, the ICAO uses the tagline "uniting aviation" [5]. Like the BIPM, the ICAO decries the nonuse of metric units. To see this, its *Annex 5 to the Convention on International Civil Aviation: Units of Measure to Be Used in Air and Ground Operations, 2010* plays a crucial role. In this document, the ICAO writes the following: "The International System of Units . . . shall . . . be used as the standard system of units of measurement for all aspects of international civil aviation and ground operations." The ICAO then goes to make this statement: "The non-SI units [of the knot, the nautical mile, and the foot] have been retained temporarily for use as alternative units because of their widespread use and to avoid potential safety problems which could result from the lack of international coordination concerning the termination of their use." The ICAO makes clear its desire to discontinue utilizing these non-metric units, when it writes the following: "The non-SI units . . . shall be permitted for temporary use as alternative units of measurement . . . It is intended that the use of the non-SI alternative units . . . will eventually be discontinued. . . ." Tellingly, the ICAO defines these non-metric units in terms of their metric equivalents. Thus, a foot is defined to be exactly 0.3048 meters, a knot is defined to be one nautical mile per hour, and a nautical mile is defined to equal exactly 1852 meters. *Annex 5* provides an extensive list of exclusively SI units to be used in every aspect of aviation and aeronautical engineering. These measurements involve direction, space, and time; mass; force; mechanics; flow; thermodynamics; electricity and magnetism; light and electromagnetic radiations; acoustics; and nuclear physics and ionizing radiation [6].

5. WHY KNOTS ARE USED TO MEASURE AIRSPEED

Two factors led to the prevalence of using knots to measure airspeed. One of these is the highly important American role in aircraft manufacturing during the early history of aviation, as a number of aviation histories have detailed [2], [8]. The second factor is the well-known refusal of the United States to fully implement the metric system, or to use the informative term favored by the U.S. Metric Association — to metricate. Quoting from its website, the U.S. Metric Association "is a national non-profit organization that was founded in 1916." The U.S.

Metric Association "advocates completing the [U.S.] conversion to the International System of Units, known by the symbol SI (ess-eye) and also called the metric system. The process of changing measurement units to the metric system is called metric transition or metrication" [17].

Thus, the United States — a country so much in the forefront of aircraft production — was simultaneously the very same country lagging in its transition to the metric system. And so the greatest advances in aviation technology ever witnessed in human history were built upon a system of weights and measures deprecated by both the BIPM and the ICAO.

Actually, however, the knot was deployed for aerial navigation as a continuation of its use in nautical navigation, in which capacity it had been used for centuries. That is to say, a third factor contributed to the widespread use of the knot in aerial navigation. The knot relates directly to the mostly spherical shape of the Earth.

6. DEFINING A KNOT AND A NAUTICAL MILE

What is a knot, and why does it continue to hold sway? A knot is a unit of speed equal to one nautical mile per hour. And what is a nautical mile? A nautical mile equals one minute of arc on the surface of the Earth.

Imagine a point on the equator, and we want to measure the distance from that point to the North Pole. In angular measurement that distance equals 90 degrees, and since each degree contains 60 minutes, the distance from the equator to the North Pole amounts to 5400 minutes. The distance intercepted (or subtended) by each one of those arcminutes is called a nautical mile. Thus, in angular measurement the distance from the equator to the North Pole equal 5400 minutes, and in linear measurement that same distance is 5400 nautical miles. (Figure 1 demonstrates this relationship.) A nautical mile, which measures approximately 6076 feet, is the arc length of one arcminute, whereas a statute mile, which measures 5280 feet, is the chord length of one arcminute.

The natural relationship of a nautical mile to the size and shape of the Earth makes this unit of measurement appealing to navigators. For hundreds of years — well before the modern age — sailors have been able to measure latitude. This is important to grasp. Experienced sailors 500 years ago, and even longer than that, knew how to determine how far north or south they were from somewhere else, and thus they would have clearly understood the meaning of a nautical mile. (However, measuring longitude — measuring how far east or west you are — is not easily accomplished, and mariners mastered that skill only within the last 250 years or so [12].)

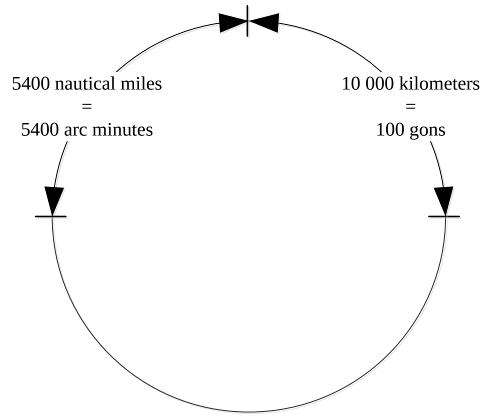

Figure 1. Measuring the Earth along a great circle.

Using non-metric units, 1 knot = 1 nautical mile per hour = 1 arc minute per hour. Using metric units, 1 kilometer per hour = 1 centigon per hour.

7. IDENTIFYING A GREAT CIRCLE

A nautical mile is one minute of arc along any great circle of the Earth. A great circle of the Earth is the circle that would be formed on a plane if that plane sliced through the center of the Earth cutting the planet into two halves. (A great circle can also be called an orthodrome.) The distance along the entire length of a great circle is the length of the circumference of the Earth [16].

Thus, any plane going through the Earth from the North Pole to the South Pole forms a great circle. For example, the prime meridian, which passes through Greenwich, England, lies on a great circle. But any of the infinitely many meridians also form great circles.

A great circle does not have to extend in a north-south direction. The equator forms a great circle. However, the equator is the only line of latitude that lies on a great circle. Any line of latitude to the north or the south of the equator does not form a great circle because it does not pass through the Earth's center. These are called small circles.

Finally, a great circle can also be formed obliquely. The only requirement is that the imaginary plane slicing through the Earth needs to go through the center of the planet. As long as that condition is met, then the plane can lie non-orthogonally to the equator.

8. GRADIANS AS AN ALTERNATIVE TO DEGREES

Since it measures speed on the basis of the nautical mile, the knot seems to be the natural choice to use for navigation. Indeed, the knot's relationship to planet Earth helps account for its long-standing use right down to the present day, even in countries where the metric system is firmly entrenched. A nautical mile equals one minute of arc length along a great circle, and counting knots tells the navigator how long it takes to travel that distance.

Yes, the knot would be the best unit of measurement if the degree were the only way to measure angles. But it is not. Enter the gradian.

To be clear, the author is not talking about the radian. In the modern metric system the radian is the SI unit for measuring plane angles, and the steradian is the SI unit for measuring solid angles [3]. The gradian measures angles using a method similar to using degrees, but based upon multiples of ten. Historically, the gradian came into existence in France at the same time that the French revolutionary government established the metric system in the 1790s.

Before proceeding, some necessary terminology. In German and in Germanic Scandinavian languages, the word *grad* means degree, and in certain Romance languages (Spanish and Italian) *grado* also means degree. Therefore, using the word gradian would lead to confusion. To avoid it, a gradian is now called a gon, taken from the Greek word *gonia*, which means angle. Information regarding the Greek origin of the word comes from the extraordinarily useful website *How Many? A Dictionary of Units of Measurement*. As a mathematical symbol, the word gon should be placed after the number with a space in between, or the superscript 'g' can be used [10].

As we know, a quarter distance around a circle equals 90°, a half distance equals 180°, a full turn is 360°, and so on. Using gons, a quarter distance around a circle equals 100 gon, a half distance equals 200 gon, a full turn is 400 gon, and so on. So each quadrant on the Cartesian plane measures 100 gon.

A few equations are provided to show how angular measurement works with gons:

$$25 \text{ gon or } 25^{g} = 22.5°$$
$$50 \text{ gon or } 50^{g} = 45°$$
$$75 \text{ gon or } 75^{g} = 67.5°$$
$$100 \text{ gon or } 100^{g} = 90°$$
$$150 \text{ gon or } 150^{g} = 135°$$
$$200 \text{ gon or } 200^{g} = 180°$$
$$400 \text{ gon or } 400^{g} = 360° \text{...............(1)}$$

9. GONS AND KILOMETERS

What do gons have to do with navigation? To answer that, understanding why the meter has the length that it does has paramount importance. That length is not arbitrary, but rather it is based in nature. The French government, upon setting up the metric system in the 1790s, commissioned a study of the length of the meridian passing from Dunkirk to Barcelona, and the government established that one ten-millionth of the arc extending from the North Pole to the Equator along this meridian would constitute a meter. Hence, a quarter circumference of the globe measures ten million meters, or 10 000 kilometers [1], [4], [7]; [9], [18], [19]. Again, see Figure 1.

Thus, 10 000 kilometers measures one quarter of a great circle, and 40 000 kilometers is the circumference of the Earth using metric units. This relates linear measurement (kilometers) to angular measurement (gons). Here are a few equations demonstrating that relationship:

$$10\,000 \text{ km} = 100 \text{ gon}$$
$$100 \text{ km} = 1 \text{ gon}$$
$$1 \text{ km} = \text{cgon, where cgon} = \text{centigon....(2)}$$

Here are analogous equations using nautical miles and degrees:

$$5400 \text{ nautical miles} = 90°$$
$$60 \text{ nautical miles} = 1°$$
$$6076 \text{ feet} \approx 1 \text{ arcminute.........................(3)}$$

The BIPM provides a lengthy footnote summarizing this information concerning the gon. The information in that footnote is worth repeating: "The gon (or grad, where grad is an alternative name for the gon) is an alternative unit of plane angle to the degree, defined as (π/200) rad. Thus there are 100 gon in a right angle. The potential value of the gon in navigation is that because the distance from the pole to the equator of the Earth is approximately 10 000 km, 1 km on the surface of the Earth subtends an angle of one centigon at the center of the Earth. However the gon is rarely used" [3].

10. THE SI WAY (KILOMETERS AND GONS) VERSUS THE NON-SI WAY (NAUTICAL MILES AND DEGREES)

The author has praised R. Rowlett's website on measurement units. However, the author disputes its statement under the entry for the grad: "Although many calculators will display angle measurements in grads as well as degrees or radians, it is difficult to find actual applications of the grad today" [10]. (Incidentally, modern sophisticated calculators, such as the Texas Instruments TI-89 Titanium and the TI-Nspire, do indeed have an option for showing grads [13], [14].)

No, gons do have a modern, practical application. *The gon establishes the connection between kilometers — the SI unit of linear measurement — and angular measurement*. Using gons, navigators can free themselves from having to use non-SI units to draw the connection in the natural world between linear and angular measurement. Using gons, navigators do not have to remind themselves that a nautical mile equals one minute of arc length along a great circle of the Earth. Instead, they can make the same connection by reminding themselves that a kilometer equals one centigon of arc length along a great circle. So, by knowing how many kilometers per hour they are traveling, they simultaneously know how long it takes them to travel one centigon of arc length on the surface of the Earth. This is as natural a way to think about navigational distance as using the knot. And it is better, because it conforms to the International System of Units advocated by both the BIPM and the ICAO.

11. CONCLUSIONS

The author is not seriously proposing that angular measurement during navigation be carried out using gons. Nor is the author suggesting that the world's airport runways be relabeled to reflect their geographical orientation using gons instead of degrees.

Now, if mathematical study were to start fresh, then gons probably would be a more convenient method to measure angles. Admittedly, gons are not perfect. For example, since a day is divided into 24 hours, then each hour corresponds to a 15-degree rotation of the Earth on its axis — a convenient integer.

Overall, however, gons would be more convenient for an aviator while navigating an aircraft. Consider the pilot who intends to land on a runway oriented at 147°, just to make up a hypothetical but entirely realistic scenario. Let's say that the pilot is flying left traffic downwind parallel to the runway, and intends to make several 45° turns until the final approach. First, therefore, the pilot is flying 327° and then subtracts 45° to obtain 282° and flies in that direction. Then, the pilot subtracts another 45° to obtain 237°, which is the base leg. Next, the pilot subtracts another 45° to obtain 192°, and then does so one last time to obtain the final approach heading of 147°. This can be summed up as follows:

$$327° \rightarrow 282° \rightarrow 237° \rightarrow 192° \rightarrow 147° \ldots (4)$$

Using gons, these would be the equivalent values:

$$363^{g} \rightarrow 313^{g} \rightarrow 263^{g} \rightarrow 213^{g} \rightarrow 163^{g} \ldots .(5)$$

These values are easier to calculate, the author contends. (A diagram of this situation is shown in Figure 2.)

Still in all, one way or the other, degrees are just too firmly embedded in popular as well as scientific thought and practice for them to be replaced by gons. Recall the BIPM's statement cited earlier that certain non-SI units, such as the units of angle, "are so deeply embedded in the history and culture of the human race that they will

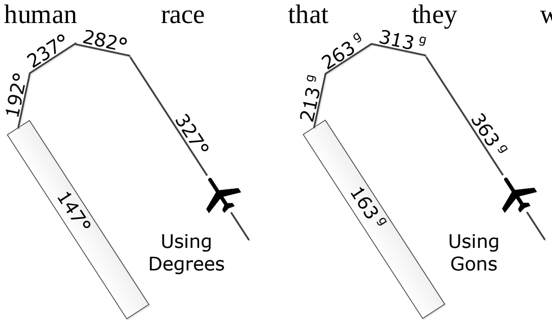

Figure 2: Approaching the runway, and calculating the approach using degrees and using gons. Using gons leads to easier calculations.

continue to be used for the foreseeable future" [3]. But knowing that a circle can be considered to contain 400 gons instead of 360 degrees provides a theoretical justification for measuring navigational speed using kilometers per hour instead of using knots.

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